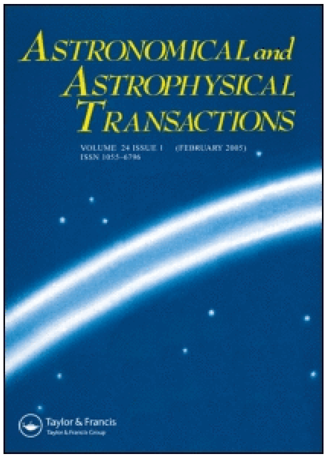

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RADIATING SHELLS AND ASTROPHYSICAL PROPERTIES

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A classical model with post-newtonian correction of X-ray burster or X-ray recurrent nova is studied.

The condition for a oscillating behaviour and the corresponding period are computed.

KEY WORDS Post-newtonian corrections, neutron stars, white dwarf stars

1 INTRODUCTION

We introduce and study a highly simplified model of X-ray burster or recurrent nova (Gallagher and Starfield, 1978; Hoffman *et al.*, 1978; Lewin and Clark, 1980; Liang and Petrosian, 1984), with spherical symmetry, in order to describe the luminosity fluctuations of these astronomical objects. The model consist of a central nucleus, a white dwarf star or a neutron star, surrounded by a gas, enclosed in a spherical dust shell, in thermal equilibrium with the gas. The shell emits blackbody radiation, and the luminosity fluctuations are caused by oscillations of a shell. Although this model is extremely simple, it predicts quite well the observational data of X-ray bursters and recurrent novae (Aquilano *et al.*, 1987). The gravity forces, the pressure of the radiation force the shell to oscillate if the relevant parameter lies between certain bounds (otherwise the shell will collapse into the white dwarf or will be ejected). We shall compute these bounds and show that, in this problem, a primary bifurcation exists when we describe the solutions in terms of the ratio of the shell mass and the gas mass.

2 THE SHELL DYNAMICS

We shall suppose that the gas mass is conserved and that the density of the gas is constant, therefore the gas density is

$$\rho(R) = \frac{3}{4\pi}\frac{m_g}{R^3}, \tag{1}$$

where m_g is the mass of the gas and R the radius of the shell. The density will obey the state equation of a perfect gas $P = \rho KT$, where P is the pressure, T the absolute temperature and K the general gases constant. We shall consider that when the shell oscillates the gas undergoes adiabatic evolutions $P = \alpha\rho^{5/3}$, where α is a parameter determined by the initial conditions of the motion.

From the previous works (Aquilano *et al.*, 1987, 1995) the shell classical equation of motion is:

$$R(t) = f(R) = \frac{A_2}{R^2} + \frac{A_1}{R^6} + \frac{A_0}{R^3}, \tag{2}$$

where A_0, A_1 and A_2 are constant (Aquilano *et al.*, 1987); the first term on the right hand side of equation is caused by the attraction of the central mass (white dwarf star), and the selfgravity of the shell; the second one is originated by the emitted radiation and the last one is the internal gas pressure. The equation shows the balance of two attractive terms (the first and the second) and the expansive term (the third).

In order to simplify these equations we introduce the following scale factors, a space factor R_0 and a time factor t_0 (Aquilano *et al.*, 1987), where R_0 is the singular point of Eq. (2) neglecting the radiation term, and t_0 is the inverse of the oscillation frequency. There is no radiation around the singular point R_0.

Using R_0 and t_0 as units of length and time respectively, the dynamical equation of the shell is:

$$x'' = f(x, x'), \tag{3}$$

where x is the radius shell and the primes are times derivations. Using classical mechanics it is easy to show that

$$f(x, x') = f_0(x, x') = -\frac{1}{x^2} + \frac{1}{x^3} - \frac{\Omega}{x^6}, \tag{4}$$

where Ω is a parameter defined by Aquilano *et al.* (1987)

$$\Omega = \frac{4}{243}\frac{b}{K^4}\frac{G^3M^2}{d}\left(\frac{m_s}{m_g}\right)^3, \tag{5}$$

where G is the gravitational constant, $b = 4\sigma/c$, σ is the Stefan–Boltzmann constant, M is the mass of the white dwarf and $d = m_g/M$.

Also if we use the post-newtonian approximation (Weinberg, 1972) we obtain the corrected function:

$$f(\delta, x, x') = -\left[1 - \frac{3}{4}\delta x'^2\right]\frac{1}{x^2} + \left[\delta + \left(1 - \frac{1}{4}\delta x'^2\right)^{1/2}\right]\frac{1}{x^3} - \Omega\left[1 - \frac{1}{4}\delta x'^2\right]\frac{1}{x^6}, \tag{6}$$

where the corrective (post-newtonian) term is

$$\delta = \frac{G}{c^2}\frac{(4\pi K)^{4/3}}{b^{1/3}}M^{2/3}\left(\frac{\Omega}{d}\right)^{1/3}\frac{1}{\alpha} = \frac{1}{\tilde{c}^2}, \tag{7}$$

where $\tilde{c}$ is the adimensionalized light velocity and c the light velocity.

In previously works (Aquilano *et al.*, 1987, 1995) we showed that the solutions of this equation fits quite well the behaviour of the astronomical objects – we studied bursters and recurrent novae – if we use reasonable physical parameters.

In this work we are interested in the study of the mathematical properties of Eq. (3) in the phases space, and to see how we can obtain a periodic luminosity with astrophysical interest.

3 PROPERTIES OF THE DYNAMICAL EQUATION

From Eq. (4) and (6) we deduce:

Property 1: If $f(\delta = 0, x, x') = f_0(x, x')$ then

$$x'' = f_0(x, x') \tag{8}$$

is the classical newtonian equation of motion of the shell.

Property 2:

Via the transformation:

$$x \to \tilde{x}, \quad \Omega \to \tilde{\Omega}, \tag{9}$$

where

$$\tilde{x} = \frac{1 - (3/4)\delta x'^2}{\delta + (1 - (1/4)\delta x'^2)^{1/2}}x, \tag{10}$$

$$\tilde{\Omega} = \frac{(1 - (1/4)\delta x'^2)^{1/2}(1 - (3/4)\delta x')^3}{[\delta + (1 - (1/4)\delta x'^2)^{1/2}]^4} \tag{11}$$

function $f(\delta, x, x')$ becomes function $f_0(\tilde{x}, \tilde{x}')$. Therefore, the singular points of Eq. (6) can be obtained solving the classical problem, i.e. (4). Besides this equation has a very simple analytical solution.

Property 3: Therefore is easy to verify that Eq. (4) has at most two real singular points, a *port* $x^-(\Omega)$ and a *centre* (a stable singular point) $x^+(\Omega)$ given by:

$$x^{\pm}(\Omega) = \frac{1}{4}(1 - g(y))[1 \pm (1 - h(y))^{1/2}], \tag{12}$$

where

$$g(y) = -(1 + 8y)^{1/2}, \tag{13}$$

$$h(y) = 16\frac{y}{g(y)[g(y) - 1]}, \tag{14}$$

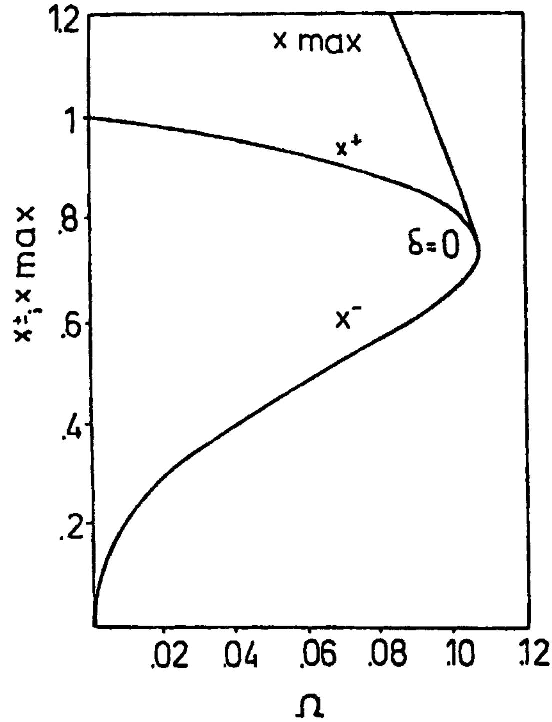

Figure 1 Singular points how Ω function, for $\delta = 0$.

$$y = \left(\frac{\Omega}{16}\right)^{1/3}\left\{\left[1+\left(1-\frac{\Omega}{\Omega_c}\right)^{1/2}\right]^{1/3}+\left[1-\left(1-\frac{\Omega}{\Omega_c}\right)^{1/2}\right]^{1/3}\right\} \tag{15}$$

and $\Omega_c = 0.10$.

Figure 1 shows the real singular points $x^{\pm}$ as a function of Ω. The parameter Ω_c is a bifurcation point for Ω, because if $\Omega > \Omega_c$ there are no real singular points and the solution of Eq. (4) yield a collapse of the shell. Solution $x^+(\Omega)$ corresponds to a *centre*, which goes to value 1 when $\Omega = 0$ (in this case there is no radiation term in Eq. (4), $x^+(\Omega_c)$ lies between 3/4 and 1 (i.e. $x^+(\Omega)$ and $x^+(0)$ respectively). And $x^-(\Omega)$ correspond to a *port*, and lies between 0 and 3/4 (i.e. $x^-(0)$ and $x^-(\Omega_c)$ respectively).

In Figures 2, 3 and 4 we show some orbits for $\delta = 0$ and different values of Ω.

In Fgure 5 (break line), we represent the bands where the acceleration keeps its sign i.e.,

* for $\Omega < \Omega_c$, $x'' > 0$ if $x \in [x^-(\Omega); x^+(\Omega)]$, and $x'' < 0$ if $x < x^-(\Omega)$ or $x > x^+(\Omega)$
* for $\Omega > \Omega_c$ is $x'' < 0\ \forall x$.

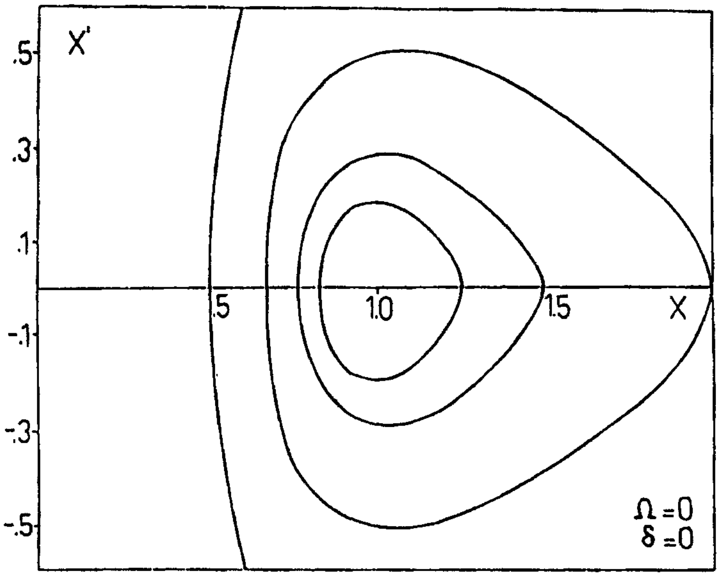

Figure 2 Phase diagram for $\Omega = 0$ (i.e. no radiation). All trajectories are stable oscillations.

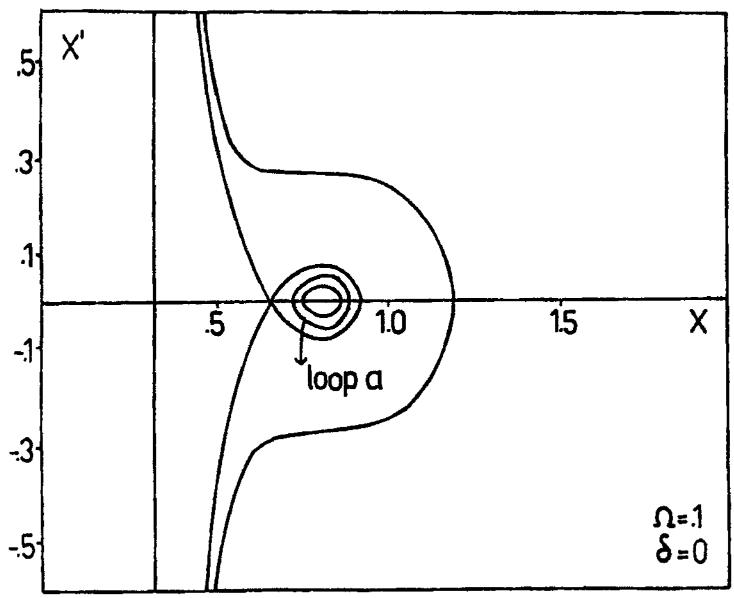

Figure 3 Phase diagram for $\Omega = 0,1$ (i.e. $\Omega < \Omega_c = 0,10$). There are two singular points x^+ and x^-. For $0,67 < 0,92$ the oscillations may occur. For x outside this interval the shell always collapse.

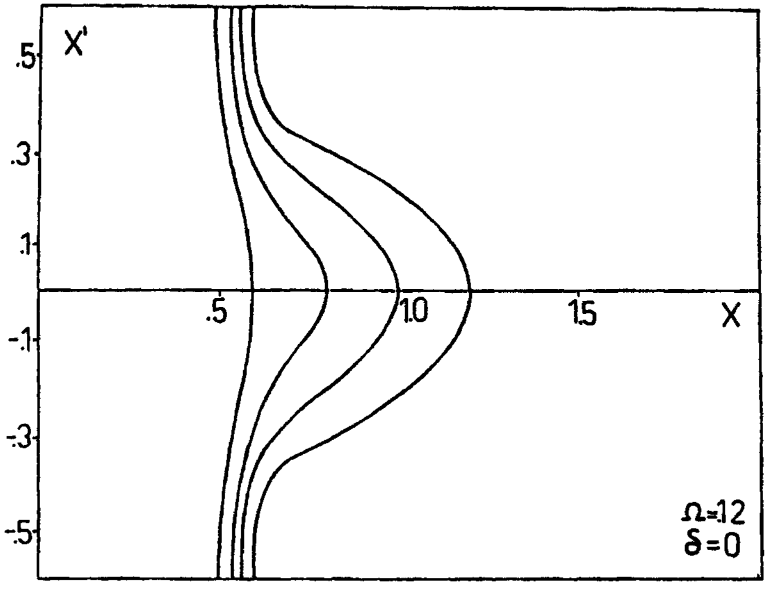

Figure 4 Phase diagram for $\Omega = 0,12$ (i.e. $\Omega > \Omega_c = 0,10$). There are no singular points. All trajectories yield the shell collapse.

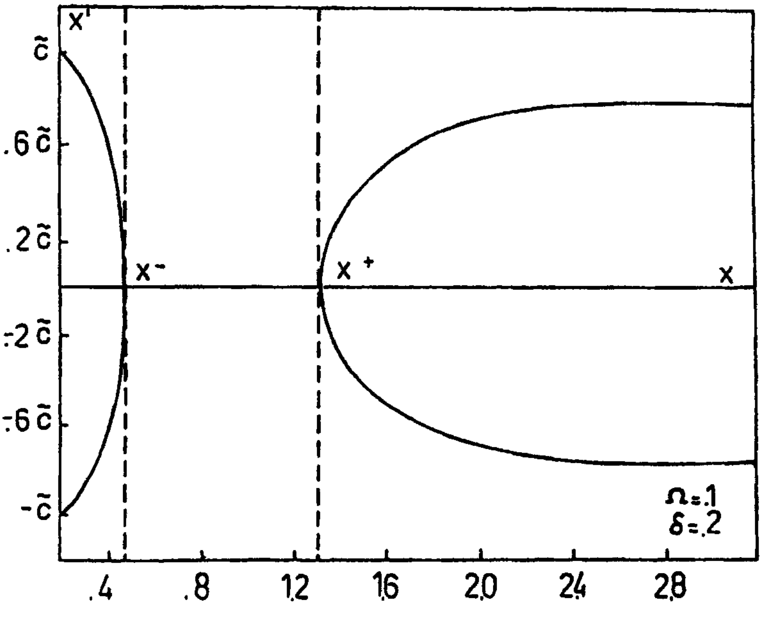

Figure 5 Representation for the bands where the acceleration keeps different signs.

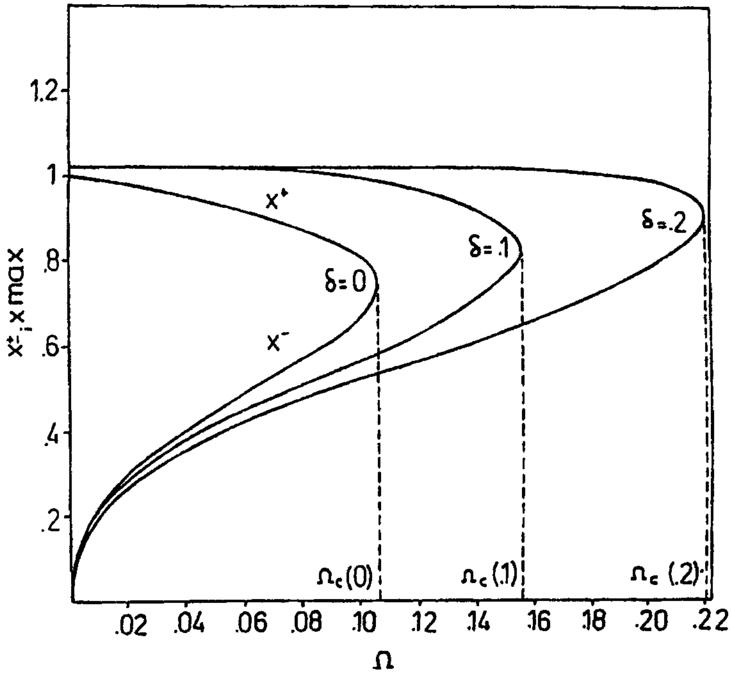

Figure 6 Singular points how Ω function, for different δ.

The bifurcation point, defined by Ω_c, yields an upper bound to the ratio m_s/m_g, if the shell oscillates

$$\left(\frac{m_s}{m_g}\right)^3 \leq \Omega_c \frac{243}{4}\frac{K^4}{\pi b}\frac{d}{G^3 M^2}. \tag{16}$$

From properties 2 and 3 we obtain,

Property 4: When $\delta \neq 0$ the real singular point of Eq. (6) are

$$x^{\pm}(\Omega, \delta) = (1+\delta)x^{\pm}(\tilde{\Omega}). \tag{17}$$

Therefore, the bifurcation point is now

$$\Omega_c(\delta) = (1+\delta)^4 \Omega_c. \tag{18}$$

In Figure 6 we represent the curves $x^{\pm}(\Omega, \delta)$ for several values of δ. In Figure 5 it is shown (complete line) the phases space for $\Omega < \Omega_c$, where the roots of x'' are defined by:

$$x^{\pm}(x', \Omega, \delta) = x^{+}(\tilde{\Omega})\frac{\delta + (1-(1/4)\delta x'^2)^{1/2}}{1-(3/4)\delta x'^2}. \tag{19}$$

Now we can compute the oscillation period:

Property 5: The oscillation period around the centre when $\delta = 0$ is

$$p(\Omega) = \frac{2\pi}{(x^{+}(\Omega))^{-1}[3x^3 - 2x^4 - 6\Omega]|_{x=x^{+}(\Omega)}}. \tag{20}$$

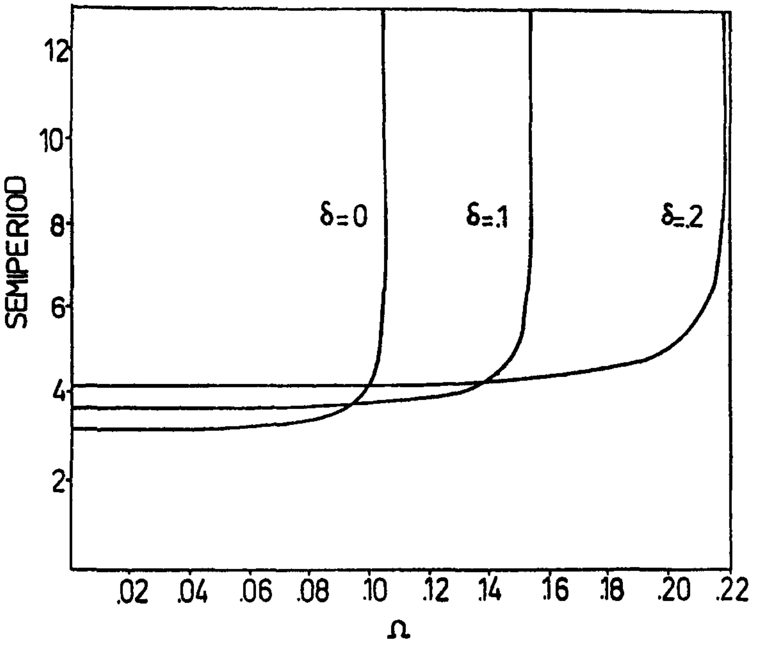

Figure 7 The semiperiod as a function of Ω, for different δ.

For $\delta \neq 0$ if we use transformations (9, 10) the corresponding time transformation is $t \to \tilde{t}$ where,

$$\tilde{t} = t\frac{(1-(3/4)\delta x'^2)^{3/2}}{[\delta + (1-(1/4)\delta x'^2)^{1/2}]^{3/2}} \tag{21}$$

from this transformation (bound around of $x^+(\Omega, \delta)$), we can obtain the period for $\delta \neq 0$.

In Figure 7 we represent the semiperiods of oscillation as a function of Ω and δ.

4 LUMINOSITY

The luminosity of a star is defined by the energy radiated by unit of time; and considering the spherical symmetry of the shell and that the radiation law obeys the vision of a blackbody, the luminosity dimensionless is

$$L(t) = \frac{1}{x^6(t)} \tag{22}$$

being $L(t) = \tilde{L}(t)/\tilde{L_0}$, where $\tilde{L}_0$ is the scale factor defined by Aquilano *et al.* (1995)

$$\tilde{L}_0 = \frac{\sigma K^4}{b^2}(4\pi)^{1/3}3^{8/3}d^{2/3}M^{2/3}\frac{\Omega}{\alpha}. \tag{23}$$

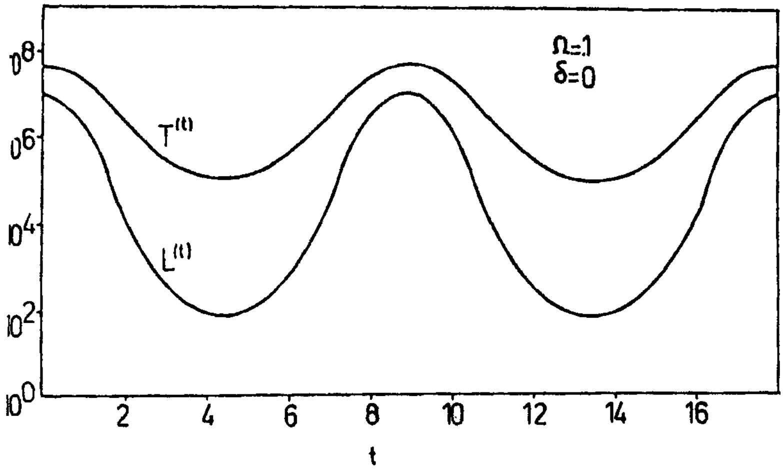

Figure 8 Dependence of the luminosity L in function of time t, and the temperature T with the time t.

In the same way we introduce the temperature,

$$T(t) = \frac{\tilde{T}(t)}{\tilde{T}_0} = \frac{1}{x^2(t)}, \tag{24}$$

where $\tilde{T}_0$ is the scale factor,

$$\tilde{T}_0 = \left(\frac{3}{b}\right)^{2/3} K^{5/3}\frac{\Omega^{2/3}}{\alpha}. \tag{25}$$

In Figure 8, the periodical fluctuation of luminosity and the shell temperature for classical oscillations around the centre have been shown ($L(t)$ and $T(t)$ correspond to loop a in Figure 3).

5 CONCLUSIONS

In this work, and in previously works (Aquilano *et al.*, 1987, 1995), we see that it is interesting to remark the coincidence between our model and several astrophysical phenomena. The model can be thus be improved with a more realistic density law; the coincidence with observational data is suggestive. We wish to note the simplicity of the model, the simple equations and the importance of applying the post-newtonian corrections in astrophysical scenarios.

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