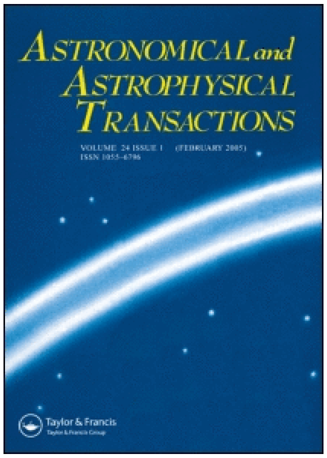

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THE FAST GALACTIC DYNAMO

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THE FAST GALACTIC DYNAMO

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The α-effect and the coefficient of eddy diffusivity are calculated for the magnetic field in a random flow with recovery. Such a flow loses its memory abruptly at random times that form a Poisson flow of events. Interstellar turbulence sustained by supernova outbursts is one of the physical realizations of such a flow. The growth rates and configurations of large-scale galactic magnetic fields for this situation are close to those predicted by simple galactic dynamo models. At the same time, the model of a flow with recovery makes it possible to trace the role of the effective "forgetting" of correlations. The presence of this forgetting distinguishes interstellar turbulence from other types of random flows.

Keywords: Fast dynamo; α-effect; Interstellar turbulence

1 INTRODUCTION

In recent years, views of mechanisms for the formation of the large-scale magnetic fields of spiral galaxies have converged appreciably (see, *e.g.*, [1–5]). At the same time, some fundamental issues in the theory of galactic dynamos raised in the early 1980s (see, *e.g.*, [6]) have not been satisfactorily resolved. The problem of a fast dynamo in a steady flow is one of such issues. Its essence is as follows [6, 7]. The equations for galactic (as well as stellar and, to some extent, planetary) dynamos underlying galactic-dynamo models can be obtained by averaging the microscopic induction equation over the turbulent pulsations of the velocity field. This procedure naturally involves some assumptions about the properties of interstellar turbulence, which are not entirely realistic. The traditional experience of mathematical physics recommends substantiation of these constructions through the analysis of some problem, in which the velocity field is sufficiently simplified such that it admits an exact solution that reproduces properties of the solutions to the averaged equations. Early in the 1980s, it was recognized that exact dynamo solutions for the induction equation behave differently from solutions of the mean-field equations for the case of galactic dynamos. It is not possible to achieve an efficiency of dynamo generation in these exactly solvable problems as high as that for mean-field dynamo. In the simplest case, the growth rate of the magnetic field is very low, hopelessly insufficient for the generation of the magnetic fields of galaxies within the lifetime of the Universe. This problem thus came to be known as the fast dynamo

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problem. There is currently little doubt that the fast growth of galactic magnetic fields is associated with the effective loss of memory due to interstellar turbulence. However, this belief has thus far remained conventional, since it has not been possible to fill the gap between turbulent-dynamo models that predict a fast growth of the magnetic field and models for dynamos in steady flows, where the growth of the magnetic field involves various complications. We have been able to fill this gap via a rather native extension of the class of turbulent-field models that can yield mean-field dynamo equations.

2 THE VELOCITY FIELD AND GOVERNING EQUATIONS

The simplest models for a turbulent medium that admits derivation of the mean-field equations are the short-correlation model, in which the memory time is considered to be infinitely short [8], and the renewal model, in which the random flow loses its memory at some predetermined, equally spaced renewal times [9, 10]. From the standpoint of the general theory of turbulence, these models are highly specialized; nevertheless, they are widely used in the transport theory of turbulent media (see, *e.g.*, [11]). Both models are quite natural in the context of interstellar turbulence sustained by supernova outbursts; it can be assumed that a supernova explosion erases the memories of the velocity-field evolution virtually instantaneously. Both models suggest the possibility of the fast generation of the galactic magnetic field for appropriate values of the parameters describing the turbulence. However, these models have not yet been applied to studies of the relationship between turbulent and laminar generation mechanisms. Note that the existence of prescribed renewal moments breaks down the uniformity of time. This uniformity can be recovered if the times of memory loss are assumed to be random Poissonian, rather than prescribed, events. It is important that such modification describes interstellar turbulence better than previous models, because supernovae explosions follow Poisson law.

Such a model with recovery for the transport of a passive scalar (dust) in the interplanetary medium was considered in [12]. The calculations in [12] to make the transition from a description of the meanfield transport in renewal models to the analogous description in recovery models, can be done quite similarly for the transport of a vector (magnetic field). We omit these routine calculations and write the resulting transport equation for the mean (large-scale) magnetic field $\mathbf{B}(t, \mathbf{x}) = \langle \mathbf{H}(t, \mathbf{x}) \rangle$

$$B_n(t, \mathbf{x}) = \int\int_0^\infty P_{nm}(\sigma, \mathbf{x}, \mathbf{y}) B_m(t - \sigma, \mathbf{y}) \lambda e^{-\lambda\sigma}\, \mathrm{d}\sigma\, \mathrm{d}^3\mathbf{y}, \tag{1}$$

where $\langle \ldots \rangle$ denotes averaging over the ensemble of velocity-field realizations, λ is the parameter of the Poisson process ($\tau = \lambda^{-1}$ being the mean time between recovery times), and P_{nm} is the kernel of the integral transport equation for the mean field in the renewal model. Let us compare (1) with the analogous equations obtained for other exactly solvable models for the velocity field. In a short-correlation approximation, the equation for the mean concentration turns out to be differential [8], while for velocity field with renewal, this equation is integral in space [9]. Equation (1) is integral in both space and time. However, analysis shows that properties of integral equations of type (1) are very similar to those of the traditional equations for the mean magnetic field (see [9, 12] and the next section).

An explicit (although very complex) functional-integral expression for the kernel P_{nm} in terms of the velocity field is given in [9]. The kernel can be explicitly calculated if we assume that the mean time between the recoveries is short. For definiteness, we present here the result for a statistically uniform, isotropic flow without reflection symmetry. Calculations lead to the famous equation obtained by Steenbeck, Krause and Rädler

$$\frac{\partial \mathbf{B}}{\partial t} = \mathrm{rot}(\alpha \mathbf{B}) + (v_m + \beta)\Delta \mathbf{B}, \tag{2}$$

where α is the helicity, β is the eddy diffusivity.

In general case the integral Eq. (1) cannot be reduced to the differential equation.

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Let us assume the velocity field to be statistically uniform, so that $P_{nm}(\sigma, x, y) = P_{nm}(\sigma, x - y)$, and the Fourier transform of Eq. (1) is algebraic in the spatial variables:

$$B_n(t, \mathbf{k}) = \int_0^\infty P_{nm}(\sigma, \mathbf{k}) B_m(t - \sigma, \mathbf{k}) \lambda e^{-\lambda\sigma} \, \mathrm{d}\sigma. \tag{3}$$

We reformulate the fast dynamo problem in terms of P_{nm} $(\sigma, \mathbf{k})$. From the standpoint of traditional mathematical physics, it would be desirable to construct examples of the velocity field in which, for a given $\mathbf{k}$, the operator $P_{nm}(\sigma, \mathbf{k})$ has a positive eigenvalue γ_0 that does not decrease for $v_m \to 0$ and $\sigma \to \infty$. There is no doubt that the solution to Eq. (1) will grow in this case at rate γ_0, even if $\lambda = 0$. However, in practice, this behavior of operator P_{nm} can easily be achieved only in the bounded interval $0 < \sigma < t^*$. The growth exponent γ_0 for larger σ normally decreases with decreasing v_m, or even with increasing σ.

To assess the role of λ in the given situation, we can assume that the magnetic field always has the form of an eigenvector of the operator P_{nm}, although its eigenvalue is positive and equal to γ_0 only when $\sigma < t^*$, later it vanishes. In this case, Eq. (1) reduces to

$$B(t) = \lambda \int_0^{t^*} e^{(\gamma_0 - \lambda)\sigma} B(t - \sigma) \, \mathrm{d}\sigma, \tag{4}$$

where $B(t)$ is taken to mean the projection of the mean magnetic field onto the fundamental eigenvector of operator P_{nm}. Substituting $B(t)$ by $e^{\gamma t}$ in Eq. (4) one can obtain the dispersion relation determining the growth rate γ of the magnetic field:

$$e^{t^*(\gamma_0 - \gamma - \lambda)} = \frac{\lambda}{\gamma_0 - \gamma} \tag{5}$$

If $\lambda \gg 1/t^*$, the root of Eq. (5) which we are interested in has the form

$$\gamma = \gamma_0 - \lambda e^{-\lambda t^*}. \tag{6}$$

If, however, $\lambda \ll 1/t^*$, the desired solution has the form:

$$\gamma = \gamma_0 - \frac{1}{t^*} \ln \frac{1}{\lambda t^*}. \tag{7}$$

Both asymptotics intersect not far from the point $\lambda t^* = 0.7058$.

We can see from Eq. (7) that, if λ is large (*i.e.*, the memory is lost frequently), the magnetic field grows at a rate that nearly coincides with γ_0. As λ decreases, the growth rate of the magnetic field decreases and, at some $\lambda = \lambda_{\rm cr}$, vanishes. We can easily verify that $\lambda_{\rm cr}$ can be obtained as non-trivial root of the following equation:

$$e^{t^*(\gamma_0 - \lambda_{\rm cr})} = \frac{\lambda_{\rm cr}}{\gamma_0} \tag{8}$$

4 DISCUSSION

Thus, in the framework of exactly solvable transport models for random media, we have traced for the first time the transition from a dynamo in a medium with a finite memory time (in our case, the recovery time) to a dynamo in a steady flow. We have verified that the properties of this transition are quite natural (cf. [13]).

Note that memory loss due to interstellar turbulence associated with supernova explosions occurs at different times in different regions of the Galaxy. Such flows can be considered as the flows with local recovery. In this case, spatial regions in which recovery has not taken place for a very long time due to random circumstances are of special interest. If the lifetime of such a region of stagnation is $\hat{t} \gg t^*$, mechanisms for the generation of magnetic field could be substantially weaker there, and even decay of magnetic field is possible. If such a correlation should actually be revealed by observations, this would imply that abrupt loss of memory is indeed important for the operation of fast dynamos, and the theory of stellar cycles requires some revision.

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