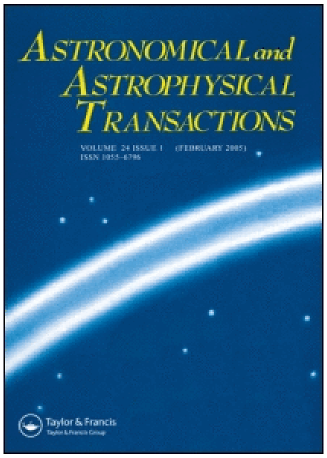

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

ORBITAL TRENDS IN GALAXY PAIRING AND MERGER RATES

Tapan Chatterjee [a]; V. B. Magalinsky
[a] Faculty of Science (FCC), Puebla A. University, Apartado Postal 1316, Puebla, México.

Online Publication Date: 01 January 2002
To cite this Article: Chatterjee, Tapan and Magalinsky, V. B. (2002) 'ORBITAL TRENDS IN GALAXY PAIRING AND MERGER RATES', Astronomical & Astrophysical Transactions, 21:1, 79 - 93
To link to this article: DOI: 10.1080/10556790215574
URL: http://dx.doi.org/10.1080/10556790215574

Astronomical and Astrophysical Transactions, 2002, Vol. 21(1–3), pp. 79–93

ORBITAL TRENDS IN GALAXY PAIRING AND MERGER RATES

TAPAN K. CHATTERJEE[a,]* and V. B. MAGALINSKY[a,b]

[a]*Faculty of Science (FCC), Puebla A. University, Apartado Postal 1316, Puebla, México;*
[b]*Department of Theoretical Physics, University of the Friendship of Peoples, Moscow, Russia.*

(Received 6 April 2001)

Low energy collisions are the most frequent, and are characterized by high eccentricities. The orbital trends of such collisions are now studied and the order of magnitude of their frequency is determined. Results support the analytical results of a previous paper that the eccentric orbit is a preferred state, and indicate that the system tends to maintain that state. The merger theory, in its extreme form, postulates that the ellipticals are the product of mergers of spirals. The frequency of the most frequent collisions is found to be extremely low to account for the formation of ellipticals. Results favor the traditional view of the formation of ellipticals by gravitational contraction (involving a single burst of star formation at a high redshift followed by passive evolution) and indicate that the merger process seems to be a sporadic one.

Keywords: Galaxies: formation; Orbital evolution and interactions; Methods: analytical

INTRODUCTION

The analytical results of a previous paper (Magalinsky and Chatterjee, 2000) indicated that galaxy pairing favors high eccentricities in an effort of the system to minimize its energy, such that the most frequent encounters will be characterized by high eccentricities. The formation and evolution of elliptical galaxies seem to be characterized by galactic interactions. A study of the nature and frequency of the most common galactic encounters in the environment where the formation process occurs is necessary to analyze their role in the formation and evolution of ellipticals.

Marginally bound collisions (MBCs) are the most frequent encounters between galaxies. This is easily inferred on grounds of larger available phase space with decreasing binding energy. This follows most instructively from the correspondence principle, taking the classical limit in the quantum mechanical description. This argument has been very nicely elucidated by Hut (1985) for binary stars; we find it instructive to summarize his arguments in our context as follows. In the classical limit, the population of (quantum) energy levels for the simplest case (the hydrogen atom), has the parametric form, $f(E) \propto e^{-E/kT}$, where the constant of proportionality is determined by the density of discrete energy levels (ρ) and a

*Corresponding author. FCC, UAP, Apartado Postal 1316, Puebla, Mexico. Fax: +22 443466

ISSN 1055-6796 print; ISSN 1476-3540 online
DOI: 10.1080/10556790290018890

measure of the degeneracy within an energy value, denominated as the statistical weight function (w). For an energy, E, in the classical limit (principle quantum number $n \gg 1$), $E \propto 1/n^2$ implying, $\rho = \mathrm{d}n/\mathrm{d}E \propto E^{-3/2}$. The degeneracy per energy level, given by n^2, then assumes the form, $w = n^2 \propto 1/E$, such that the first relation assumes the form, $f(E) = E^{-5/2}\mathrm{e}^{-E/kT}$: A high frequency of MBCs is also favored as proportionately more phase space volume is available for higher values of eccentricities, for an initially homogeneous distribution of positions and velocities of component galaxies (*e.g.*, Jeans, 1929; Heggie, 1975; Hut, 1986; Spitzer, 1987). In fact, an explicit integration over the other five classical variables of Keplerian orbits also leads to this conclusion. The existence of very wide, physical galaxy pairs (~ 1 Mpc separation) in near parabolic orbits, representing MBCs, is supported strongly by current observational evidence (Chengalur, Salpeter and Terzian, 1993).

The kinetic description of gravitating systems has acquired a vital importance. In the work of Magalinsky, 1972 (see also Bisnovatyi-Kogan and Zeldovich, 1970), the Vlasov equation was applied to study small perturbations (considered as protogalaxies) of the exact solution corresponding to a spacially homogeneous medium in expansion. An integral equation was obtained for the reduced density perturbation (in terms of the initial density), taking the effect of the thermal motion into account,

$$y(\eta,\tau) = y_s(\eta,\tau) + 6\int \mathrm{d}\xi[(\tau-\xi)/(\xi^2-\varepsilon)]\langle y[\eta + T(\tau-\xi)\mathbf{P},\xi]\rangle_o$$

[where η is a reduced coordinate (in terms of separation scale), $\varepsilon = 1$, for open universe, ξ and τ denote the initial and final values of the parameter in question, T is the characteristic time (corresponding to the characteristic frequency), $y_s(\eta,\tau)$ denotes the initial (averaged) value of $y(\eta,\tau)$]. The point to note is that $T(\tau-\xi)\mathbf{P}$ represents a reduced displacement in the position of an internal constituent, treated as a mass point, due to thermal motion over a time interval; and that the kernel of this equation is integrable, indicating that the amplitude of the inhomogeneity or density perturbation remains bounded with the passage of time. The further evolution was investigated by expanding the condensation as a Fourier integral. During the initial phase the inhomogeneity decays due to thermal motion. However it was found that the kinetic energy (K.E.) varies as the square of the distance between neighboring masses, r^{-2}, in this phase (as a consequence of the thermal momentum dispersion in accordance with the adiabatic law), while the potential energy (P.E.) varies as r^{-1}; such that, with the passage of time, thermal repulsion is replaced by Newtonian attraction and the decay is replaced by growth. If the initial kinetic temperature is low (*i.e.*, in regions of low relative velocity), the condensation attains a saturated size; given by,

$$Y(1) = P_2(\tau_o)\exp[T\beta P_o(1-\tau_o)],$$

[where, P_2 is Legendre function of the first kind, β is the reduced (in terms of separation scale) wave vector and P_o is the initial momentum]. This is such that we can expect an environment of galaxies of comparable mass and size with low velocity dispersion following the formation process. The environmental study of galaxy interactions by Hashimoto and Oemler (2000), indicates that galaxies with companions are more frequent in poor clusters in comparison to the field or rich clusters, indicating that once the local velocity dispersion exceeds a critical value the pairing is no longer favorable. Numerical simulations (*e.g.*, Jones and Efstathiou, 1979), and studies of paired galaxies (Karachentsev, 1990, and references therein) also indicate enhanced frequency of galaxy interactions for equally massive and similar galaxies.

If galaxies are formed without large peculiar velocities, that is, in regions of low kinetic temperature, then it is natural to suppose that in many cases a galaxy and its nearest neighbor will form a bound pair. Gravitational interactions will cause the galaxies to fall together and collide; if the collision is close enough to a head-on direction then mergers will occur. Since the first simulations of galactic interactions it was noticed that the rapidity of the merging process decreases sharply with increasing relative velocity and distance of closest approach between the galaxies, and that the probability is at a maximum for equally massive and similar galaxies (*e.g.*, Alladin and Narasimhan, 1982). But, as indicated by the results of Chatterjee, 1992 (hereafter Paper I) the frequency of close encounters between galaxies is extremely low. Toomre (1977) estimates the observational value by identifying 11 candidates in which a galaxy merger is in progress out of 4000 odd NGC galaxies, giving a frequency of 0.3%–0.1% for a galaxy merger in the present epoch.

In Paper I, MBCs were not studied, as in any numerical scheme the perturber does not actually escape but recedes to an enormous distance; we propose to study the frequency of such collisions and their dynamical implications in this paper. As mentioned before the analytical results of Magalinsky and Chatterjee (2000), indicate that galaxy pairing favors high eccentricities on account of a tendency of the system to minimize its energy; we also intent to throw light on low energy orbits and test numerically their eccentricities and orbital evolution, using simple methods. In particular we expect the orbital eccentricities to almost maintain their values for a long time, decreasing extremely slowly; very late circularization is expected. We use the method of impulsive approximation, as the MBCs which we study have near parabolic orbits and so will not be affected by not using *n*-body simulations; moreover as our aim is the study of many encounters with a view of determining order of magnitude statistical results on the orbital shape and the frequency, this is the most suitable method. In dense regions, where collisions are common, the halos are expected to be truncated (*e.g.*, Ostriker, Peebles and Yahil, 1974; Richstone, 1976; Krumm and Salpeter, 1977; Storm and Storm, 1978); thus it is sufficient to use a model with a small implicit halo for the galaxies. As the merger theory postulates that the progenitors of elliptical galaxies are spiral galaxies, we study MBCs between spiral pairs. The planes of the two disks are always mutually parallel and normal to the trajectory of the collision. The results of MBCs between elliptical pairs and spiral – elliptical pairs are also studied to throw light on the results of Magalinsky and Chatterjee, 2000. Their frequency is found to be less than that of spiral pairs.

THEORY

We modify the method developed in Paper I (to which we refer for the details and references). Either of the two spirals is modeled as an exponential model disk, given by,

$$\sigma(r) = \sigma_c \exp[-4r/R] \tag{1}$$

where $\sigma_c = M_D/(2\pi R^2 b)$ is the central density, M_D and R are the mass and radius of the disk (b is a constant pertaining to the density distribution of the disk). The disk is thickened by a method indicated by Rohlfs and Kreitchmann (1981) and a polytropic bulge, containing a third of the mass of the galaxy as a superposition of polytropes $n = 0, 3, 4$ (with the mass equally distributed amongst the components), is superposed on it. We model the elliptical galaxy identically as the bulge of the spiral. In the case of spiral – elliptical pairs, the trajectory of the collision is taken to be a normal on-axis but off center trajectory of the elliptical galaxy with respect to the disk of the spiral, and the elliptical galaxy is taken to be the perturber.

The Lagrangian equations are used to determine the relative orbit (of the perturber, of mass M_p, about the test of mass M_t),

$$dt = [(2/\mu)\{E - W(r) - L^2/(2\mu r^2)\}]^{-0.5}\ dr \tag{2}$$

$$d\theta = [L/(\mu r^2)]dt \tag{3}$$

μ, L, E and $W(r)$ being the reduced mass, angular momentum, energy of orbital motion and the instantaneous potential energy of interaction between the two galaxies; the last one being given by,

$$W(t) = W(r) = -(G\ M_p M_t/R)\chi(r) \tag{4}$$

$\chi(r)$ being the value of the mutual potential energy function for the pair in question that takes the extended nature of the galaxies into account; (*e.g.*, Chatterjee, 1987). The relative orbit is determined by numerical integration; the instantaneous relative velocity being derived from,

$$V(t) = dr/dt = [(2/\mu)\{E_i - W(t) - \Delta U_p(t) - \Delta U_t(t)\}]^{0.5}\ dt \tag{5}$$

where $\Delta U_p(\mathbf{t})$ and $\Delta U_t(t)$ are the instantaneous internal energy changes of the perturber and test galaxy, respectively, and E_i is the initial value of the orbital energy, E. $\Delta U\mathbf{t}(t)$ is determined by dividing the test galaxy into shells or rings of stars, each shell or ring being characterized by a common distance r' (in terms of the radius of the galaxy) from the center of the galaxy; the increase in kinetic energy in the time interval t, in the impulsive approximation, is,

$$\Delta U_t(t) = \int_0^1 (1/2)\langle[\Delta v'(r')]^2\rangle(dM_t/dr')dr' \tag{6}$$

where, $(1/2)\langle[\Delta v'(r')]^2\rangle$ is the average value of the increase in internal energy, per unit mass, for the representative stars at distance r' from the center of M_t, in the time interval t. $\Delta U_p(t)$ is found by interchanging the galaxies and integrating the relative motion in the vicinity of minimum separation (for details see Paper I).

We study many collisions between galaxies of equal dimensions and mass. We characterize the encounters by the initial closest approach between the pair components, p (normalized to the radius of the test galaxy), and the relative velocity therein, $V(p)$ (normalized to the escape velocity therein). We begin the study of the orbit near the pericentric approach, choosing the initial position and relative velocity such that the maximum relative velocity attained by the galaxies due to tidal acceleration corresponds to the value of the escape velocity at the pericenter, $V(\max) \approx V_e(p)$, before dynamical friction is sufficient to cause a decrease of the relative velocity and the minimum separation, p (of the initial orbit), is near the value desired. This is achieved by an iteration procedure. In case the galaxies converge towards a second encounter, we study the orbital evolution to merger and give the merger timescale (T_{merge}). Time is measured in dimensionless units, normalized to the circular orbital velocity corresponding to the radius of the test galaxy; the origin of time is reckoned from the instant when the galaxies begin to feel their mutual influence (as indicated by the beginning of a convergence in the trajectory of the perturber). We maintain $V(\max) \approx V_e(p)$ (implicitly) and vary p (implicitly) up to about $2R$, as dynamical friction is insufficient to cause a convergence of the relative orbit beyond about $2R$. As our main aim is an order of magnitude frequency determination, we extract the critical values of these parameters [p and $V(p)$] corresponding to merger in an enormous timescale and escape.

We choose the dimensionless system of units $M = 1$, $R = 1$ and $G = 4.50 \times 10^{-4}$, such that a translation to physical units for the specific collision, $M_t = M_p = M = 10^{11}$ solar masses, $R = 10$ kpc, gives the unit of velocity as 1.0×10^3 km/s.

We have not considered the different mutual orientations of the planes of the disks as well as different inclinations of their planes to the direction of relative motion. We consider only collisions in which the direction of relative motion is along the axis of the disk and the planes of disks are mutually parallel; that is normal on-axis, off-center collisions. It is extremely difficult to consider all possible trajectories during disk–disk collisions, as the parameter space becomes enormous. To make an order of magnitude estimate of the frequency of mergers, it is not essential to investigate the entire parameter space; but it is sufficient to select a typical orientation and trajectory of relative motion, which are typical averages in terms of mergers. We selected this trajectory and mutual orientation of the disks because we found it to be typical in terms of merger time in comparison with a few other mutual orientations of disks and trajectories of collisions. Results of Farouki and Shapiro (1982) also indicate that for such an orientation the merger is affected in an average time, neither too great nor too less, as compared to other collisions (see Figs. 3 and 4 in the mentioned article; our collision corresponds to case 4).

RESULTS AND DISCUSSION

We give our results schematically in Table I; the columns being serial number, pericentric distance in units of the common radius of the galaxies (and in brackets in units of the median radius R_h, of the test galaxy), the eccentricity acquired by the initial relative orbit, the merger time in dimensionless units (and in brackets scaled to the mentioned typical collision) and comments, respectively. In general we can define a MBC by two important characteristics – the timescale and the apocentric distance acquired by the initial orbit, and merger in an enormous timescale and an apocentric distance greater than of the order of the distance between neighboring galaxies in dense regions.

Previous work on merging galaxies does not focus explicitly on marginally bound orbits, as the interest was directed to collisions giving rise to mergers in compatible timescales. We find it convenient to compare our results with Farouki and Shapiro, 1982, as the parameters which characterize each collision are similar in both simulations. They classify collisions in which, after the closest approach, the galaxies continue to recede even after attaining a distance of $6R$ as "escape", though the "orbit after the encounter may be bound and hence would eventually lead to a merging given sufficient time". They give their results graphically in terms of the pericentric separation and the velocity therein; from these graphical results we find that one of their collisions corresponds to $p \approx 4.8\ R_h$ and $V(p) \approx V_e(p)$. This collision is termed "escape", implying that the galaxies are likely to converge towards a second

TABLE I Spiral Pairs Encounters.

No.	*P*	*e*	T_{merge}	*Comments*
1	≈ 2.01 (5.6)	1.04	–	Slow escape
2	≈ 1.90 (5.3)	0.93	474 (5.30×10^{10})	Merger in enormous timescale
3	≈ 1.81 (5.1)	0.90	90 (9.24×10^9)	Merger in enormous timescale
4	≈ 1.72 (4.8)	0.60	19 (1.94×10^9)	Slow merger

encounter after attaining an apocentric distance $\geq 6R$. We find that this collision is very similar to our collision No. 4 for spiral pairs (see Tab. I) in which the progenitors converge towards a second encounter at an apocentric distance $r_a \approx 6.5R$.

If we take these collisions into account and conduct the frequency determinations of merging galaxies, as in Paper I, under favorable conditions, assuming that in dense regions the average distance between neighboring galaxies is $p_\infty \sim 10R$ (R being the average radius of a galaxy) (*e.g.*, Ogorodnikov, 1965; Mitton, 1977), and take the range of variation of the initial relative velocity to be from 0 to $4V_e$, as collisions with initial velocity greater than four times the escape velocity seldom occur, then the probability of a galaxy merger is given by,

$$P = [(r_p)_m/p_\infty]^2 \cdot [V_e/4V_e]^3 \tag{7}$$

where $(r_p)_m$ is the maximum value of the initial minimum separation for which the merger, in an enormous timescale, still takes place. This gives the value of the expected frequency of merging galaxies as $\sim 0.056\%$ for disk-disk progenitors, taking $(r_p)_m \approx 1.9R$. This indicates that taking these marginally bound collisions into account, the expected frequency of merging galaxies is $\sim 0.056\%$ of all galaxies, when measured with respect to densely populated regions of the Universe where galactic mergers are common. Hence assuming that $\sim 10\%$ of all galaxies are ellipticals, $\sim 0.56\%$ of all ellipticals can be expected to be merger remnants.

The results of collisions between elliptical pairs and spiral-elliptical pairs are given in Tables II and III. We find $(r_p)_m \approx 1.6R$ for elliptical pairs and spiral-elliptical pairs in collision. Using relation (7) this gives the frequency of mergers as $\sim 0.04\%$ for elliptical and spiral-elliptical pairs.

We have studied the orbital evolution of these collisions. Tables Ia, IIa and IIIa give the orbital evolution of the eccentricities for the collisions corresponding to Tables I, II and III. The first column gives the serial number of the collision corresponding to the previous tables;

TABLE II Elliptical Pairs Encounters.

No.	*P*	*e*	T_{merge}	*Comments*
1	≈ 2.00 (6.3)	1.08	–	Abrupt escape
2	≈ 1.71 (5.1)	1.04	–	Slow escape
3	≈ 1.61 (5.3)	0.91	110 (1.18×10^{10})	Merger in enormous timescale
4	≈ 1.49 (5.0)	0.90	66 (6.66×10^9)	Merger in enormous timescale
5	≈ 1.40 (4.7)	0.70	35 (3.82×10^9)	Slow merger
6	≈ 1.31 (4.3)	0.65	29 (2.28×10^9)	Slow merger
7	≈ 1.21 (4.0)	0.56	15 (1.30×10^9)	Relatively fast merger

TABLE III Spiral-Elliptical Pairs Encounters.

No.	*P*	*e*	T_{merge}	*Comments*
1	≈ 1.71 (4.8)	1.04	–	Slow escape
2	≈ 1.61 (4.5)	0.90	102 (1.06×10^{10})	Merger in enormous timescale
3	≈ 1.50 (4.2)	0.67	30 (3.60×10^9)	Slow merger
4	≈ 1.41 (3.9)	0.59	19 (2.16×10^9)	Relatively fast merger

the second column gives the pericentric distance; the next two columns give the evolution of the eccentricity as a function of time; and the last column comments on the encounter.

We find that the MBCs leading to a merger in an enormous timescale maintain their eccentricities for a very long time, until they have a spiraling infall to merger. Let us consider the extreme MBC, collision No. 2 in Table Ia. Notice that this collision is on the verge of escape. (The distinction between merger in an enormous timescale and slow escape in an expanding orbit is quite controversial as a very slight change in dynamical friction can cause a change from one domain to another. This collision illustrates how a collision on the verge of escape can lead to a merger in favorable situations.) We find that for this collision there is hardly any change in eccentricity during the first two orbital periods (the average value being 0.9). After executing the first two orbits of enormous proportions, the orbit shrinks rapidly due to dynamical friction and the orbital eccentricity falls to 0.63 for the third orbit followed by circularization in the next orbit and infall to merger in about of a 1/30 of the fifth orbital period. Notice that the eccentricity almost maintains its initial value from $T = 0$ to $T \approx 400$, and falls to a value of 0.78 at $T \approx 460$, after which there is a spiraling infall to merger in which the eccentricity virtually cascades to circularization. This is such that the eccentricity begins to fall at about the end of the second orbit and at about the end of the third orbit there is a spiraling infall to merger. Thus the high eccentricity state in which the tidal capture occurs is maintained for more than 4/5 of the entire orbital evolution and the cascading fall of eccentricity leading to spiraling infall to merger occurs only after 97% of the orbital

TABLE Ia Orbital Evolution of Spiral Pairs.

No.	*T*	*e*	*Comments*
1	0	1.04	Slow escape in expanding spiral trajectory
2	0	0.93	
	100	0.93	
	200	0.93	
	300	0.93	Extreme MBC with an initial orbit of enormous proportion;
	400	0.92	the very high initial eccentricity is almost maintained,
	420	0.90	until an infall to merger in an exceptionally enormous
	430	0.88	timescale. This collision is on the verge of escape and
	440	0.87	may not result in a merger if conditions are not favorable
	450	0.85	
	460	0.78	
	470	0.11	
3	0	0.90	
	40	0.90	
	50	0.88	
	60	0.85	MBC in which the initial high eccentricity is almost
	70	0.83	maintained until an infall to merger in an
	80	0.76	enormous timescale
	85	0.68	
	87	0.60	
	88	0.15	
4	0	0.60	
	16	0.57	The merger is relatively quick amongst slow mergers.
	17	0.39	It is a transitional case between slow and quick mergers.
	18	0.16	The initial eccentricity is moderate and falls

evolution. For MBC collision No. 3 in Table IIa the eccentricity falls very little up to $T \approx 90$, and falls to a value of 0.68 at $T \approx 100$, after which there is a spiraling infall to merger in which the eccentricity virtually cascades to circularization. Here the eccentricity starts falling at about the end of the second orbit and the circularization is achieved in the fifth orbit followed by infall to merger in about of a third of the sixth orbit. Thus the high eccentricity state in which the tidal capture occurs is maintained here also for more than 4/5 of the entire

TABLE IIa Orbital Evolution of Elliptical Pairs.

No.	*T*	*e*	*Comments*
1	0	1.08	Abrupt escape
2	0	1.04	Slow escape in expanding spiral trajectory
3	0	0.91	MBC with an initial orbit of enormous proportion; the high eccentricity is almost maintained until an infall to merger, in an enormous timescale
	20	0.91	
	40	0.90	
	50	0.88	
	60	0.86	
	70	0.85	
	80	0.81	
	90	0.79	
	100	0.68	
	103	0.65	
	104	0.48	
	108	0.13	
4	0	0.90	Very slow merger. The eccentricity falls off more rapidly and the merger timescale is less extreme compared to extreme MBCs
	20	0.88	
	30	0.85	
	40	0.77	
	50	0.72	
	55	0.68	
	62	0.46	
	64	0.11	
5	0	0.70	Slow merger. The initial orbit is of normal proportions and the initial eccentricity is not very high and falls off slowly
	22	0.68	
	25	0.63	
	27	0.58	
	30	0.52	
	32	0.26	
	33	0.07	
6	0	0.65	Slow merger. The initial orbit is of normal proportions and the initial eccentricity is not very high and falls off slowly
	17	0.63	
	18	0.60	
	19	0.55	
	20	0.50	
	22	0.41	
	28	0.08	
7	0	0.56	Transitional case between slow and quick mergers
	12	0.43	
	13	0.19	
	14	0.02	

orbital evolution, and the cascading fall of eccentricity leading to spiraling infall to merger occurs only after 91% of the orbital evolution. For the MBC collision No. 2 in Table IIIa the eccentricity falls very little up to $T \approx 75$, and falls to a value of 0.55 at $T \approx 98$, after which there is a spiraling infall to merger in which the eccentricity virtually cascades to circularization. Here the eccentricity starts falling at the beginning of the second orbit and the circularization occurs in the fourth orbit followed by infall to merger in about a fifth of the fifth orbit. Thus the highly eccentric state in which the tidal capture occurs is maintained for about 3/4 of the entire orbital evolution and the cascading fall of eccentricity leading to spiraling infall to merger occurs only after 96% of the orbital evolution. Thus high eccentricity states are favored. This supports our previous work (Magalinsky and Chatterjee, 2000), on the analytical study of orbital evolutionary trends in synchronous binary motion, using a condition which favors minimization of energy of the system, wherein we found that if we consider binaries to be subsystems of a microcanonical ensemble, after tidal capture the equilibrium thermal distribution function favors high eccentricities.

On the other hand if we consider a slow merger like collision No. 6 in Table IIa, we find the initial eccentricity is moderate and falls off comparatively quickly. In the first orbital period its average value is 0.6, in the second 0.4, followed by circularization in the third and spiraling infall to merger in about a third of the next orbital period. Here the initial orbit is of normal proportions and starts shrinking rapidly due to dynamical friction; in the initial orbit itself the eccentricity decreases substantially followed by more rapid decrease of the same in subsequent orbits. If we consider a relatively faster merger like collision No. 7 of Table IIa, we find the orbit is of comparatively small proportions and the merger is affected

TABLE IIIa Orbital Evolution of Spiral-Elliptical Pairs

No.	*T*	*e*	*Comments*
1	0	1.04	Slow escape in expanding spiral trajectory
2	0	0.90	
	20	0.90	
	40	0.85	
	60	0.83	
	70	0.82	
	75	0.80	MBC with an initial orbit of enormous proportion; the high eccentricity is almost maintained, until an infall to merger, in an enormous timescale
	80	0.79	
	85	0.74	
	90	0.72	
	95	0.62	
	98	0.55	
	101	0.12	
3	0	0.67	
	25	0.60	
	27	0.61	Slow merger. The initial orbit is of normal proportions and the initial eccentricity is not very high and falls off slowly
	28	0.47	
	29	0.12	
4	0	0.59	
	12	0.55	
	15	0.45	Transitional case between slow and quick mergers
	17	0.28	
	18	0.03	

in a timescale which is very short compared to MBCs; the eccentricity falls off comparatively quite rapidly. In the initial orbit its average value is 0.6, in the second 0.4 followed by circularization in the next and infall to merger in the about a fourth of the subsequent orbital period.

Chengalur, Salpeter and Terzian (1993, 1994, 1995, 1996) find that, for close pairs as for wide pairs, the velocity difference distribution has a small median value. The low impact parameter and slow velocity suggests that close pairs have orbits similar to wide ones and thus indicates, as proposed by them, that high eccentricity orbits do not circularize easily and that high eccentricity pairs remain eccentric and go through gradually shrinking radial bounces and evolve to eccentric close pairs. Our results support their contention as is evident from the orbital evolution of the MBCs discussed above. For example for collision No. 2 of Table Ia, we find that the initial eccentricity of 0.93 falls to only 0.88 in the second orbit and the orbit has shrunk to about 4/5 of its initial size. For collision No. 3 in Table IIa, the initial eccentricity of 0.91 falls to 0.81 in the second orbit and the orbit has shrunk to about 1/3 of its initial size. For collision No. 2 in Table IIIa, the initial eccentricity of 0.90 falls to 0.82 in the second orbit leading to shrinkage of the initial orbit by a factor of about 1/3. On the other hand for the relatively quick merger in collision No. 7 in Table IIa, the initial eccentricity of 0.56 falls to 0.19 in the second orbit leading to a shrinkage of the initial orbit to about 1/25. Thus it is evident that the collision corresponds to a spiraling infall to merger, in which the initial state of high eccentricity is missing.

From the tables, two types of encounters are evident: the MBCs characterized by initial orbits of enormous proportions with an extremely slow rate of fall in eccentricity until an infall to merger in a spiraling trajectory of rapidly decreasing eccentricity in an enormous timescale or slow escape in an expanding spiral trajectory; and the slow mergers characterized by orbits of smaller proportions with an infall to merger in a spiraling trajectory of gradually decreasing eccentricity is a much reduced timescale. Thus in slow mergers the state characterized by high eccentricity is missing and they mimic the final stages of the merger in MBCs. But, as mentioned in the introduction and reinforced by the frequency determinations, MBCs are far more abundant than the collisions leading to slow mergers or quick mergers (see Paper I for quick mergers). This seems to indicate that MBCs are the collisions preferred by nature in an effort towards energy minimization and close encounters are a chance perturbation process. Thus the state characterized by high eccentricities is a preferred state and binary systems tend to linger in that state. Our results also support the contention of Ostriker and Turner (1979) that, as close pairs merge, there is a feedback by captures at high separations (MBCs), due to the overwhelmingly high frequency of MBCs as compared to close encounters.

These results indicate that only a fraction of a percentage of ellipticals may be expected to be merger remnants. The existence of different types of ellipticals supports these results. Ellipticals and bulges of spirals are hot components (high velocity dispersions compared to the virial velocity), while disks of spirals are cool components; it is more likely that these hot components were formed by gravitational collapse than reheating of cool disk mergers. The properties of ellipticals, from giants to compacts, vary smoothly with mass, *e.g.*, rotation, degree of velocity anisotropy, radio properties; with increasing mass, ellipticals become more anisotropic, surface brightness declines and the frequency of faint stellar disks decreases. All types of ellipticals define fundamental planes with small offsets between classes (*e.g.*, Dressler *et al.*, 1987; Kormendy and Djorgovski, 1989). A deeper physical significance is provided by principle component analysis of mixed samples of galaxies, where we get indications that two dimensionality is probably universal for all galaxies, as spirals and ellipticals seem to have similar principle component solutions; spirals and ellipticals even seem to lie on the same fundamental parameter plane. A calculation of the correlation coefficients after principle component analysis of mixed samples indicates that one factor determines the global structure of the ellipsoidal component, whether they are ellipticals or bulges of spirals and the other factor determines the global structure of the disk component (*e.g.*, Watanabe, Kodaira and Okamura, 1985; and references therein). Neither dwarf ellipticals nor dwarf spheroidals can be produced by mergers (*e.g.* Kormendy, 1989); the core densities of dwarf ellipticals (like M32 or NGC 3377) are extremely high and to form them by mergers of spirals requires an increase of central density by $\sim 10^4$. On the other hand dwarf spheroidals are extremely diffuse and have anomalously low binding energies (*e.g.*, Writh and Gallagher, 1984). Also, as spirals of the present epoch have gas to star ratios smaller than 1/5 and most stars show solar element ratios, ellipticals cannot be produced (in a massive scale) by the mergers of spirals, in the present epoch. The specific frequency of globular clusters in ellipticals is an order of magnitude higher than in spirals (*e.g.*, van den Bergh, 1980, 1984); also the specific frequency of globular clusters is higher in cluster ellipticals as compared to field ellipticals.

The merger theory does not account for color gradients seen in most elliptical galaxies (*e.g.*, Sandage and Visvanathan, 1978). The central velocity dispersion plays the prominent role in determining the stellar population of ellipticals. Metalicity increases with luminosity among ellipticals (*e.g.*, Faber, 1973), as best displayed by the Magnesium abundance to central velocity dispersion relation ($Mg_2 - \sigma$), which is obeyed from bright ellipticals to the faint dwarf spheroidals. The Mg_2 indices for the central part of the ellipticals are correlated, one to one, with the central $(B - V)_o$ colors (Burstein *et al.*, 1988); the tight $Mg_2 - (B - V)_o$ relationship implies that the stellar populations in the centers of ellipticals are strongly coupled to the entire stellar population throughout the system. As different types of ellipticals follow this relationship, there is a great degree of homogeneity among the stellar populations of these systems. If the central velocity dispersion, σ_o, is assumed to be an indicator of the mean potential well, then the increase in metalicity with σ_o implies that the deeper potential wells retain more gas and hence more heavy elements; and thus the local metalicity is determined by the local escape velocity (Franx and Illingworth, 1990). But in the merger scenario, the remnant of cumulative mergers cannot maintain a link between local escape velocity and local metalicity. On the other hand Schweizer *et al.* (1990, 1992) find that some morphologically peculiar galaxies are bluer than normal ones; the Mg_2 index is weakened and the weakness correlates positively with the degree of peculiarity; this only indicates that recent interactions trigger starbursts that cause the blueing and dilute Mg_2. In such galaxy, interactions, not necessarily leading to mergers, have played a prominent role. Paired galaxies display higher star formation rates and infrared emissions (*e.g.* Sulentic, 1989; Keel, 1991). Kennicutt *et al.* (1985) find from a large sample of interacting spirals and irregular galaxies and a control sample of isolated galaxies (using Hα emission and far-infrared observations) that interactions constitute a small percentage ($< 15\%$) of active systems; star formation burst models involving $< 2\%$ of the total mass of a galaxy can reproduce the observed Hα equivalent widths and broadband colors. Galactic interactions are common and the most frequent hyperbolic fly-bys will induce star formation.

An optical-infrared study of clusters in the redshift range $z = 0.5$ to 0.9 was conducted by Aragon-Salamanca *et al.* (1993); from color–magnitude diagrams of ellipticals a blueing trend with increasing redshift is noticed. By examining a uniform subset of early type galaxies in rich clusters it was found (O'Connell, 1998) that in the rest frame color-luminosity plane a red envelope exists (populated by ellipticals drawn mostly from the cluster core), whose spectral properties are almost unchanged upto $z \sim 1/2$. Aragon-Salamanca *et al.* tracked the evolution in this envelope. Color distribution studies for clusters at different redshifts show that the red envelope in clusters moves substantially bluewards with increasing redshift, indicating significant color evolution. They also find uniform evolutionary trends from cluster to cluster (for a given z) indicating that the stellar population of ellipticals formed within a narrow time interval. The monotonic trends in the observed colors suggest a coeval homogeneous population; using population synthesis evolutionary models (Bruzaual, 1983), they find an early epoch for the initial burst of star formation in ellipticals, indicating a formation epoch of $z_f > 2$. An upper limit is highly model and cosmology dependent, but $z_f < 5$ is indicated (corresponding to ages greater than $\sim 10^{10}$ years). Such high redshifts of formation are also indicated by the low evolution of mass to light ratios (M/L) of ellipticals with redshift (van Dokkum and Franx, 1996); (the M/L ratio of the distant cluster C1 $0024+16$ ($z=0.39$) is only $\sim 30\%$ lower than that of the Coma ($z=0.023$), a factor expected from the passive evolution of stellar populations alone), as well as the low redshift evolution of the bright end of the galaxy luminosity function (Lilly *et al.*, 1995). Studies of spectral energy distribution evolution of ellipticals by Broadhurst and Bouwens (2000), using optical-infrared images of Hubble deep fields, indicate that the mean spectral energy distribution evolves passively (toward a mid-F star dominated spectrum) upto $z \sim 2$.

Bender *et al.* (1993) found, from the observed scatter in the Magnesium abundance to central velocity dispersion relation ($Mg_2 - \sigma$) and using stellar population synthesis models (*e.g.*, O'Connel, 1986; Bruzual and Charlot, 1993; Worthy, 1994), that the combined scatter in age and metalicity for luminous ellipticals, at a fixed σ is smaller than 15%. Measurements of $Mg_2 - \sigma$ relation at different redshifts indicate that at a given σ the strongest Mg absorption is found at $z=0.4$ and is significantly weaker at $z=0$, indicating an evolution of stellar populations with redshift for ellipticals. Translating this difference into relative age differences, using stellar population synthesis models (*e.g.*, Worthy, 1994), implies that the bulk of the star formation in luminous ellipticals occur at $z > 2$. This obviously implies that ellipticals have not formed continuously over the Hubble time; consequently the merger process is a sporadic one. Stranford, Eisenhardt and Dickson, 1998, from an extensive study spanning a very large redshift range, find that the colors of combined early type galaxies in cluster cores show changes only expected from passive spectral evolution out to $z \sim 1$. De Propris *et al.* (1999) determine the characteristic infrared luminosity evolution model in which the main star formation episode occurs at $z_f \sim 2$–3 and the assembly of the galaxies is almost complete by $z \sim 1$.

The projected density profile power laws r^{-2} for merger remnants and ellipticals coincide, but to maintain the mass-metalicity relation this merging must involve protogalactic clumps (*i.e.*, before the epoch of star formation). The morphology density relationship (*e.g.*, Dressler, 1980; Postman and Gellar, 1984), indicating that early type galaxies dominate high density regions and spirals low density regions, does not necessarily support the extreme form of the merger theory as collision cross sections are lower in high density regions. As this relationship is a very slow function of density, it could be indicative of the dependence of disk growth on local density; Ostriker (1977) finds that the mass of the intercluster gas in Coma is about what would remain if disks of the size of field spirals did not form. Also a field protospiral which has evolved to a spiral, almost without collisions, should be older than ellipticals; but field spirals are bluer than spirals in the cores of nearby clusters (Butcher and Oemler, 1984). These discrepancies arise because, if the merger theory is invoked to account for the formation of ellipticals (instead of only their evolution), the assumption that spirals are the fundamental blocks must follow to account for the morphology–density relationship.

The main discrepancies come when the merger theory is extrapolated from mergers of fully formed galaxies to mergers of protogalactic clumps and the epoch of peak merger rates extrapolated back to the epoch of galaxy formation, and the present mergers are considered as the extremity of the process. These are two different processes because protogalactic mergers are markedly different from those of fully formed galaxies, as they form an integral part of the galaxy formation processes involving violent collisions of gaseous clumps.

The traditional view of the formation of ellipticals (*e.g.*, Eggen, Lynden-Bell and Sandage, 1962) involving a single burst of star formation at a high redshift followed by passive evolution, is more likely than hierarchical cosmological merger driven growth of ellipticals by a process of gradually merging smaller stellar systems over a wide range of redshifts (*e.g.*, Cole *et al.*, 1994) or the hybrid approach to galaxy formation (where dissipative collapse and merger of subunits function simultaneously). Though a negative luminosity evolution of early type field galaxies (deficit of bright, high redshift field galaxies) has been sighted as evidence supporting hierarchical models (*e.g.*, Kauffmann and Charlot, 1998), the stellar mass growth of a factor ~ 4 to 5 in BCMs in massive clusters since $z = 1$ to the present epoch, predicted in such models is not detected from an analysis of the K-band Hubble diagram of BCMs (Bruke, Collins and Mann, 2000); whence a growth factor ~ 1.5–2 is detected.

The percentage of ellipticals in which mergers have played a critical role in their secular evolution are the BCMs, as displayed by signs of ongoing mergers. The great majority of the BCMs are of the D or cD type; the main characteristic of D type galaxies is that their surface brightness profiles are more extended than giant ellipticals, such that they are enlarged for a given surface brightness and the slope of their profile is less at the plate limiting magnitude ($V \approx 24$ mag sec^{-2}). A cD has an extra envelope and nuclei of merging galaxies are generally visible (*e.g.*, see Dressler, 1984, for a review). Their luminosities are too bright to be consistent with the luminosity function of other ellipticals. These galaxies are positioned on local cluster density enhancements (Beers and Geller, 1983); they have a much lower rms velocity with respect to the cluster mean in comparison to other galaxies (Quintana and Lawrie, 1982); thus they are almost stationary at the bottom of the cluster potential well. The envelope of a cD can be modeled by a two component model of a galaxy potential on top of a more extended cluster potential (*e.g.*, Schombert, 1988), indicating that the cluster potential plays a role in sustaining the envelope. Several factors indicate that these galaxies are very likely to have evolved secularly due to cumulative mergers. In general, low luminosity BCMs have low velocity merging companions, but high velocity BCMs do not; suggesting that BCMs evolve until they have consumed their merging companions. BCMs span a small range in luminosity. The D galaxies fit the luminosity relation, $L \propto \sigma^{3.3}$, when any excess halo luminosity (above the $r^{1/4}$ law) is excluded (Torny, 1986). However, the properties of BCMs indicate that they do not form by mergers, but have an evolutionary history modified by mergers. For example, the envelope luminosities of these galaxies are comparable to their core luminosities, indicating that the mergers are an insufficient source to build their envelopes and suggestive of a common formation process for the core and envelope (*e.g.*, Schombert, 1988); their isophotes tend to align with the cluster major axis (*e.g.*, van Kampen and Rhee, 1990); they have colors and color gradients in their envelopes and mass to luminosity ratios similar to giant ellipticals (*e.g.*, Schombert, 1988); their projected velocity dispersion increase with radius and approach the cluster velocity dispersion (*e.g.*, Dressler, 1979).

For the fundamental plane in which BCMs lie the exponent corresponding to the kinetic temperature (σ) decreases from 2 to 1, as compared to the fundamental plane of normal ellipticals, while the exponent corresponding to the projected density (I) is hardly affected. This indicates a shallower dependence of the kinetic temperature and that the size to projected density relationship is tightened (*e.g.*, Hoessel *et al.*, 1987). The scale length (R) to surface brightness (μ) correlation shows only very marginal improvement with the introduction of σ as a second parameter, indicating that the dimensionality of the data manifold decreases to nearly one. Repeated episodes of violent relaxation (Lynden-Bell, 1967), caused by cumulative merger effects, seem to drive the BCMs toward a preferred kinetic temperature.

These results indicate that mergers of fully formed galaxies do not play a prominent role in the formation of elliptical galaxies, but only in their secular evolution.

CONCLUSIONS

The results of this paper throw considerable light on our previous work (Magalinsky and Chatterjee, 2000), on the analytical study of orbital evolutionary trends in synchronous binary motion, using a condition which favors minimization of energy of the system, where we found that considering binaries to be subsystems of a microcanonical ensemble, after tidal capture the equilibrium thermal distribution function favors high eccentricities. Here our computations indicate that the most frequent type of collision (MBCs) with high eccentricities maintain their eccentricities for a long time.

This is to be expected not only on a theoretical basis but also considerable observational evidence has surfaced in its favor; this being the detection of very wide, physical galaxy pairs in near parabolic orbits as mentioned in the introduction. Chengalur, Salpeter and Terzian (1996) find from an analysis of observational data for close and wide pairs, that if wide pairs and close pairs constitute a single population, then close pairs are formed by the radial infall of wide ones (due to dynamical friction) in which the orbital trajectory continues to be highly eccentric. The results of this paper indicate that after tidal capture in an eccentric orbit favoring energy minimization, the pair is reluctant to abandon that natural state. On the basis of these results and that of our mentioned paper it is evident that celestial bodies when they feel their mutual two-body force, fall towards each other in orbits of high eccentricities because eccentric orbits appear to drive the system towards a favored energy state in terms of energy minimization. This may be compared to the process of violent relaxation (Lynden-Bell, 1967) which leads to a preferred state.

A study of the frequency of the most frequent type of collision characterized by low energy, which in the limit are marginally bound collisions, indicate that their frequency is too low to account for the formation of ellipticals by the merger of spirals, as they can account for the formation of only $\sim 0.5\%$ of ellipticals. This, coupled with spectral evolution studies of ellipticals implying that the bulk of the star formation in luminous ellipticals occur at high redshifts, indicates that the merger process is a sporadic one, which accounts only for the evolution of ellipticals. It is the BCMs that are affected most by this phenomena, and the merger frequency indicates that BCMs should constitute $\sim 0.5\%$ of ellipticals.

Acknowledgements

It is a matter of great pleasure to thank all those with whom we had helpful discussions, especially Prof. Arcadio Poveda.

References

Alladin, S. M. and Narasimhan, K. S. V. S. (1982) *Phys. Rep.*, **92**, 339.
Aragon-Salamanca, A., Ellis, R. S., Couch, W. J., Carter, D. (1993) *MNRAS*, **262**, 764.
Beers, T. C. and Geller, M. J. (1983) *ApJ*, **274**, 491.
Bender, R., Burstein, D., Faber, S. M. (1993) *ApJ*, **411**, 153.
Bisnovatyi-Kogan, G. S. and Zel'dovich, Ya. B. (1970) *Azh*, **47**, 942 [*Sov. Astron. — AJ*, **14**, 758 (1971)].
Broadhurst, T. and Bouwens, R. J. (2000) *ApJ*, **530**, L53.
Bruzual, G. and Charlot, S. (1993) *ApJ*, **405**, 538.
Burstein, D., Davis, R. L., Dressler, A., Faber, S. M., Lynden-Bell.
Butcher, H. and Oemler, A. (1984) *ApJ*, **285**, 426.

Burstein, D., Davis, R. L., Dressler, A., Faber, S. M., Lynden-Bell, D., Terlevich, R., Wegner, G. (1988) In: Kron, R. G. and Renzini, A. (eds.) *Towards Understanding Galaxies at Large Redshifts* (Kluwer), p. 17.
Chatterjee, T. K. (1987) *Ap&SS*, **139**, 243.
Chatterjee, T. K. (1992) *Ap&SS*, **196**, 283 (Paper I).
Chengalur J., Salpeter E. E., Terzian Y. (1993) *ApJ*, **419**, 30.
Chengalur J., Salpeter E. E., Terzian Y. (1994) *ApJ*, **107**, 1984.
Chengalur J., Salpeter E. E., Terzian Y. (1995) *ApJ*, **110**, 167.
Chengalur J., Salpeter E. E., Terzian Y. (1996) *ApJ*, **461**, 546.
Dressler, A. (1979) *ApJ*, **231**, 659.
Dressler, A. (1980) *ApJ*, **236**, 351.
Dressler, A. (1984) *ARA&A*, **22**, 185.
Dressler A., Lynden-Bell, D., Burstein, D., Davis, R. L., Faber, S. M., Terlevich, R. J., Wegner, G. (1987) *ApJ*, **313**, 42.
Eggen, O. J., Lynden-Bell, D., Sandage, A. R. (1962) *ApJ*, **136**, 748.
Faber, S. M. (1973) *ApJ*, **179**, 423.
Farouki, R. T. and Shapiro, S. L. (1982) *ApJ*, **259**, 103.
Franx, M. and Illingworth, G. (1990) *ApJ*, **359**, L41.
Hashimoto, Y. and Oemler, A. O. (2000) *ApJ*, **530**, 652.
Heggie, D. C. (1975) *MNRAS*, **173**, 729.
Hoessel, J. G., Oegerle, W. R., Schneider, D. P. (1987) *AJ*, **94**, 1111.
Hut, P. (1985) In: Goodman, J. and Hut, P. (eds.) *IAU Symp. 113, Dynamics of Star Clusters* (Reidel), p. 103.
Hut, P. (1986) In: Smoluchowski, R. (Ed.) *The Galaxy and the Solar System*.
Jeans, J. H. (1929) *Astronomy and Cosmogony* (Cambridge University Press), p. 302.
Jones, B. J. T. and Efstathiou, G. (1979) *MNRAS*, **189**, 27.
Karachentsev, I. D. (1990) In: Sulentic, J. W., Keel, W. C. and Telesco, C.M. (eds.) *IAU Coll. 124, Paired and Interacting Galaxies* (NASA, Washington, DC), p. 3.
Kauffmann, G. and Carlot, S. (1998) *MNRAS*, **294**, 705.
Kennicutt, R. C., Keel, W.C., van der Hulst, J. M., Hummel, E., Roettiger, K. A. (1987) *AJ 93*, 1011.
Keel, W. C. (1991) In: Combes, F. and Casoli, F. (eds.) *IAU Symp. 146, Dynamics of Galaxies and their Molecular Cloud Distribution* (Kluwer), p. 243.
Kormendy, J. and Djorgovski (1989) *ARA&A*, **27**, 235.
Lilly, S. J., Tresse, L., Hammer, F., Crampton, D., Le Fevre, O. (1995) *ApJ*, **455**, 108.
Lynden-Bell, D. (1967) *MNRS*, **136**, 101.
Magalinsky, V. B. (1972) *Azh*, **49**, 1017 [*Sov. Astron. – AJ*, **16**, 830 (1973)].
Magalinsky, V. B. and Chatterjee, T. K. (2000). *Astron. Astrophys. Trans.*, **18**, 807.
Mitton, S. (ed.) (1977) *The Cambridge Encyclopedia of Astronomy* (Jonathan Cape, Ltd, UIK.), p. 345.
Ogorodnikov, K. F. (1965) *Dynamics of Stellar Systems* (Pergamon Press), p. 100.
O'Connell, R. (1988) In: Kron, R. G. and Renzini, A. (ed.) *Towards Understanding Galaxies at Large Redshifts* (Kluwer), p. 243.
Ostriker, J. P. (1977) In: Tinsley, B. M. and Larson, R. B. (eds.) *The Evolution of Galaxies and Stellar Populations* (Yale University Observatory), p. 369.
Ostriker, J. P., Peebles, P. J. E., Yahil, A. (1974) *ApJ*, **193**, L1.
Ostriker, J. P. and Turner, E. L. (1979) *ApJ*, **234**, 785.
Postman, M. and Geller, M. J. (1984) *ApJ*, **281**, 95.
Quintana, H. and Lawrie, D. G. (1982) *AJ*, **87**, 1.
Rohlfs K. and Kreitchmann J., (1981) *Ap&SS*, **79**, 289.
Richstone, D. O. (1976) *ApJ*, **204**, 642.
Sandage, A. and Visvanathan, N. (1978) *ApJ*, **223**, 707.
Schombert, J. M. (1988) *ApJ*, **328**, 475.
Schweizer, F., Seitzer, P., Faber, S. M., Burstein, D., Dalle Ore, C. M., Gonzalez, J. J. (1990) *ApJ*, **364**, L33.
Schweizer, F. and Seitzer, P. (1992) *AJ*, **104**, 1040.
Spitzer, L. (1987) *Dynamical Evolution of Globular Clusters* (Princeton University Press), p. 123.
Stanford, S. A., Eisenhardt, P. R., Dickson, M. (1998), *ApJ*, **492**, 461.
Storm K. M. and Storm, S. E. (1978) *AJ*, **83**, 1293.
Sulentic, J. W. (1989) *AJ*, **98**, 2066.
Toomre, A. (1977) In: Tinsley, B. M. and Larson, R. B. (eds.) *The Evolution of Galaxies and Stellar Populations* (Yale University Observatory), p. 401.
Torny, J. L. (1986) *AJ*, **90**, 243.1
van den Bergh, S. (1984) *PASP*, **96**, 329.
van den Bergh, S. (1980) *PASP*, **92**, 122.
van Dokkum, P. G. and Franx, M. (1996) *MNRAS*, **281**, 985.
van Kampen, E. and Rhee, G. (1990) In: Sulentic, J. W., Keel, W. C. and Telesco, C. M. (eds.) *IAU Coll. 124, Paired and Interacting Galaxies* (NASA, Washington, DC), p. 255.
Watanabe, M., Kodaira, K., Okamura, S. (1985) *ApJ*, **292**, 72.
Worthy, G. (1994) *ApJ (S)* , **95**, 10.
Writh, A. and Gallagher, J. S. (1984) *ApJ*, **282**, 85.