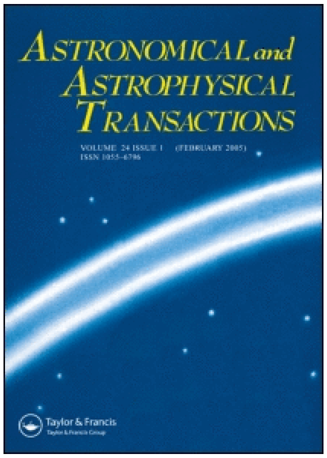

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IMPACT OF THE TIDAL FORCE FROM THE PLANETS ON THE BRIGHTNESS OF THE CENTRAL STAR

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Gravitational influence of planetary systems on "stellar activity" (the brightness variations process) is considered. Expressions for the tidal forces affecting the stellar atmosphere are obtained taking into account the degree of freedom of the gravitating bodies. We show that periodicities in solar cycles may be related to the tidal forces from the big planes of the Solar System.

Keywords: tidal force, solar activity, perturbation components, orbital motion, gravitational impact

INTRODUCTION

Explicit expressions for the components of the tidal forces affecting a random probing particle (local area) were obtained by Gerasimov *et al.* (2000) in the frame of the 2D version of three-body-problem (the Sun–Jupiter–Saturn) on the base of tidal force general determination; variability of the components with time was investigated. It was established that former problems of the gravitational-dynamic approach to the interpretation of the mechanism of solar activity induction (sunspots generation) were due to incompleteness of the tidal force description. Generally, the tidal force has, besides its non-perturbed component that characterize coupled interactions and are described by the Laplass formula, perturbation components related, in particular, with solar orbital moment changing. The tidal forces computed according to the classical Laplass algorithm, are, strictly speaking, adequate only in the case of unperturbed Keplerian motion of gravitating bodies, because perturbations by "odd bodies" (other planets) that lead to shifting of the mass center of the bodies interacting by pairs (Sun – planet) are not taken into account. The perturbing components of the tidal force may be formally derived from the equation for the orbital moment L of the central star (the Sun) variations because the variation δL is what characterizes deviations of the real motion from the unperturbed Keplerian one (Avsyuk, 1996).

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The present paper not restricting to the 2D case, discuss direct joint tidal effects (and relevant variations of the central body luminosity) from $N > 2$ planets P_i of the mass $M_i(i = \overline{1, N})$ upon a random particle P with a mass δm in the convection zone of the atmosphere of the central star P_0 with a mass M_0 that is considerably greater than that of the planets:

$$\mu_i = \frac{M_i}{M_0 + \sum_{j=1}^{i} M_j} \ll 1 \quad (i = \overline{1, N}). \tag{1}$$

The general expression for the tidal force valid in any reference system and time unit being selected so that the gravitational constant f be equal to 1, is (Avsyuk, 1997):

$$\vec{F} = \sum_{i=1}^{N} \frac{M_i \delta m}{\rho_i^3} \vec{\rho}_i - \delta m \vec{a}, \tag{2}$$

where $\vec{\rho}_i$ is the vector joining the particle $P \subset P_0$ and the center of mass P_i, $\vec{a}$ – acceleration of the orbital motion of the star center of mass P_0 in respect to the barycenter of the system under consideration, which, the motion of the whole system in respect to the center of the Galaxy disregarded, is adopted as the origin of the inertial reference system. In (2), the distances ρ_i are assumed to be of a considerable length so the planet deviations $P_i(i = \overline{1, N})$ from spherically-symmetrical distribution may be neglected. We shall also assume that in the initial moment t_0 the particle under consideration P, together with P_0, the differential axial rotation being taken into account, participates in translation–rotatory motion; therefore, denoting as $\vec{l}$ the radius-vector of the particle P located somewhere inside the convective zone of the star P_0 atmosphere, we obtain $\vec{l} = \vec{l}(t)$.

Besides, conserving the general character of the analysis, let us arrange all the distances R_i between $P_i(i = \overline{1, N})$ so that at the initial moment $R_1 \leq R_2 \leq \cdots \leq R_N$ (Fig. 1). The bodies $P_i(i = \overline{1, N})$, represent, in general, the centers of mass of the planets, their satellites taken into account.

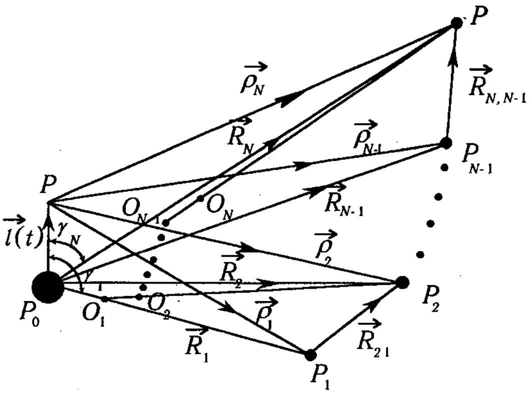

FIGURE 1 Configuration of the system: P_0 is the mass center of the central star; $P_i(i = \overline{1, N})$ – tide generating planets; P – the probe particle in the stellar atmosphere.

THE 2D VERSION OF THE PROBLEM

Let us consider first the case of all the planets $P_i(i = \overline{1, N})$ motion restriction to one and the same plane understood as the equatorial plane of the star P_0. The motion of the mass center P_0 in respect to the barycenter O_N of the system under investigation formed by $N + 1$ bodies may be depicted, at a random moment, as P_0 motion in respect to the subsystem P_0 and P_1 mass center O_1, which, in its turn moves in respect to the mass center O_2 of the subsystem $P_0 + P_1$ and P_2 (see Fig. 1). Similarly, O_2 moves in respect to the mass center O_3 of the subsystem $P_0 + P_1 + P_2$ and P_3 and so on; finally, O_{N-1} moves around the barycenter O_N of the whole system $P_j(j = \overline{0, N})$. Such breaking up of a complex motion is based on an obvious vector equation:

$$\vec{r}_\delta = \vec{r}_1 + \vec{r}_2 + \cdots + \vec{r}_N,$$

where $\vec{r}_1 = \overrightarrow{P_0 O_1}, \vec{r}_k = \overrightarrow{O_{k-1} O_k}(k = \overline{2, N}), \vec{r}_\delta = \overrightarrow{P_0 O_N}$ or, taking into account the designations (1),

$$\vec{r}_1 = \overrightarrow{\mu_1 R_1}, \vec{r}_k = \overrightarrow{\mu_k R_{0k}}, \vec{R}_1 = \overrightarrow{P_0 P_1}, \vec{R}_{0k} = \overrightarrow{O_{k-1} P_k} \quad (k = \overline{2, N}). \tag{3}$$

Let us consider the description of star P_0 motion, which, according to (1) considerably exceeds in mass the planets $P_i(i = \overline{1, N})$, that, as I. Newton (1936) has pointed out is the optimal $N + 1$ bodies problem breaking up into N two-bodies "quasi-problems", because it meets the condition of quasi-ellipticity of the orbits. The acceleration $\vec{a}$ of P_0 orbital motion in the expression (2) may be considered as a sum of the unperturbed component $\vec{a}_0$, answering to Keplerian motion in the frame of paired interactions $P_0' - P_i$ $(i = \overline{1, N}; P_0' \to O_{i-1}$ if $i > 1)$, and the perturbation component $\vec{a}'$, that is, $\vec{a} = \vec{a}_0 + \vec{a}'$.

In the frame of two-bodies problem, the Keplerian motion is characterized with acceleration of only the radial component, while under gravitational perturbations from two planets or more, the resulting acceleration vector consists of radial and tangential (and, in 3D case, normal) components. The perturbation component $\vec{a}'$, reveals itself in variation of the sectoral velocity and the orbital moment P_0 in respect to the planetary system barycenter.

To tell apart the patent harmonics related to mutual perturbations and revelations of the corresponding degrees of freedom, the dependence of the vectors $\vec{R}_k(k = \overline{2, N})$ that appears in the expression describing the multi-vector acceleration of the particle P

$$\vec{a} = \sum_{i=1}^{N} \frac{M_i}{R_i^3} \vec{R}_i, \tag{4}$$

from $\vec{R}_1, \vec{\tau}_1 \perp \vec{R}_1$ as well as from $\vec{R}_{0k}$ and $\vec{\tau}_k \perp \vec{R}_{0k}$ $(k = \overline{2, N})$, where $\vec{\tau}_i$ stay for the ort-vectors $(i = \overline{1, N})$ tangential to $\vec{R}_1$ and, respectively, to $\vec{R}_{0k}$.

Let us introduce the angles (see Fig. 2.)

$$D_l = \pi - (\lambda_{l+1} - \lambda_1), \quad l = \overline{1, N-1}$$

with vertices at the mass centers O_l. Here λ_l and λ_{l+1} are the longitudes of P_l and P_{l+1}s viewed from O_l. From the expression (3) and Fig. 2, and denoting $\vec{R}_1$ as $\vec{R}_{01}$ we deduce, that

$$\vec{R}_k = \vec{R}_{0k} + \sum_{l=1}^{k-1} \mu_l \vec{R}_{0l}, \quad k = \overline{2, N}, \tag{5}$$

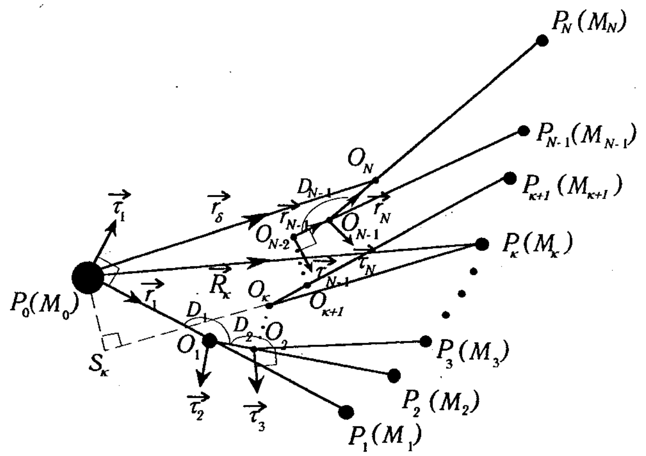

FIGURE 2 Determination of non-radial components of acceleration $\vec{a}$ of the mass center of the star P_0. Here $\overrightarrow{P_0O_1} = \vec{r}_1$, $\overrightarrow{O_{k-1}O_k} = \vec{r}_k$, $\overrightarrow{O_{k-1}P_k} = \vec{R}_{ok}$ $(k = \overline{2, N})$.

and

$$R_k^2 = |P_0S_k|^2 + |S_kP_k|^2,$$

or

$$\begin{aligned} R_k^{-3} &= R_{0k}^{-3} + 3\mu_{k-1}R_{0k}^{-4}R_{0,k-1}\cos D_{k-1} + K + (-1)^k 3\mu_1 R_{01}R_{0k}^{-4}\cos(D_1 + K + D_{k-1}) \\ &\quad - \frac{3}{2}\mu_{k-1}^2 R_{0,k-1}^2 R_{0k}^{-5}(1 - 5\cos^2 D_{k-1}) - K - \frac{3}{2}\mu_1^2 R_{01}^2 R_{0k}^{-5}[1 - 5\cos^2(D_1 + K + D_{k-1})] \\ &\quad + 3\mu_{k-1}\mu_{k-2}R_{0k}^{-5}R_{0,k-1}R_{0,k-2}[\cos D_{k-2} - 5\cos(D_{k-2} + D_{k-1})\cos D_{k-1}] + K \\ &\quad + (-1)^{j+m-1}3\mu_j\mu_m R_{0k}^{-5}R_{0j}R_{0m}\left[\cos\left(\sum_{l=m}^{j-1} D_l\right) - 5\cos\left(\sum_{i=m}^{k-1} D_i\right)\cos\left(\sum_{p=j}^{k-1} D_p\right)\right] + K \\ &\quad + 3\mu_1\mu_2 R_{0k}^{-5}R_{01}R_{02}[\cos D_1 - 5\cos(D_1 + K + D_{k-1})\cos(D_2 + K + D_{k-1})] \\ &\quad + O[\mu_q\mu_s\mu_n], \quad k = \overline{2, N}; \quad q, s, n = \overline{1, N}. \end{aligned} \tag{6}$$

Representing the vector components (5) as

$$\begin{aligned} \frac{\vec{R}_{0m}}{R_{0m}} &= -\cos D_{m-1}\frac{\vec{R}_{0,m-1}}{R_{0,m-1}} + \vec{\tau}_{m-1}\sin D_{m-1}, \quad m = \overline{2, k}, \\ \frac{\vec{R}_{01}}{R_{01}} &= (-1)^{k+1}\cos(D_1 + D_2 + \cdots + D_{k-1})\frac{\vec{R}_{0k}}{R_{0k}} + \vec{\tau}_k(-1)^k\sin(D_1 + D_2 + \cdots + D_{k-1}), \end{aligned} \tag{7}$$

we get for the acceleration of inertia the expression

$$\vec{a} = \sum_{i=1}^{N}\frac{M_i}{R_{0i}^3}\vec{R}_{0i} + \vec{a}_{1\mu} + \vec{a}_{2\mu} + O[\mu_i^3], \tag{8}$$

where $\vec{a}_{1\mu}$ and $\vec{a}_{2\mu}$ are, respectively, the linear and the square components over minor parameters $\mu_i(i = \overline{1, N})$:

$$\vec{a}_{1\mu} = \sum_{k=2}^{N} \frac{M_k}{R_{0k}^3} \Bigg\{ \mu_1 \vec{R}_{01} + \cdots + \mu_{k-2} \vec{R}_{0,k-2} + \frac{\vec{R}_{0,k-1}}{R_{0,k-1}} [\mu_{k-1} R_{0,k-1} - 3(\mu_{k-1} R_{0,k-1} \times \cos D_{k-1} - \mu_{k-2} R_{0,k-2} \cos(D_{k-2} + D_{k-1}) + \cdots + (-1)^k \mu_1 R_{01} \times \cos(D_1 + D_2 + \cdots + D_{k-1})) \cos D_{k-1}] + 3\vec{\tau}_{k-1} [\mu_{k-1} R_{0,k-1} \cos D_{k-1} + \cdots + (-1)^k \mu_1 R_{01} \cos(D_1 + D_2 + \cdots + D_{k-1})] \sin D_{k-1} \Bigg\},$$

$$\vec{a}_{2\mu} = -3 \sum_{k=2}^{N} \frac{M_k}{R_{0k}^4} \vec{\tau}_k \sum_{j=1}^{k-1} \mu_j^2 R_{0j}^2 \cos(D_j + \cdots + D_{k-1}) \sin(D_1 + \cdots + D_{k-1}) - \frac{3}{2} \sum_{k=2}^{N} \frac{M_k}{R_{0k}^5} \vec{R}_{0k} \times \sum_{j=1}^{k-1} \mu_j^2 R_{0j}^2 [1 - 3\cos^2(D_j + \cdots + D_{k-1})] + 3 \sum_{i=3}^{N} \frac{M_i}{R_{0i}^4} \sum_{l=1}^{i-1} \mu_l \vec{R}_{0l} \sum_{j=1 (j \neq i-l)}^{i-1} (-1)^{j+1} \times \mu_{i-j} R_{0,i-j} \cos \sum_{m=1}^{j} D_{i-m} + 3 \sum_{i=3}^{N} \frac{M_i}{R_{0i}^5} \vec{R}_{0i} \sum_{j=2}^{i-1} \sum_{m=1}^{j-1} (-1)^{j+m+1} \mu_j \mu_m R_{0j} R_{0m} \times [\cos(D_m + \cdots + D_{j-1}) - 5\cos(D_j + \cdots + D_{i-1}) \cos(D_m + \cdots + D_{i-1})].$$

It follows from (8), that in correspondence to the subsystems degree of freedom in respect their mass centers, the inertia acceleration of the stellar atmosphere particle P under investigation possess, in respect to the gravitating bodies $P_i(i = \overline{1, N})$ both the radial and the tangential components.

THE GENERAL CASE

Now, let the orbits of the planets $P_i(i = \overline{1, N})$ form a 3D pattern, so, that the planet radius-vectors $\vec{R}_{0i} = \overrightarrow{O_{i-1}P_i}$ measured from the mass center of the subsystems, are characterized, in the local coordinate system $x_j O_j y_j z_j (j = \overline{0, N-1})$ with the axis parallel to those of an unknown Laplassian system $(x_0 O_0 y_0 z_0)$, by the angles λ_i, θ_i (see Fig. 3). The prime Laplassian plane $x_0 O_0 y_0$ perpendicular to the kinetic vector of the orbital momentum of the whole system under consideration will be understood as the Sun equator plane.

Taking into account that

$$R_k^2 = |P_k P_k'|^2 + |P_0 P_k'|^2, \quad k = \overline{2, N},$$

introducing the angles between the vectors $\overrightarrow{O_{l-1}O_l}$ and $\overrightarrow{O_l O_{l+1}}$ $(l = \overline{1, N-1})$, determined by the expressions of the type of

$$\begin{aligned} \cos\psi_l &= -\cos\theta_l \cos\theta_{l+1} + \sin\theta_l \sin\theta_{l+1} \cos D_l, \\ \cos\psi_{l,l+1} &= \cos\theta_l \cos\theta_{l+2} + \sin\theta_l \sin\theta_{l+2} \cos(D_l + D_{l+1}), \\ &\vdots \\ \cos\psi_{l,l+1,\ldots,l+m} &= (-1)^{m+1} \cos\theta_l \cos\theta_{l+m+1} + \sin\theta_l \sin\theta_{l+m+1} \\ &\quad \times \cos(D_l + D_{l+1} + \cdots + D_{l+m}), \end{aligned} \tag{9}$$

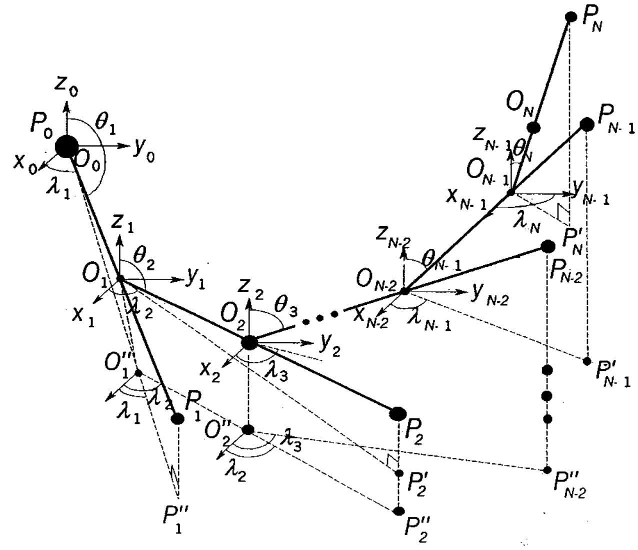

FIGURE 3 Determination of local coordinate systems in the 3D case. The points P_i'' are the mass centers of the planets projections $P_i(i = \overline{1, N})$ on the Laplassian plane $x_0O_0y_0$.

and making use of the results obtained in the previous section, after some simple transformations of the magnitude R_k^{-3}, from the expression for the inertia acceleration (4) we get the expression (6), where the following substitutions are to be made: D_i for $\psi_i, D_i + D_j + \cdots + D_m, D_i + D_j + \cdots + D_m$ for $\psi_{i,j,\ldots,m}(i, j, \ldots, m = \overline{1, N-1})$.

Let us now express the vectors $\vec{R}_{0k}$ through $\vec{R}_{0,k-1}, \vec{\tau}_{k-1}, \vec{n}_{k-1}(k = \overline{2, N})$, and $\vec{R}_{01}$ through $\vec{R}_{0N}, \vec{\tau}_N, \vec{n}_N$, where $\vec{\tau}_i, \vec{n}_i$ are the corresponding ort-vectors to $\vec{R}_{0i}(i = \overline{1, N})$. We obtain (similar to Gerasimov, 2000):

$$\frac{\vec{R}_{0k}}{R_{0k}} = \frac{\vec{R}_{0,k-1}}{R_{0,k-1}} \cos(\pi - \psi_{k-1}) + \vec{\tau}_{k-1} \cos L_k\breve{B}_{k-1} + \vec{n}_{k-1} \cos L_k\breve{B}'_{k-1}, \tag{10}$$

where, as it follows from Figure 4,

$$\cos L_k\breve{B}'_{k-1} = \sin I_{k,k-1} \sin \Lambda_k, \cos^2 L_k\breve{B}_{k-1} = \sin^2 \psi_{k-1} - \sin^2 \Lambda_k \sin^2 I_{k,k-1}$$

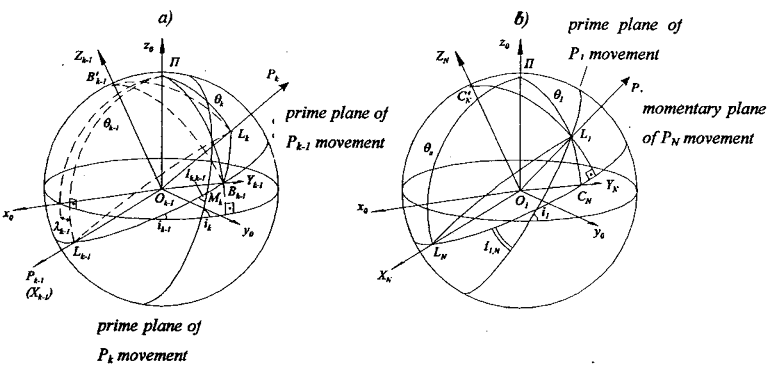

FIGURE 4 Location on the celestial sphere of the osculating orbit projections: *a*) for the planets P_{k-1} and $P_k(k = \overline{2, N})$; *b*) for the planets P_1 and $P_N(\overrightarrow{O_1X_N} \| \overrightarrow{O_{N-1}P_N})$.

or

$$\cos L_k\breve{B}_{k-1} = -\sin^{-1}\Lambda_{k-1}(\cos\Lambda_k + \cos\psi_{k-1}\cos\Lambda_{k-1}).$$

Here, in its turn, $L_k\breve{M}_k$ and $L_{k-1}\breve{M}_k$ are, respectively, the longitudes Λ_k and Λ_{k-1} of the planets P_k and P_{k-1} in respect to their shared orbital nod, and $I_{k,k-1}$ is the inclination angle between the osculating orbits of P_k and P_{k-1}.

At the same time

$$\frac{\vec{R}_{01}}{R_{01}} = \frac{\vec{R}_{0N}}{R_{0N}}\cos L_1\breve{L}_N + \vec{\tau}_N\cos L_1\breve{C}_N + \vec{n}_N\cos L_1\breve{C}'_N, \tag{11}$$

where $R_{01} = R_1$ and, out of Figure 4b,

$$\cos L_1\breve{L}_N = (-1)^{N+1}\cos\psi_{1,2,\ldots,N-1}, \quad \cos^2 L_1\breve{C}_N = \sin^2\psi_{1,2,\ldots,N-1} - \sin^2\Lambda_1\sin^2 I_{1,N},$$
$$\cos L_1\breve{C}'_N = \sin\Lambda_1\sin I_{1,N}.$$

In general case, the vector $\vec{R}_{0i}/R_{oi}$ expansion in terms of the basis vector $\vec{\tau}_j \perp \vec{n}_j \perp \vec{R}_{0j}/R_{0j}(i \neq j;\, i,\, j = \overline{1,N})$ has the form:

$$\frac{\vec{R}_{0i}}{R_{oi}} = (-1)^{i+j}\cos\psi_{i,i+1,\ldots,j-1}\frac{\vec{R}_{0j}}{R_{0j}} + \vec{\tau}\cos L_i\breve{C}_j + \vec{n}_j\cos L_i\breve{C}'_j. \tag{12}$$

$\cos\psi_{i,\ldots,j-1}$ is determined by (9) and

$$\cos^2 L_i\breve{C}_j = \sin^2\psi_{i,\ldots,j-1} - \sin^2\Lambda_i\sin^2 I_{i,j}, \quad \cos L_i\breve{C}'_j = \sin\Lambda_i\sin I_{i,j}. \tag{13}$$

Taking (9)–(13) into account, with accuracy up to power index 3 in terms of minor parameters μ_i in the form (1) for the acceleration of the orbital motion of the star P_0, conforming the degree of freedom in respect to mass centers $O_i(i = \overline{1,N})$ we obtain the expression (8), for the items $\vec{a}_{1\mu}$ and $\vec{a}_{2\mu}$ of which the following substitutions should be performed:

$$\begin{aligned}
D_{k-1} \to \psi_{k-1},\ D_{k-2} + D_{k-1} &\to \psi_{k-2,k-1}, \ldots, \\
D_j + D_{j+1} + \cdots + D_{k-1} &\to \psi_{j,j+1,\ldots,k-1}, \ \ldots, \\
D_1 + D_2 + \cdots + D_{k-1} &\to \psi_{1,2,\ldots,k-1}, \\
\vec{\tau}_{k-1}\sin D_{k-1} &\to \vec{\tau}_{k-1}\cos L_k\breve{B}_{k-1} + \vec{n}_{k-1}\cos L_k\breve{B}'_{k-1}, \\
\vec{\tau}_k\sin(D_1 + \cdots + D_{k-1}) &\to (-1)^{k+j}(\vec{\tau}_k\cos L_j\breve{C}_k + \vec{n}_k\cos L_j\breve{C}'_k).
\end{aligned}$$

Then, the tidal force affecting a random particle P (with mass δm) of the atmosphere of the star P_0 will be determined, according to (2) by expressions of the type

$$\vec{F} = \sum_{j=1}^{N}\left[\frac{M_j}{\rho_j^3}\vec{\rho}_j - \frac{M_j}{R_{0j}^3}\vec{R}_{0j}\right]\delta m + \vec{F}_2^{(1)} + \vec{F}_3^{(1)} + \vec{F}_\mu^{(2)} + O[\mu_q\mu_s\mu_p], \quad q,s,p = \overline{1,N}, \tag{14}$$

where the items linear in terms of minor parameters are:

$$\vec{F}_2^{(1)} = \frac{M_2\delta m}{R_{02}^3}\mu_1\vec{R}_{01}(3\cos^2\psi_1 - 1) + \frac{3M_2\delta m}{R_{02}^3}\mu_1 R_{01}\cos\psi_1 \times [\vec{\tau}_1 \sin^{-1}\Lambda_1(\cos\Lambda_2 + \cos\psi_1\cos\Lambda_1) - \vec{n}_1\sin\Lambda_2\sin I_{2,1}],$$

$$\vec{F}_3^{(1)} = -\sum_{i=3}^{N}\frac{M_i\delta m}{R_{0i}^3}\{\mu_1\vec{R}_{01} + \mu_2\vec{R}_{02} + \cdots + \mu_{i-2}\vec{R}_{0,i-2} + R_{0,i-1}^{-1}\vec{R}_{0,i-1}[\mu_{i-1}R_{0,i-1} - 3(\mu_{i-1}R_{0,i-1}\cos\psi_{i-1} - \mu_{i-2}R_{0,i-2}\cos\psi_{i-2,i-1} + \cdots + (-1)^i\mu_1 R_{01}\cos\psi_{1,2,\ldots,i-1}) \times \cos\psi_{i-1}] + 3[\mu_{i-1}R_{0,i-1}\cos\psi_{i-1} - \mu_{i-2}R_{0,i-2}\cos\psi_{i-2,i-1} + \cdots + (-1)^i\mu_1 R_{01} \times \cos\psi_{i,2,\ldots,i-1}][\vec{\tau}_{i-1}\sin^{-1}\Lambda_{i-1}(\cos\Lambda_i + \cos\psi_{i-1}\cos\Lambda_{i-1}) - \vec{n}_{i-1}\sin\Lambda_i\sin I_{i,i-1}]\},$$

and the square item has the form

$$\vec{F}_\mu^{(2)} = \frac{3}{2}\sum_{k=2}^{N}\frac{M_k\delta m}{R_{0k}^5}\vec{R}_{0k}\sum_{j=1}^{k-1}\mu_j^2 R_{0j}^2(1 - 3\cos^2\psi_{j,j+1,\ldots,k-1}) + 3\sum_{k=2}^{N}\frac{M_k\delta m}{R_{04}^4}\sum_{j=1}^{k-1}(-1)^{k+j}\mu_j^2 R_{oj}^2 \times \cos\psi_{j,\ldots,k-1}[\vec{\tau}_k(\sin^2\psi_{j,\ldots,k-1} - \sin^2\Lambda_j\sin^2 I_{j,k})^{1/2} + \vec{n}_k\sin\Lambda_j\sin I_{j,k}] + 3\sum_{k=3}^{N}\frac{M_i\delta m}{R_{0i}^4}\sum_{l=1}^{i-1}\mu_l\vec{R}_{0l}\sum_{j=1}^{i-1}(-1)^j\mu_{i-j}R_{0,i-j}\cos\psi_{i-j,\ldots,i-1} + 3\sum_{i=3}^{N}\frac{M_i\delta m}{R_{0i}^5}\vec{R}_{0i} \times \sum_{j=2}^{i-1}\sum_{k=1}^{j-1}(-1)^{j+k}\mu_j\mu_k R_{0j}R_{0k}[\cos\psi_{k,\ldots,j-1} - 5\cos\psi_{j,\ldots,i-1}\cos\psi_{k,\ldots,i-1}].$$

Here $\vec{R}_{0i}$, and $\vec{R}_{0l}$ are given in the form (12).

As $\mu_j(j = \overline{1,N})$ are minor parameter, according to the initial assumption, the mass centers O_j do not deviate considerably from the mass center P_0; accordingly, $(R_j - R_{0j})/R_j \ll 1$ and therefore, the expression (14) for the tidal force first items that correspond to paired interactions

$$\sum_{j=1}^{N}\left(\frac{M_j\delta m}{\rho_j^3}\vec{\rho}_j - \frac{M_j\delta m}{R_{0j}^3}\vec{R}_{0j}\right),$$

are of quasi-Laplassian nature. However, as it follows from (14), the tidal force is characterized, apart from the items, by other additional components, including tangential and normal ones.

CALCULATION OF TIDAL FORCE COMPONENTS

Let us introduce the angles $\alpha(t)$ and $\delta(t)$ corresponding to right accession and declination of the particle under investigation P in the atmosphere of the star (see Fig. 5). As it follows from Figures 1 and 5, denoting γ_j the angles between radius-vector $\vec{l}(t)$ and $\vec{R}_j(j = \overline{1,N})$ we obtain:

$$\cos\gamma_j = \cos LL_j$$

Consequently,

$$\cos\gamma_j = \cos\theta_j^*\sin\delta(t) + \sin\theta_j^*\cos\delta(t)\cos[\alpha(t) - \lambda_j^*]. \tag{15}$$

Unlike the case in the Figure 3, here the spherical angles $\lambda_j^* = \gamma\check{N}_j$, $\theta_j^* = \Pi\check{L}_j(j = \overline{1,N})$ are determined in the coordinate system with the center at P_0.

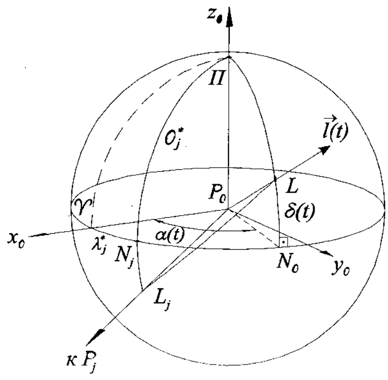

FIGURE 5 Projections of the particle P of the stellar atmosphere and of the mass center of the planet P_j on the "celestial sphere" the center of which coincides with the star.

As (see Fig. 1)

$$\vec{\rho}_j = \vec{R}_j - \vec{l} \quad (j = \overline{1,N}), \tag{16}$$

where, if $k = \overline{2,N}$,

$$\vec{R}_k = \vec{R}_{0k} + \mu_{k-1}\vec{R}_{0,k-1} + \cdots + \mu_1 \vec{R}_{01},$$

we have

$$\rho_j = (l^2 - 2lR_j\cos\gamma_j + R_j^2)^{1/2} \quad (j = \overline{1,N}). \tag{17}$$

Assuming (as we did in the case of the Solar System) that $(l/R_j)_{\max} \propto \mu^{2/3}$, we obtain, out of (17):

$$\rho_j^3 = R_j^{-3} + 3\frac{l}{R_j^4}\cos\gamma_j - \frac{3}{2}\frac{l^2}{R_j^5}(1 - 5\cos^2\gamma_j) - \frac{15}{2}\cos\gamma_j\left(1 - \frac{7}{3}\cos^2\gamma_j\right)\frac{l^3}{R_j^6} + O\left[\frac{l^4}{R_j^7}\right].$$

But, according to (6) and (9), we have, with the adopted accuracy of the order of square in terms of minor parameters $\mu_j (j = \overline{1,N})$:

$$\begin{aligned}
\rho_j^{-3} = {} & R_{0j}^3 + \frac{3}{R_{0j}^4}\{l\cos\gamma_j + \mu_{j-1}\xi_j R_{0,j-1}\cos\psi_{j-1} - \mu_{j-2}\xi_j R_{0,j-2}\cos\psi_{j-2,j-1} \\
& + \cdots + (-1)^j \mu_1 \xi_j R_{01}\cos\psi_{1,2,\ldots,j-1}\} + \frac{3}{R_{0j}^5}\left\{ -\frac{1}{2}l^2(1 - 5\cos^2\gamma_j) \right. \\
& - \frac{1}{2}\mu_{j-1}^2 R_{0,j-1}^2(1 - 5\cos^2\psi_{j-1}) - \frac{1}{2}\mu_{j-2}^2 R_{0,j-2}^2(1 - 5\cos^2\psi_{j-2,j-1}) \\
& - \cdots - \frac{1}{2}\mu_1^2 R_{01}^2(1 - 5\cos^2\psi_{1,2,\ldots,j-1}) + \mu_{j-1}\mu_{j-2}R_{0,j-1}R_{0,j-2} \\
& \times (\cos\psi_{j-2} - 5\cos\psi_{j-2,j-1}\cos\psi_{j-1}) - \mu_{j-1}\mu_{j-3}R_{0,j-1}R_{0,j-3} \\
& \times (\cos\psi_{j-3,j-2} - 5\cos\psi_{j-3,j-2,j-1}\cos\psi_{j-1}) + \cdots + (-1)^{m+k-1}\mu_m\mu_k R_{0m}R_{0k} \\
& \times (\cos\psi_{k,k+1,\ldots,m-1} - 5\cos\psi_{k,k+1,\ldots,j-1}\cos\psi_{m,m+1,\ldots,j-1}) + \mu_1\mu_2 R_{01}R_{02} \\
& \left. \times (\cos\psi_1 - 5\cos\psi_{1,2,\ldots,j-1}\cos\psi_{2,3,\ldots,j-1}) \right\} - \frac{15}{2}\cos\gamma_j\left(1 - \frac{7}{3}\cos^2\gamma_j\right)\frac{l^3}{R_{0j}^6}.
\end{aligned}$$

Here $\xi_j = 1 + 4(1/R_{0j})\cos\gamma_j$.

Therefore, taking (16) into account, we obtain, out of (14), for the tidal force:

$$\frac{\vec{F}}{\delta m} = \sum_{j=1}^{N} \frac{M_j}{R_{0j}^3} \left\{ \frac{3l}{R_{0j}} \cos\gamma_j \vec{R}_{0,j} - \vec{l} - \frac{3l}{R_{0j}} \cos\gamma_j \vec{l} - \frac{3}{2}(1 - 5\cos^2\gamma_j) \frac{l^2}{R_{0j}^2} \vec{R}_{0j} - \frac{15}{2}\cos\gamma_j \right.$$
$$\times \left(1 - \frac{7}{3}\cos^2\gamma_j\right) \frac{l^3}{R_{0j}^3} \vec{R}_{0j} + \frac{3}{2}(1 - 5\cos^2\gamma_j) \frac{l^2}{R_{0j}^2} \vec{l} + \frac{15}{2}\cos\gamma_j \left(1 - \frac{7}{3}\cos^2\gamma_j\right) \frac{l^3}{R_{0j}^3} \left.\vec{l}\right\}$$
$$+ \sum_{i=2}^{N} \frac{3M_i}{R_{0i}^3} \left\{ \frac{\cos\psi_{i-1}}{R_{0,i-1}} \vec{R}_{0,i-1} \sum_{k=1}^{i-1} (-1)^{i-k+1} \mu_k R_{0k} \cos\psi_{k,k+1,\dots,i-1} + [\vec{n}_{i-1} \sin I_{i,i-1} \sin\Lambda_i \right.$$
$$- \vec{\tau}_{i-1} \sin^{-1}\Lambda_{i-1} (\cos\Lambda_i + \cos\psi_{i-1} \cos\Lambda_{i-1})] \left[\sum_{k=1}^{i-1} (-1)^{i+k} \mu_k R_{0k} \cos\psi_{k,k+1,\dots,i-1} \right]$$
$$+ \frac{l\cos\gamma_i}{R_{0i}} \sum_{k=1}^{i-1} \mu_k \vec{R}_{0k} + \frac{\xi_i}{R_{0i}} \vec{R}_{0i} \sum_{k=1}^{i-1} (-1)^{i+k+1} \mu_k R_{0k} \cos\psi_{k,k+1,\dots,i-1} - \sum_{k=1}^{i-1} (-1)^{i-k} \mu_k^2$$
$$\times \frac{R_{0k}}{R_{0i}} \vec{R}_{0k} \cos\psi_{k,\dots,i-1} + 4\frac{l\cos\gamma_i}{R_{0i}^2} \sum_{k=1}^{i-1} \mu_k \vec{R}_{0k} \sum_{j=1}^{i-1} (-1)^{i+j+1} \mu_j R_{0j} \cos\psi_{j,j+1,\dots,i-1}$$
$$+ \frac{\xi_i \vec{l}}{R_{0i}} \sum_{k=1}^{i-1} (-1)^{i+k} \mu_k R_{0k} \cos\psi_{k,\dots,i-1} + \sum_{k=1}^{i-1} (-1)^{k+i} \mu_k^2 \frac{R_{0k}^2}{R_{0i}} \cos\psi_{k,\dots,i-1} [\vec{\tau}_i (\sin^2\psi_{k,\dots,i-1}$$
$$- \sin^2\Lambda_k \sin^2 I_{k,i})^{1/2} + \vec{n}_i \sin\Lambda_k \sin I_{k,i}] + \frac{\vec{R}_{0i}}{R_{oi}^2} \sum_{k=1}^{i-1} \mu_k^2 R_{0k}^2 \cos^2\psi_{k,\dots,i-1}$$
$$+ \frac{\vec{l}}{R_{0i}^2} \sum_{j=2}^{i-1} \sum_{k=1}^{j-1} (-1)^{j+k} \mu_j \mu_k R_{0j} R_{0k} [\cos\psi_{k,\dots,j-1} - 5\cos\psi_{j,j+1,\dots,i-1} \cos\psi_{k,\dots,i-1}]$$
$$\left. + \frac{\vec{l}}{2R_{0i}^2} \sum_{k=1}^{i-1} \mu_k^2 R_{0k}^2 (1 - 5\cos^2\psi_{k,k+1,\dots,i-1}) \right\} + O[\mu_q \mu_s \mu_p^{1/3}], \quad q, w, p = \overline{1, N}. \tag{18}$$

In the case of the Solar System ($N = 9$), under the assumption of the minuteness of the angles between the planet orbits and the Sun equatorial planes (*e.g.*, the 2D case) and taking into account the predominance of the mass of Jupiter and Saturn over the mass of the other giant planets, we can deduce directly out of (18) the conclusions described in (Gerasimov, 2000), namely: in the frame of 2D version of the problem of now 10 (the Sun + 9 big planets) bodies, the expression for the tidal force affecting a probe particle (local area) in the Solar atmosphere contains the main perturbation harmonics with periodicities equal to the syderic periods of Saturn and Jupiter as well as to the periods combinations, including the resonance period of ~ 60 years. The quasi-laplassian component of the tidal force (paired interactions) may determine the dynamical shape of the Solar photosphere formation, while the perturbation component would respond for the loss of stability of quasistationary fluxes of the Solar plasma, causing the corresponding rotations in the photosphere that are related with the spots formation process. So then, the mechanism that induces the Solar cycles may be related to the big planets dynamical impact on the Solar atmosphere.

However, in the general 3D case (with absence of degeneration in the system), the perturbation harmonics amplitude, as it follows from (18), increases considerably by a magnitude of an order of $R_{0j}/l(j = \overline{1, N})$, which, in the case of the Solar system amounts to $\sim 10^3$. To obtain, in 3D case, direct expressions for the tidal force components (18) as projected onto the coordinate system (x_0, y_0, z_0) related to the equatorial plane of the star P_0 (the Laplass plane $x_0O_0y_0$ – see Fig. 3.), we should select the axis O_0x_0 that pass through the longitude of the ascending nod of the equator P_0 (see Fig. 5). Naturally, the true equator plane P_0 perpendicular to the rotation axis will not coincide with the invariable Laplassian plane.

Projecting the vectors $\vec{l}$, $\vec{\tau}_j$, $\vec{n}_j$ and $\vec{R}_{0j}(j = \overline{1, N})$ from (18) over the axis x_0, y_0, z_0 with the center in the point O_0 of P_0 mass center and denoting as s_0 the stellar time that is determined through the star P_0 proper rotation speed, so that $\alpha(t) = \alpha - s_0$ and by u_j, Ω_j and i_j, correspondingly, the latitude argument, the ascending nod longitude and the angle between the planet $P_j\,(j = \overline{1, N})$ osculating orbit and the plane $(x_0O_0y_0)$ of the equator P_0 we obtain (see Abalakin, 1971):

$$\begin{gathered} l_x = l\cos\delta\cos(\alpha - s_0),\quad l_y = l\cos\delta\sin(\alpha - s_0),\quad l_z = l\sin\delta, \\ (R_{0j})_x = R_{0j}\sin\theta_j^*\cos\lambda_j^*,\quad (R_{0j})_y = R_{0j}\sin\theta_j^*\sin\lambda_j^*,\quad (R_{0j})_z = R_{0j}\cos\theta_j^*, \\ \tau_{jx} = \cos\Omega_j\sin u_j - \sin\Omega_j\cos u_j\cos i_j,\quad \tau_{jy} = \sin u_j\sin\Omega_j + \cos u_j\cos\Omega_j\cos i_j, \\ \tau_{jz} = \sin i_j\cos u_j,\quad n_{jx} = \sin\Omega_j\sin i_j,\quad n_{jy} = -\cos\Omega_j\sin i_j,\quad n_{jz} = \cos i_j \quad (j = \overline{1, N}). \end{gathered} \tag{19}$$

Calculating all the distances

$$\begin{aligned} &R_{01} = R_1,\quad R_{02} = R_2 + \mu_1 R_{01}\cos\psi_1 + O[\mu_1^2], \ldots \\ &R_{0k} = R_k + \mu_{k-1}R_{0,k-1}\cos\psi_{k-1} - \mu_{k-2}R_{0,k-2}\cos\psi_{k-2,k-1} + \cdots + (-1)^k\mu_1 R_{01} \\ &\qquad \times \cos\psi_{1,2,\ldots,k-1} + O[\mu_q\mu_p];\quad q, p = \overline{1, k-1} \end{aligned} \tag{20}$$

in the osculating elements $R_j = a_j(1 - e_j\cos E_j)$, $j = \overline{1, N}$, where a_j, e_j are the planet P_j orbits large semi-axis and the eccentricity and E_j is the planets eccentric anomaly, and using the expansions in terms of the orbit P_j main anomaly M_j multiples (Duboshin, 1964)

$$R_j = a_j\left\{1 + \frac{e_j^2}{2} + \left(\frac{3}{8}e_j^3 - e_j\right)\cos M_j - \frac{1}{2}e_j^2\cos 2M_j - \frac{3}{8}e_j^3\cos 3M_j + \cdots\right\},\quad j = \overline{1, N}, \tag{21}$$

we can easily obtain the target components of the tidal force affecting a random particle in the star atmosphere directly from (18), taking into account (9), (15), and (19)–(21), which make is possible for us to study the properties of the components time variations on the scale of the star P_0 several rotation periods as well as on the evolutionary scale.

The tidal force components we have obtained contain the harmonics related to the planet $P_j(j = \overline{1, N})$ orbital periods as well as to their combinations and resonance frequencies. Dynamical impact of the planets must lead to induced oscillations of the star P_0 convection areas, which, appearing at the surface of the star, may form active zones that change the integral flux of the star radiation.

Taking into account the fact that the "Sunspots formation" leads to Sun radiation variations amounting to several decimals of a percent (Vitinsky, 1986; Friins Christiansen, 1991), we may expect similar variations in brightness of stars of the same spectral class as that of the Sun and possessing planet systems.

CONCLUSIONS

Dynamical influence impact on the process of sunspots formation have been repeatedly turned to by a number of authors. Certain facts, such as absence of spots at high latitudes, equatorial symmetry of spot appearing in the Northern and Southern hemisphere of the Sun, etc., support the assumption that the mechanism that determines the Solar cycles characteristics may be regulated by the tidal impact from big planets. However, matching the observational data against simulations of the tidal force components affecting the Sun convective zone, performed according to the approximate algorithm of Laplassian "paired interactions" did not support the assumption.

The present article shows that a reason for such inconsistency may be found in the incompleteness of the classical description of the tidal force (the principal harmonics of the tidal force are not taking completely into account). Full description of the tidal force (direct tidal impact) contains both the unperturbed component (paired interactions) and the perturbing one, related with variations in the orbital momentum of the object under investigation (the central star).

Similar dynamical effects must be present as well in other planetary systems orbiting a central star Po. Gravitational impact from the planets must affect the star activity and be revealed in the form of the star Po integral brightness variations. The symmetrical, unperturbed component of the tidal force will be responsible for the dynamical shape of the star atmosphere, but it will not affect the stability of quasistationary fluxes in the atmosphere of the star. The perturbation component is related to formation of vertices in the convective zone of the atmosphere of the star, and, consequently to the "spotsformation process".

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