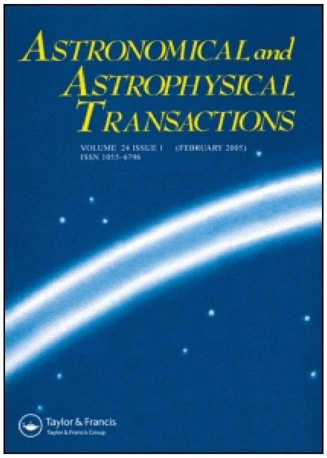

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GLOBAL MONOPOLES AS INDIRECT EVIDENCE FOR TORSION CONSTRAINT BY COBE DATA

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An approximate solution of Einstein-Cartan field equation describing a topological global monopole is presented.The gravitational field of the torsionic monopole is shown to be attractive.The deficit angle is shown to receive a contribution of torsion which is well within experimental limits in agreement with COBE data.This provides a mechanism for an indirect evidence for Cartan torsion.The presence of torsion may also lead to a stable configuration of the global monopole after inflation as happens in Brans-Dicke gravity.

KEY WORDS Global monopoles, torsion

Among all space-time topological defects (cosmic strings, domain walls, monopoles and textures) the only ones that have been not consider in the realm of Einstein–Cartan gravity have been the monopoles. Domain walls have been extensively investigated by one of us (Garcia de Andrade, 1998a, 1998b, 1999a, 1999b) while cosmic strings as line defects in spacetimes with torsion have been investigated by Letelier (1995a, 1995b) in recent years. Topological defects in Brans–Dicke theory of Gravity in the linear approximation have been considered by Barros and Romero (1995, 1997). The extension of textures to spacetimes with torsion is a more delicated issue which we shall address opportunately. In this Letter we shall use the linear approximation of the Einstein–Cartan gravity to address the global monopoles. One of the important physical consequences of this paper is that different to what happens in the case of Barriola–Vilenkin (1989) global monopoles in the context of General Relativity is that the global monopole there does not produce any gravitational effects off the monopole while here the introduction of an extra torsion term produces an extra gravitational effect outside the core of the monopole. Let us now consider the linearized metric (Landau and Lifchitz)

$$ds^2 = (1+g)\,dt^2 - (1+f)\,dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{1}$$

where g and f are only functions of the radial coordinate r. The Einstein–Cartan field equations are written in the quasi-Einsteinian form

$$G_{ij} = T_{ij} \tag{2}$$

where G_{ij} is the Einstein–Riemannian tensor and T_{ij} is the sum of the global monopole tensor

$$T_0^0 = T_1^1 = \frac{\eta^2}{r^2}, \tag{3}$$

and the torsionic monopole tensor

$$T_i^{k^{\text{torsion}}} = S_{\text{iml}} S^{\text{kml}} - \frac{1}{2}\delta_i^k S_0^2, \tag{4}$$

where $S_0 = S_{230} = a(\eta^2/r^2)$ is the only component for torsion. The equations for the EC equations are

$$\frac{f'}{r} + \frac{f}{r^2} = 8\pi G\left[\frac{\eta^2}{r^2} - S_0^2\right] = 8\pi G(1-a)\frac{\eta^2}{r^2} \tag{5}$$

and

$$\frac{-g'}{r} + \frac{f}{r^2} = 8\pi G\left(\frac{\eta^2}{r^2} - S_0^2\right) = 8\pi G(1-a)\frac{\eta^2}{r^2} \tag{6}$$

and

$$g'' - \frac{f'}{r} + \frac{g'}{r} = 0. \tag{7}$$

These three equations together yields the following solution

$$f(r) = 8\pi G(1-a)\eta^2 + \frac{C}{r}, \tag{8}$$

and

$$g(r) = -f(r) + B, \tag{9}$$

where B and C are integration constants. Assuming $B = 0$ and knowing that for $a = 0$ the solution must be reduced to the Barriola–Vilenkin solution, we have that $C = 2GM$ where M is the approximated mass of the monopole core and the final solution is

$$f(r) = 8\pi G(1-a)\eta^2 + \frac{2GM}{r} \tag{10}$$

and $g(r) = -f(r)$. The complete metric can be expressed as

$$\begin{aligned} ds^2 = {} & \left(1 - \frac{2GM}{r} - 8\pi G(\eta^2 - S_0^2 r^2)\right) \mathrm{d}t^2 \\ & - \left(1 + \frac{2GM}{r} + 8\pi G(\eta^2 - S_0^2 r^2)\right) \mathrm{d}r^2 - r^2\,\mathrm{d}\Omega^2. \end{aligned} \tag{11}$$

Note that in the absence of mass M and in the absence of torsion the metric above does not produce any gravitational field off the monopole core. To check these results in the weak field approximation of the gravitational field it is enough to check that the geodesic equation

$$\dot{v}_r = -\frac{\partial h_{00}}{\partial r} = -\left[\frac{2GM}{r^2} + 16\pi G S_0^2 r\right]. \tag{12}$$

Note that the presence of torsion does not modify the nature of the gravitational field of the global monopole and the gravitational field is attractive. The most interesting application of the solution described here is that the solution allow us to write the new metric in the form of a conical metric as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}r^2 - (1 - 8\pi G\eta'^2)r^2\,\mathrm{d}\Omega^2, \tag{13}$$

where $\mathrm{d}\Omega^2 = (\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$ and we have used the following transformation

$$\eta' = (1-a)^{1/2}\eta. \tag{14}$$

The new conical metric above implies that the curvature Ricci tensors and the Ricci scalar are respectively given by (Bezerra *et al.*, 1999)

$$R^{\theta\phi}_{\theta\phi} = R^{\theta}_{\theta} = R^{\phi}_{\phi} = \frac{8\pi G(1-a)\eta^2}{r^2} \tag{15}$$

and

$$R = \frac{16\pi G(1-a)\eta^2}{r^2} \tag{16}$$

which implies that curvatures, contrary to what happens in general relativistic global monopoles, can be negative. Note that in the case $a = 1$ the above metric is a Schwarzschild metric and from the curvature expressions one notices that they vanish for $a = 1$ though torsion does not vanish, which would represent a situation called Teleparallelism. In the formalism of condensed matter physics the conical metric of a global monopole represents a disclination. From the observational point of view the new deficit angle is

$$\Delta' = 8\pi G(1-a)\eta^2 = 8\pi G(\eta^2 - S_0^2 r^2) \tag{17}$$

receives a contribution from torsion to decrease the deficit angle, which in principle could provide an indirect evidence for torsion. The spectrum of fluctuation is

$$\left(\frac{\delta\rho}{\rho}\right)' = 30G(1-a)\eta^2. \tag{18}$$

Since $G\eta^2 = (5.9{+}1.2)\times 10^{-7}$ (Vilenkin and Shellard, 1995) and it is known that this value is maximum of the experimental acceptable value if torsion is introduced it will be of course within limits since the value of the constant $G\eta'^2 < G\eta^2$. The spectrum $\delta T/T < 10^{-4}$ from the COBE data is also compatible with the global torsionic monopole. The values obtained are very well within the COBE data (Vilenkin and Shellard, 1995). Yet another interesting application of our model is that the attractive force between monopole–antimonopole pair given by $F = \partial e/\partial R = 4\pi(1-a)\eta^2$ decreases due to the presence of torsion. This effect is similar to Trautman's mechanism to use torsion as a repulsive force to avoid gravitational collapse. This idea could be used to argued that monopoles in Einstein–Cartan gravity would be much more stable that in General Relativity. This effect was recently show to happen also in Brans–Dicke gravity (Sakai *et al.*, 1999). Recently one of us (Garcia de Andrade, in press) has computed the spin-torsion density fluctuation in de Sitter phase of the Universed also based on COBE data.

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