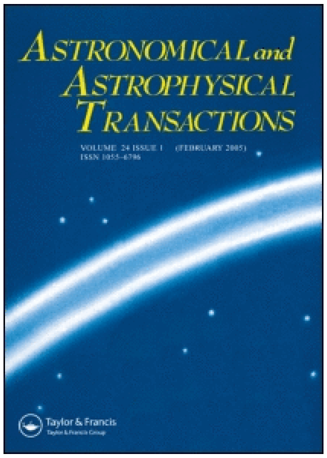

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THE SUN AS A VARIABLE STAR

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The fine structure of the solar activity index *W* (Wolf numbers) is investigated. It is suggested that the quasi-biennial oscillations are the main features of the 11-year cycles. The attempt to simulate them by analytical functions is presented. The spectral properties of the whole *W* values time series are discussed. A conception of the Sun as a magnetic variable star is suggested. A prognosis of the solar cycles 23–24 are given.

Keywords: Solar activity; 11-year cycle; Quasi-biennial oscillations; Solar cycles forecasting

1 INTRODUCTION

The Sun is a hot analogy of the BY Dra variables. The solar luminosity varies according to Frölich and Lean (1998), with the amplitude about 1.5×10^{-3} ($\approx 0.004^{\mathrm{m}}$). Foucal and Lean (1988) have noticed that this variation is correlated with the solar magnetic activity. This suggests that the cause of the solar luminosity change is the transformation of its radiative energy into the magnetic one and its subsequent return to the heat. This permits to classify the Sun as a magnetic variable star. The time scale of this variation corresponds to the well-known 11-year cycle of the solar activity (SA). But, this cycle is not a strict periodicity due to variation of the individual cycle durations. Nevertheless, the mean value of the cycle length is surprisingly constant over a rather large time interval, at least several centuries, and may be even millenniums. Thus, the SA cycle length is determined by the amount of the solar energy, stored in the form of the magnetic one, during a cycle and the physical processes of its return back as well. It is very important to reveal the physical nature of these processes and their connection with the physical structure of the Sun.

The same problem, naturally, concerns the other stars, but in this case, the unique possibility is to analyse a fine structure of the time dependence of the stellar light curve variations. Possibly, not only, in the case of the Sun, but also at least for the sun-like stars the short time variations are physically significant and provide a valuable tool for the general problem of the stellar activity solving.

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2 THE FINE STRUCTURE OF THE SOLAR ACTIVITY CYCLES

2.1 Data Processing

The solar 11-year cycles are very similar in general form and differ primarily in their amplitudes and duration. But, their fine structure is rather complicate. It includes a lot of short time variations. Nevertheless, these variations have several important features. The main one is the existence of two noticeable maxima at the beginning of each cycle. To the end of the 11-year cycle the width of the individual maxima diminishes.

Fourier analysis, performed by means of the Maximum Entropy Method (MME), reveals a lot of unstable frequencies corresponding to the range of periods from 3 to 0.7 years as it is shown in Figure 1 (Kononovich, 1999). Noisy character of these variations is very similar to that observed in the case of variable stars. So, usually one assumes them to be a noise and ignores them by smoothing.

Presumably the outlined SA cycle properties are a consequence of the main cycle peculiarity, namely, the quasi-biennial oscillations. Schuster (1906) was the first to reveal these variations in the Wolf number solar activity index *W*. He noticed a variation with 2.7 year period. Later, there were found a lot of other periods. Shapiro and Ward (1962) obtained period about 2.1 year and Sleptzov-Shevlevich (1969) obtained the 2.3 year period. Later Gnevyshev (1966, 1977) using the solar corona intensities showed a bimodal nature of the 11-year cycles 17–19 with the 2–3 year intervals between the maxima. Apostolov (1985) filtered the *W* values for 20 cycles and found up to 5 oscillations for each cycle with main period of 25.6 months. Mordvinov (1988) supported this result using the moving average of the spectral-temporal analysis.

Many geophysical processes also exhibit variations within the same time intervals, which correlate with the solar activity (*e.g.* Labitzke, 1987; Ivanov-Kholodny *et al.*, 2000). All variations, treated in these papers, are known as quasi-biennial oscillations (QBO) discovered nearly everywhere in the terrestrial atmosphere.

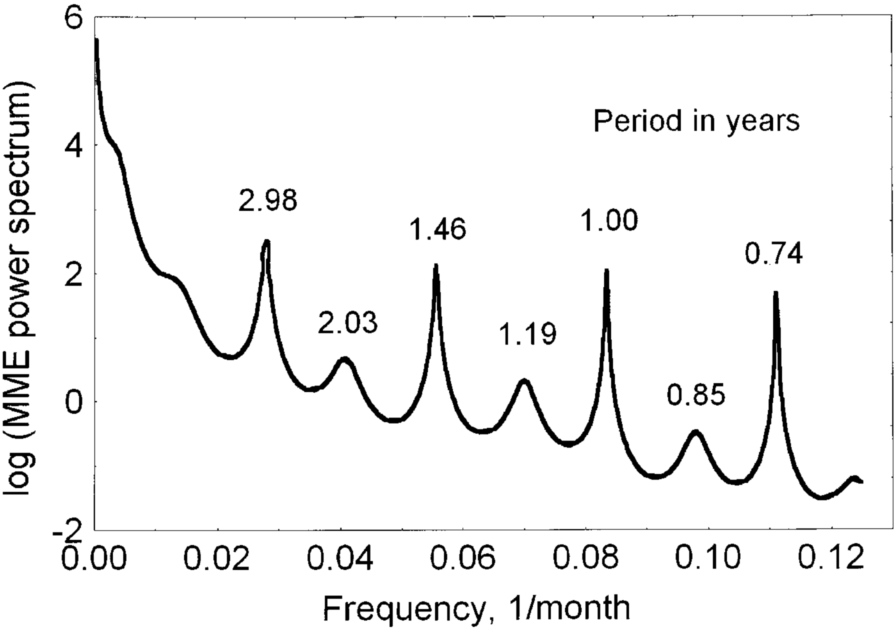

FIGURE 1 MME power spectrum of the Wolf number time series in the quasi-biennial frequency interval.

The procedure of the proper QBO extraction from a time series is very important. As it was mentioned above the QBO present the basis of the 11-year solar cycle structure. To separate this structure a special filter was used (Ivanov-Kholodny *et al.*, 2000). However, it includes too much of the noise component. It is better to decompose the original monthly mean of Wolf numbers W into several components using the Singular Spectrum Analysis (SSA) (Golyandina *et al.*, 2001).

The SSA method was especially designed for the nonstationary processes analysis and is useful to:

- decompose the time series according the basis generated by this time series itself,
- extract the time series components of both known and apriori unknown quasi-period,
- analyse and forecast the components,
- compose the time series on the basis of components chosen for the problem under consideration.

The principal parameter used in the SSA method is the lag M. The choice of this lag is determined by the problem. For the analysis of QBO in the monthly mean Wolf number time series this lag was settled to the average number of months in the 11-year solar cycle, *i.e.* $M = 132$. The same number of components was obtained and a few most interesting were selected.

The most interesting components obtained by SSA method are presented in Figure 2. The upper panel shows the W time series. The second one from above represents the centural part of the W curve (W_C), the third is the QBO component (W_{QBO}), and the last panel is the 11-year component (W_{11}).

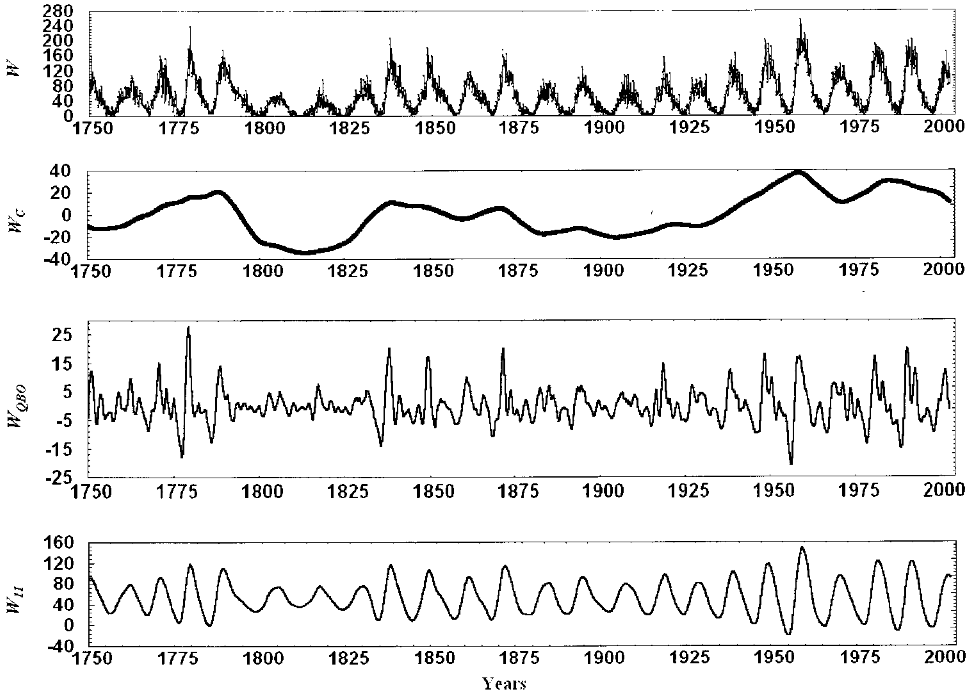

FIGURE 2 The upper panel shows the monthly mean Wolf numbers W; below are: the centural (W_C), the quasi-biennial (W_{QBO}), and the 11-year (W_{11}) principal components revealed in the Wolf number time series by means of SSA method.

The 11-year component (W_{11}) explains ~64% of Wolf number time series variability, the centural component (W_C) explains ~18%, and the quasi-biennial component (W_{QBO}) explains ~11%, *i.e.* these three components explain 93% of Wolf number time series variability, and all the others explain just ~7%. So all the components except the 11-year, the centural, and the quasi-biennial are just the noises and high frequency variations of the solar activity. The application of the SSA method to the Wolf number time series and the results were described in details by Khramova *et al.* (2001a).

It is easy to notice from the lower two panels in Figure 2 that not only the 11-year pattern of the W curves for all cycles are very alike, but also very similar are the W_{QBO} details in all cycles. Their main feature is the existence of the two prominent maxima with nearly equal amplitude and diminishing of their width. The time interval between them is about 2 years just corresponding to the bimodal character of the solar cycle mentioned above. But, the width of the second maximum is lower. These two maxima form the basis of the QBO. A more attentive comparison shows a lot of similar but perturbed details. However, it is more difficult to recognise them because of strong noise distortions. Nevertheless, it is possible to conclude that in general the W_{QBO} variations for each solar cycle present a fading wave train. This is illustrated in the Figure 3 for cycles 20–22. The same result was obtained for other cycles using the above-mentioned procedure.

2.2 Simulation Modeling

The results shown in Figure 3 for cycles 20–22 illustrate the important property of the wave trains during the cycle 20 end and the cycle 22 beginning. Namely, the last waves amplitudes

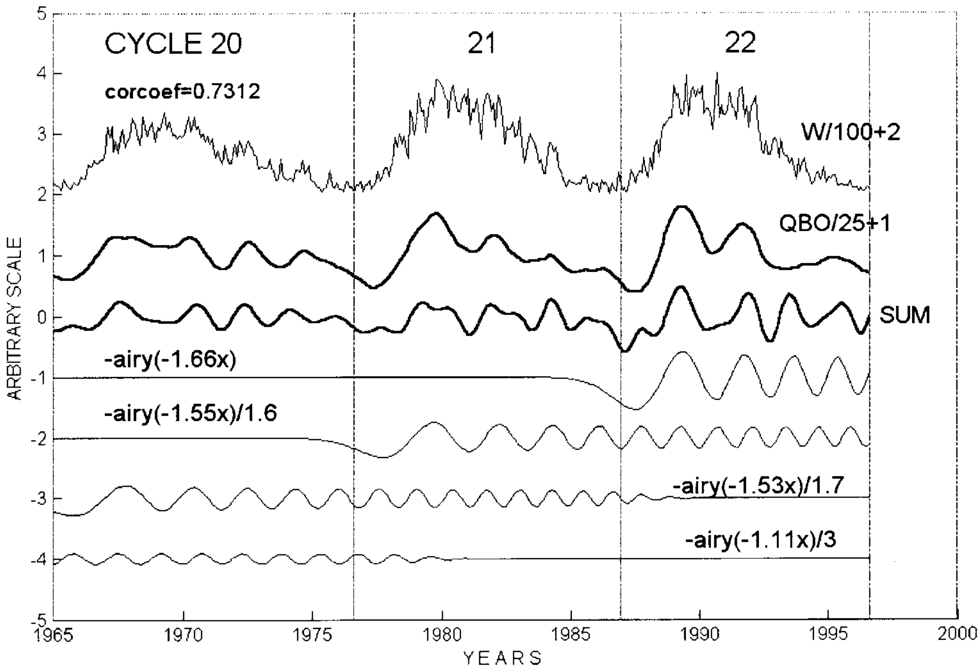

FIGURE 3 Presentation of Wolf numbers for cycles 20–22 by Airy functions. The upper curve is the Wolf numbers (W), the next one corresponds to QBO from Figure 2, below is the sum of the four Airy functions presented by four lower curves. Their parameters and amplitudes are adjusted to obtain indicated maximum correlation value 0.7312.

of the cycle 21 reveal fast decrease while the amplitudes of the first and second waves of the cycle 22 sharply increase. In all cases, the duration of the individual waves of wave trains diminishes with the cycle phase from about 3.0 to 1.5 years.

These remarkable properties can be presented by Airy functions well known in hydrodynamics, but for the first time applied for the QBO interpretation. These functions present wave trains with diminishing amplitudes and periods of consequent peaks (Abramowitz and Stegun, 1964). Each of the 4 curves in the lower part of Figure 3 (their ordinates are shifted down by integer values) is the graph of the analytical function $-Ai(-x)$, where the Airy function $Ai(x)$ is the solution of the differential equation

$$y'' - xy = 0, \tag{1}$$

and is given by the integral

$$Ai(x) = \frac{1}{\pi}\int_0^{\infty} \cos\left[\frac{v^3}{3} + vx\right] \mathrm{d}v. \tag{2}$$

The broad curve minimum corresponds to the 11-year cycle minimum, *i.e.* the beginning of the current cycle. It is evident that the left side of this slope suppresses the end of the previous cycle. At the right side all functions are truncated on about the 12th maximum. This corresponds to the end of the next cycle. As a result each wave train presented by a function $-Ai(-x)$ covers a couple of 11-year cycles. They suit the rule proposed by Ohl (1976), who noticed that the beginning of the next solar cycle is related to the end of the previous one.

All these relations suggest presenting the QBO by the superposition of the individual $-Ai(-x)$ functions. The result of such superposition is given by the non-shifted curve (marked as SUM in Fig. 3). It is evident that the resemblance of the curves SUM and W_{QBO} is not accidental. Only slightly adjusting the length and zero points of the individual Airy functions, it is possible to obtain very good similarity of small details though not always with the same amplitudes. The parameters and amplitudes of the Airy functions are adjusted to obtain maximum correlation between the both curves. The result illustrated in Figure 3 corresponds to the correlation coefficient value 0.73. This correlation shows that the QBO may be presented by quite similar functions.

There is one additional point of evidence. The interference of the wave trains explains why the first two maxima of a given cycle dominate the others. This is because they quite-well coincide with the 6th and 8th waves of the previous cycle. Meanwhile, the 7th one coincides with the minimum between those two maxima. This diminishes the contrast of the bimodal pattern of the 11-year cycle.

2.3 Forecast of the Solar Cycles

Since the solar activity nature is not clear, as yet, it is rather difficult to forecast it. So, to prognose the SA parameters, it is necessary to analyse their structure. Nowadays, there are a lot of methods of the SA forecasting. However, nearly all of them are essentially statistical and mathematical. As far as the fine structure of the 11-year solar cycles is concerned, even rather simple such a mathematical approach described in Section 2.2 permits to obtain good accuracy. For example, Khramova *et al.* (2001b) proposed and developed an original method of the 11-year cycles prognosis. This method is based on subdivision of each cycle on certain number of phases and their comparison in order to get the interconnections between them both within each particular cycle and different ones.

This method of Equal-Phase Average (EPA) was applied to the time series of sunspot Wolf numbers obtained in 1755–1996 (solar activity cycles 1–22). Each cycle was divided into $k = 1, 2, 3, \ldots, 16$ equal parts. The value of Wolf numbers in each part was averaged. Thus, the matrix W^i_{nk} of EPA values was obtained. In this matrix $i = 1, \ldots, k$ is the phase number among $k = 1, 2, 3, \ldots, 16$ phases in the solar cycle number $n = 1, \ldots, 22$. The correlation analysis has revealed that more than 70% of correlation matrix values exceed 0.85. For the preliminary analysis only the W^i_{n16} elements were used. So, each cycle was divided into 16 equal parts and the analysis was carried out on the basis of 16 time series (each time series is 22 elements long). It became clear that it is useful to divide each cycle into another number of equal parts only, in order to revise and correct the obtained regularities and to verify the self-consistence of the forecast.

A few equations which reveal some features of the 11-year solar cycle were found. Let us introduce a few variables which describe the solar cycle: W^C_M is for maximal EPA value (measured in Wolf numbers), T_R is for duration of cycle growth (years), T_D is for duration of cycle recession (years), $A = T_D/T_R$ is for cycle asymmetry (pure number), $T = T_R + T_D$ is for cycle duration (years).

It is a well known fact (Vitinsky *et al.*, 1986) that the higher the value is of Wolf number in the solar cycle maximum, the shorter the duration is of solar cycle growth:

$$\lg W^C_M = 2.603 - 0.132 T_R, \quad (r = -0.88), \tag{3}$$

where r (in brackets) is the coefficient of linear Pearson correlation.

The other way to express the same property:

$$W^C_M = 249.213 - 30.904 T_R, \quad (r = -0.84). \tag{4}$$

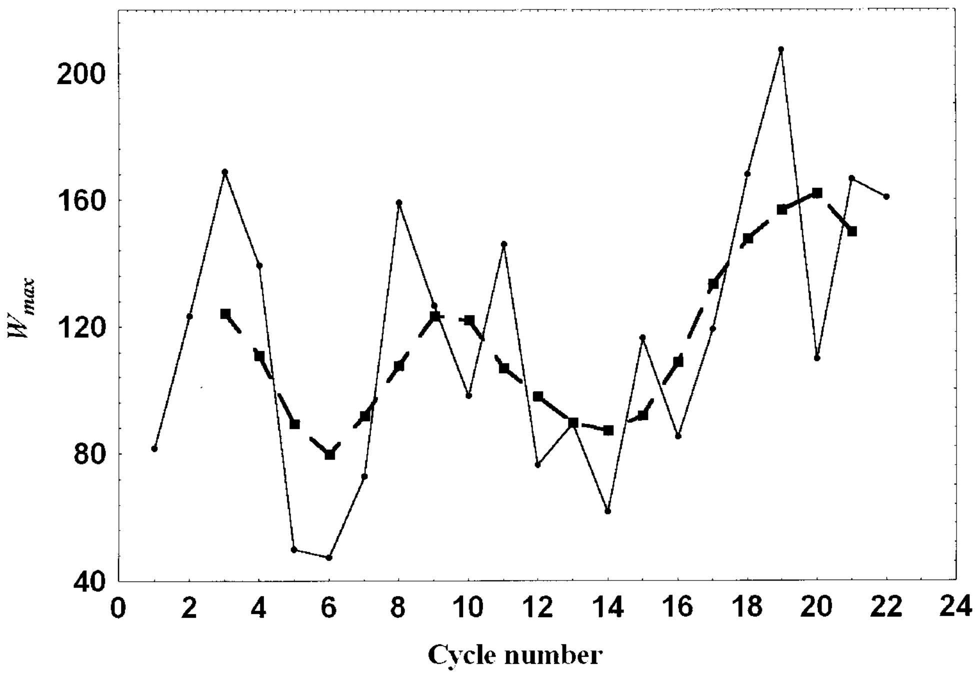

FIGURE 4 Dependence of the cycle maximal EPA values upon their number. The thin line is observed values. The full dashed line is the smoothed value over 4 cycles.

The relation between the Wolf number in the solar cycle maximum and the duration of cycle recession is not close, but still exists:

$$T_D = 5.327 + 0.011 W_M^C, \quad (r = 0.39). \tag{5}$$

Also there exists a very close relation between the maximum Wolf number of a solar cycle, and its asymmetry:

$$W_M^C = 22.277 + 57.009A, \quad (r = 0.76). \tag{6}$$

Considering different cycles, the coefficients on the right side of relations (3)–(6) turned out to vary.

The following characteristics of the 11-year solar cycle were found to vary in different manners along the century solar cycle: asymmetry A, maximal EPA value W_M^C, duration of cycle growth T_R, and the duration of cycle recession T_D. This fact is illustrated in Figures 4–7. Dashed lines in these figures represent the moving average. One can find (Fig. 6) that the duration of the cycle growth has decreased during two last centuries from 4.5 to 3.6 years. The duration of the cycle recession (Fig. 7) was found to vary very slightly between 6 years (for high cycles) and 7 years (for low cycles), and that is why the correlation found for Eq. (5) is low. Also that fact explains why the recession part of the solar cycle is usually forecasted more correctly than the growth part.

It is interesting to note that there were no over-century trend neither in the solar cycle asymmetry A (Fig. 5) nor in the recession duration T_D. But a trend was found (and even non-linear!) in maximal Wolf numbers (Fig. 4) for a giving cycle. So in the analysis of EPA values

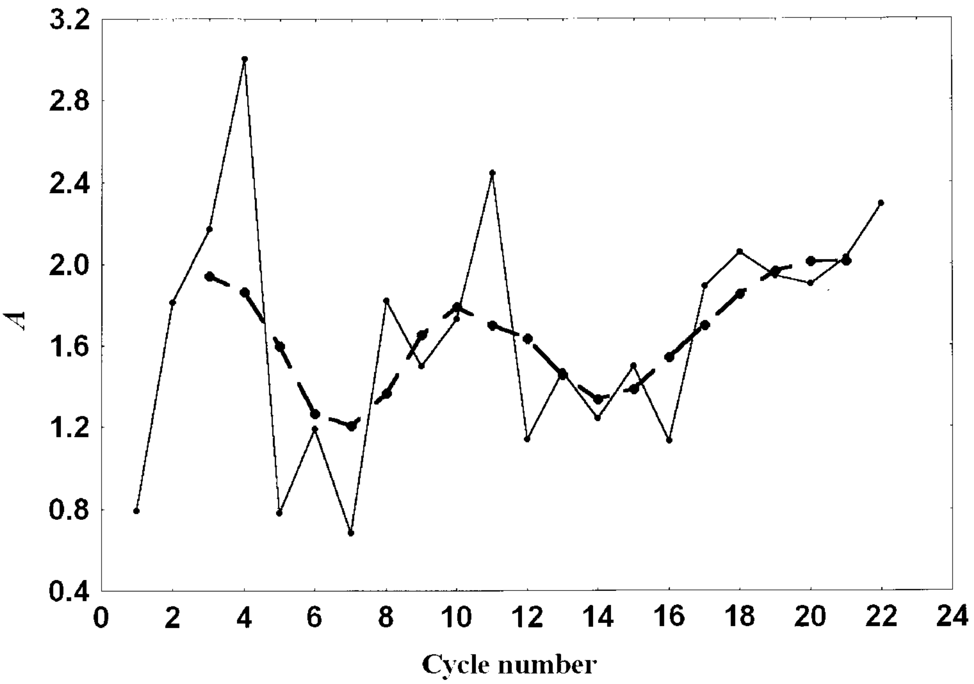

FIGURE 5 Dependence of the asymmetry of cycle upon their number. The thin line is observed values. The full dashed line is the smoothed value over 4 cycles.

it becomes necessary to find the existing long-term trend and remove it. By means of SSA algorithm, this trend was found in the following form:

$$W^i_{16\mathrm{tr}}(n) = A^i_{16} + B^i_{16} \cdot \ln\left\{(n - C^i_{16}) + \sqrt{1 + (n - C^i_{16})^2}\right\}, \quad (7)$$

where $A^i_{16}, B^i_{16}, C^i_{16}$ are the certain coefficients. We suppose them to have the following meanings: C^i_{16} determines the number of the cycle after which the W^i_{16} grow up monotonous, B^i_{16} is approximately the mean value of W^i_{16} during solar cycles 1–22, A^i_{16} as the analogue of the standard deviation.

Then the trend, described by (7), was diminished from the time series of EPA values of Wolf numbers, and for the first residuals, the century cycle was found in the following form:

$$W^i_{16\mathrm{cent}}(n) = D^i_{16} \cos\left(\frac{2\pi\, n}{T^i_{16}}\right) + E^i_{16} \sin\left(\frac{2\pi\, n}{T^i_{16}}\right), \quad (8)$$

where ${D^i_{16}}^2 + {E^i_{16}}^2 = {M^i_{16}}^2$, M^i_{16} is the amplitude of the century cycle variation, $\arctan(E^i_{16}/D^i_{16}) = \varphi^i_{16}$, φ^i_{16} is its phase, and T^i_{16} is its period. Next, both long-term trend and the century cycle were diminished from the time series of EPA values of Wolf numbers. The resulting time series of second residuals were tested for the normal distribution to reveal whether all regular components were noticed and extracted. It is necessary to note that it is impossible to reveal the 22-year component because of its frequency closeness to the corresponding Nyquist frequency.

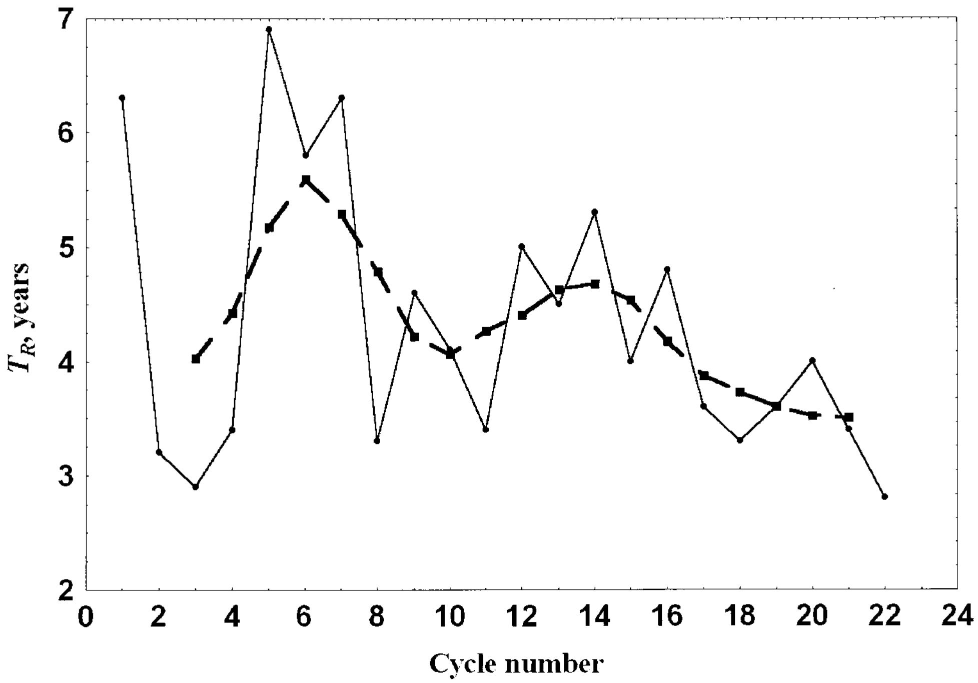

FIGURE 6 Dependence of the rising phase of the SA cycles upon their number. The thin line is observed values. The full dashed line is the smoothed value over 4 cycles.

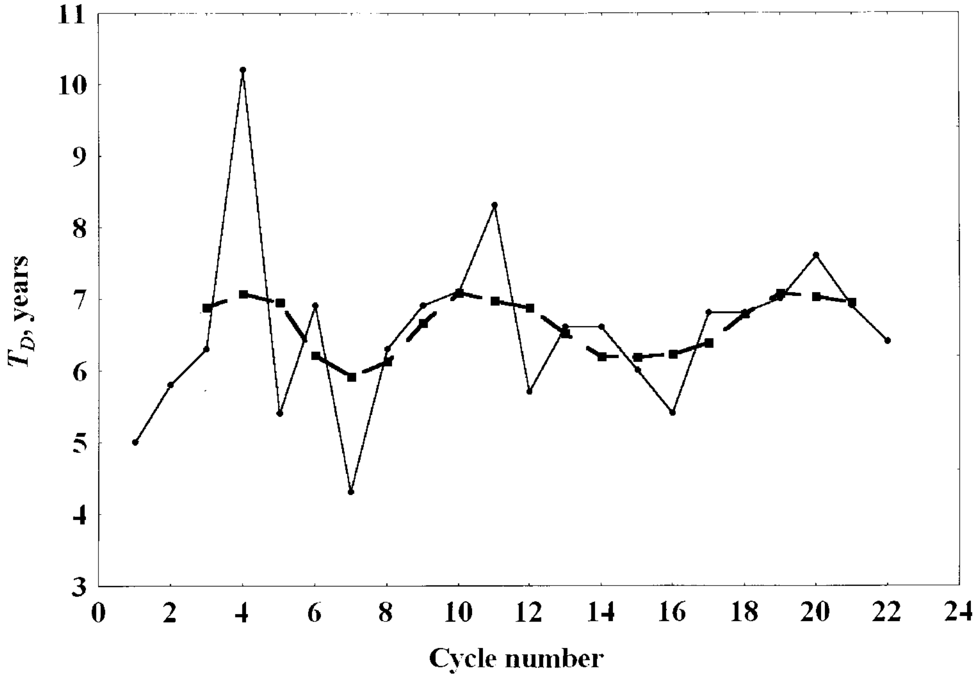

FIGURE 7 Dependence of the falling phase of the SA cycles upon their number. The thin line is observed values. The full dashed line is the smoothed value over 4 cycles.

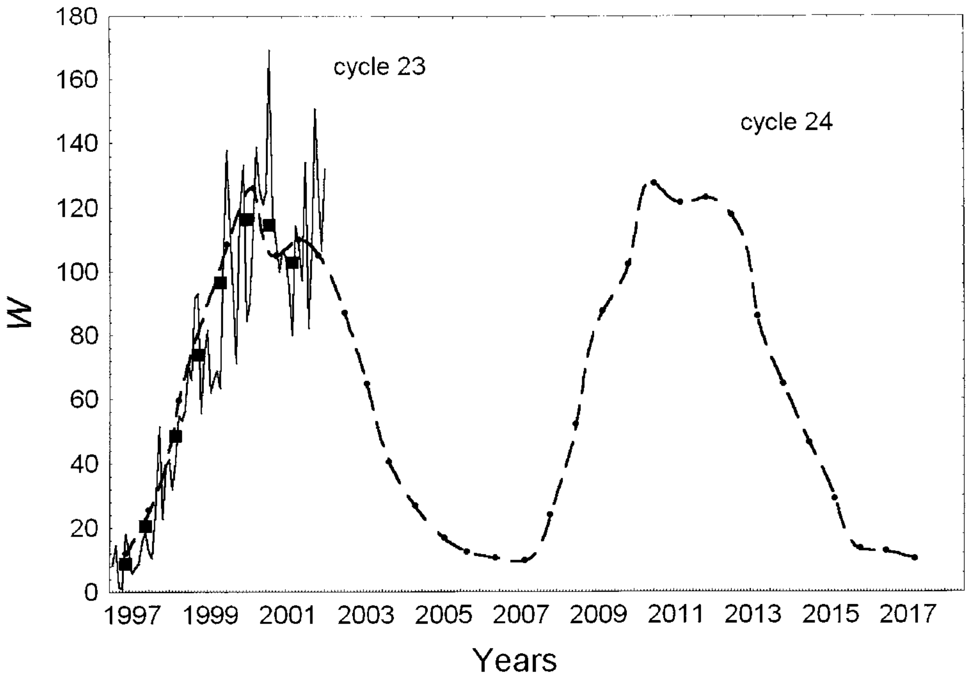

FIGURE 8 The forecast of the Wolf numbers for solar cycles 23 and 24. The dashed line aggregates the points of the forecasted Wolf numbers averaged over 8 months. The black squares are the observed Wolf numbers averaged over the same period. The thin line joints the monthly mean Wolf numbers observed in July, 1996–December, 2001.

The duration of the century solar activity cycle was estimates to be 100 ± 5 years, and the amplitude of the century cycle was estimated to be about 35 units of Wolf numbers.

The forecast of the Wolf numbers for the cycles 23 and 24 was obtained by the one-step extrapolation of the $W^i_{16}(n)$ time series using the procedure presented by Eqs. (7) and (8). To find the duration of the cycle growth and recession, the features found in (4) and (6) were used.

Figure 8 presents the prognosis of the W values based on the application of the above mentioned method. The observed W values (both monthly mean and averaged over each phase) are also shown in this figure. The prognosis can be obtained by the extrapolation of the time series of W value averaged over the particular phase. The advantage of this method is that the prognosis of the whole cycle is obtained by the one-step extrapolation of several time series. The double-peak structure of the forecasted cycles is well seen in Figure 8. This double-peak structure is the important property of each 11-year cycle. The QBO properties treated in Section 2.2 easily explain this structure and are useful to refine the prognosis method.

3 STELLAR VARIABILITY

The main result is that the similar wave trains of the quasi-biennial variations are the main property of the fine structure of the 11-year SA cycle. In each case, these variations are a superposition of a triad of consecutive, nearly equal fading wave trains appearing regularly with 10–11 year intervals. The main wave train starts just before the end of the previous cycle. Its two front maxima are amplified by the interference with the end of the previous wave train. This leads to the well-known bimodal structure of the 11-year cycles. The deep and broad minimum of the following wave train suppresses the current cycle determining the exact moment of its minimum and of the following cycle beginning. The wave duration is about 38 months at epoch of SA maximum linearly diminishing to 21 months toward the cycle end. The separate wave trains are well presented by the analytical Airy functions. The origin of the outlined structure is no doubt related to the dynamical processes in the deeper layers of the Sun and presumably its convection.

The 11-year solar cycle is apparently the result of a certain valve-like mechanism. However in contrast to Cepheids this mechanism is not connected with the helium energy absorption, but with the temporal transformation of the radiation energy into that of the magnetic field.

The puzzling problem concerning the stars of different spectral types is how the energy transformation occurring within the stellar connective zones may result in different forms of the stellar variability and instability.

Like in the solar case, the stellar activity reveals itself mostly on the chromospheric level, *i.e.* in the variation of the spectral line emissions, especially in that of the Ca II ions. Periodical structures of the long-term variations of the chromospheric emissions have been observed among a lot of stars of spectral types F6–K7. At least comprehensive data, presented by Baliunas *et al.* (1995), support this possibility. The problem is to extract these data from the original ones. It seems that, nowadays, only solar observations are able to present the unique possibility to analyse the fine structure of stellar variability. One may hope that special analysis of the existing stellar observations cited above may reveal stellar analogues to the solar QBO.

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References

Abramowitz, M. and Stegun, I. (1964). *Handbook on Mathematical Functions.* National Bureau of Standards, Washington, D. C.

Apostolov, E. M. (1985). *Bull. Astron. Inst. Czechosl.*, **36**, 97.

Baliunas, S. L., *et al.* (1995). *Astrophys. J.*, **438**, 269.

Foucal, P. V. and Lean, J. (1988). *Astrophys. J.*, **28**, 347.

Frölich, C. and Lean, J. (1998). *Geophys. Res. Lett.*, **25**, 4377.

Gnevyshev, M. N. (1966). *Uspehi Phys. Nauk.*, **90**, 291 (in Russian).

Gnevyshev, M. N. (1977). *Solar Phys.*, **51**, 175.

Golyandina, N. E., Nekrutkin, V. V. and Zhigljavsky, A. A. (2000). *Analysis of Time Series Structure: SSA and Related Techniques*, Chapman & Hall/CRC.

Ivanov-Kholodny, G. S., Nepomnjashchaja, E. V. and Chertoprud, V. Ye. (2000). *Geomagnetizm i aeronomia*, **40**, 126 (in Russian).

Khramova, M. N., Kononovich, E. V., Krasotkin, S. A. and Shefov, N. N. (2001a). *Izbrannye Problemy astronomii. Trudy konferenzii*, Irkutsk, 177 (in Russian).

Khramova, M. N., Krasotkin, S. A. and Kononovich, E. V. (2001b). *Proc. Conference Solar Cycle and Space Weather*, 25–29 September, Italy (in press).

Kononovich, E. V. (1999). *Trudy conferenzii v Pulkovo Large Scale Structure of the Solar Activity*, SPb, 115 (in Russian).

Labitzke, K. (1987). *Geophys. Res. Lett.*, **14**, 535.

Makarov, V. I. and Sivaraman, K. R. (1989). *Solar Phys.*, **123**, 367.

Mordvino, A. V. (1988). *Issledovania po geomagnetizmu, aeronomii i fizike Solnsta*, **83**, 134 (in Russian).

Ohl, A. I. (1976). *Solnechnye dannye*, **9**, 73 (in Russian).

Slepzov-Shevlevich, B. A. (1969). *Trudy AANII*, Leningrad, Gydromet, **289**, 152 (in Russian).

Shapiro, R. and Ward, F. (1962). *J. Atmos. Sci.*, **19**, 506.

Schuster, A. (1906). *Proc. Roy. Soc.*, London **77A**, 141.

Vitinsky, Yu. I., Kopecky, M. and Kuklin, G. V. (1986). *Statistika pyatnoobrazovatelnoi deyatelnosti Solnsta*, Moscow (in Russian).