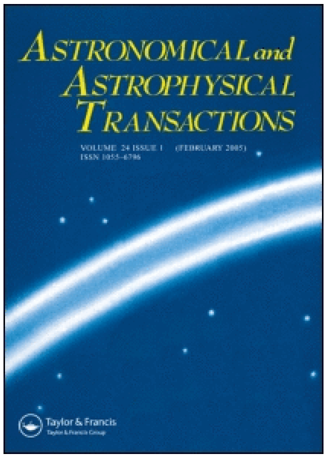

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Physics of the solar dynamo: Outstanding problems

M. Stix [a]
[a] Kiepenheuer-Institut für Sonnenphysik, Freiburg, Germany

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PHYSICS OF THE SOLAR DYNAMO: OUTSTANDING PROBLEMS

M. STIX

Kiepenheuer-Institut für Sonnenphysik,
Schöneckstr. 6, 79104 Freiburg, Germany

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Mean-field theory appears appropriate to describe the solar dynamo. The $\alpha\Omega$ dynamo yields a cyclic mean magnetic field, including the butterfly diagram. In spite of this apparent success many difficulties persist. The dynamo coefficients, especially the α effect, currently have not been determined quantitatively. The analytical treatment remains insufficient, mainly because of the large correlation time of solar convection; numerical simulation is difficult due to the large magnetic Reynolds number, as well as the extreme values of other parameters. Other problems appear less severe; they are related to the non-uniform rotation, to the overshooting velocity at the base of the convection zone, to the phase between the toroidal and poloidal mean-field components, and to the mechanisms that are responsible for the long-term variation of the solar activity.

KEY WORDS Dynamo theory, α effect, solar cycle

1 THE SUN – AN $\alpha\Omega$ DYNAMO

Solar activity is governed by the 22-year magnetic cycle. The mere fact that this time scale is so short in comparison to the evolutionary and thermal time scales is evidence that the magnetic field of the Sun cannot be fossil, but must be regenerated continuously. The convection zone in the outer third of the Sun (by radius) provides the flow of electrically conducting matter that is required for electromagnetic induction. Therefore a hydromagnetic dynamo is the obvious alternative.

Working solar dynamo models, in the form of $\alpha\Omega$ dynamos, were advanced first by Parker (1955b) and by Steenbeck and Krause (1969); a recent review of their successes has been given by Rüdiger and Arlt (2000). In this model the Coriolis force influences the turbulent convective flow (within the convection zone and overshooting from it) in such a way that the α effect occurs: namely, that the mean electric field $\langle \boldsymbol{u} \times \boldsymbol{b} \rangle$, which is derived from the velocity fluctuation $\boldsymbol{u}$ and the associated magnetic fluctuation $\boldsymbol{b}$, has a component $\alpha\langle \boldsymbol{B} \rangle$ in the direction of the mean magnetic field $\langle \boldsymbol{B} \rangle$. From an originally toroidal field this effect generates a large-scale poloidal field; the latter and the non-uniform angular velocity $\Omega(r, \theta)$ then cooperate to induce a new toroidal mean field (Figure 1). If $\alpha(r, \theta)$ and $\Omega(r, \theta)$

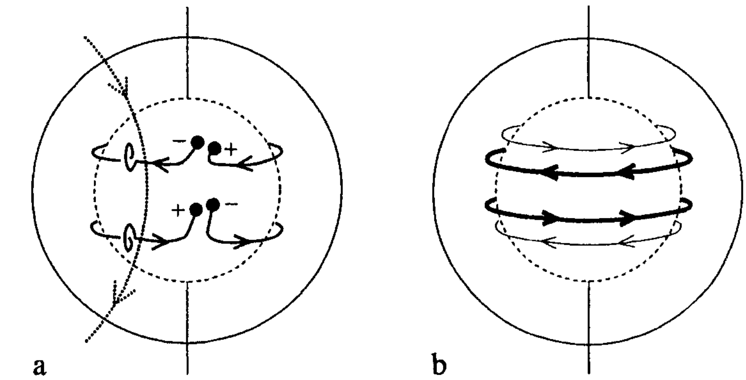

Figure 1 The scheme of an $\alpha\Omega$ dynamo. *Left*: poloidal field generation by the α effect; *right*: toroidal field generation by non-uniform rotation. Courtesy M. Ossendrijver.

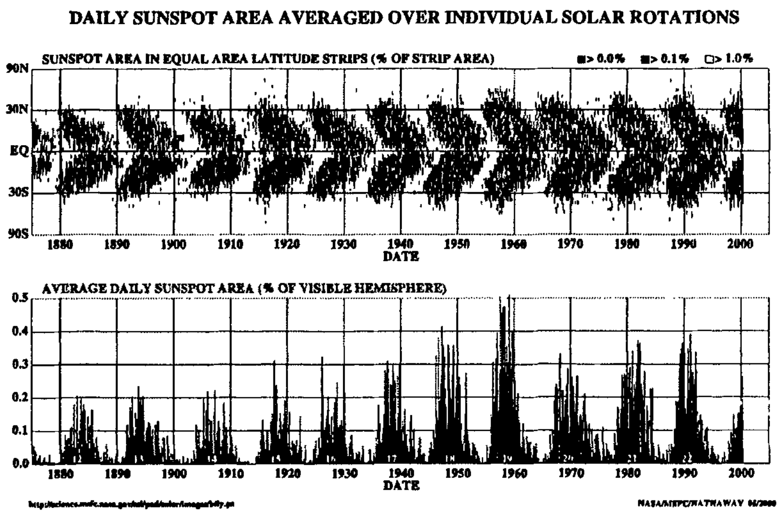

Figure 2 The butterfly diagram. Sunspot area in equal latitude strips, averaged over individual solar rotations. Courtesy D. H. Hathaway.

have a suitable spatial variation then the new mean field has a reversed sign, and a magnetic cycle can operate.

The $\alpha\Omega$ dynamo not only generates a cyclic mean field. Its greatest achievement is the field migration over latitude during the cycle, thus reproducing the solar butterfly diagram (Figure 2). The mean field migrates along the surfaces of constant angular velocity, in the direction $\alpha\nabla\Omega \times \boldsymbol{e}_\phi$, where $\boldsymbol{e}_\phi$ is an azimuthal unit vector; this result was already manifest in the pioneering work of Parker (1955b), but was more generally proved by Yoshimura (1975). No other theory has been successful in this respect, and it is mainly for this reason that the solar $\alpha\Omega$ dynamo deserves attention. In the present review I shall deal with this model only.

There are severe difficulties, nevertheless. The most outstanding, in my view, is the α effect itself, to be discussed in Sect. 2. Other problems, treated in the subsequent sections, appear minor in comparison. They are related to the Sun's differential rotation, to overshooting at the base of the convection zone, and to the origin of variations on a long time scale, such as the grand minima of solar activity.

2 MOST OUTSTANDING: THE α EFFECT

In an expansion of the mean electric field $\langle \boldsymbol{u} \times \boldsymbol{b} \rangle$ in terms of the spatial derivatives of the mean field $\langle \boldsymbol{B} \rangle$ the α effect appears as a tensor in the first term:

$$\langle \boldsymbol{u} \times \boldsymbol{b} \rangle_i = \alpha_{ij} \langle B_j \rangle + \beta_{ijk} \nabla_j \langle B_k \rangle + \cdots . \tag{1}$$

The second term contains the tensor β_{ijk} and describes the diffusion of the mean field by the turbulent flow. There are many open questions as to how this process works at high magnetic Reynolds number, in helical turbulence, etc. (e.g., Kraichnan, 1976; Drummond and Horgan, 1986; Nicklaus and Stix, 1988; Petrovay and Zsargó, 1998). Here I shall concentrate on the first term of (1), the α effect.

2.1 *First-order smoothing*

In order to evaluate the dynamo coefficients α and β the induction equation for the magnetic field fluctuation $\boldsymbol{b}$ must be solved. The traditional approach to do this is *first-order smoothing*, where only the terms of first order in $\boldsymbol{u}$ and $\boldsymbol{b}$ are retained. The electric field $\langle \boldsymbol{u} \times \boldsymbol{b} \rangle$ is then expressed in terms of second-order correlations of $\boldsymbol{u}$; hence another name for this approach is the *second-order correlation* approximation (Krause and Rädler, 1980). This approximation is justified if either of the following two conditions is met:

$$ul/\eta \ll 1 \qquad \text{or} \qquad u\tau/l \ll 1 \tag{2}$$

(Steenbeck and Krause 1969). Here η is the magnetic diffusivity, u is a typical magnitude of $\boldsymbol{u}$, and l and τ are typical scales of the variation of $\boldsymbol{u}$ (and $\boldsymbol{b}$) in space and time, respectively. The first of conditions (2) means a small magnetic Reynolds number; but on the Sun this number is very large, e.g., Figure 8.1 of Stix (1989). And the second condition is at best marginally satisfied, if one uses values of u, l, and τ as observed at the solar surface, or if one follows the ideas underlying the mixing-length theory of turbulent convection. Thus, first-order smoothing is far from being an appropriate approximation for the Sun. This is confirmed by attempts to calculate dynamo coefficients that include contributions of higher order in $u\tau/l$: the corrections are substantial (e.g., Nicklaus and Stix 1988).

2.2 *Numerical methods*

With numerical methods it is possible to go beyond first-order smoothing. Drummond and Horgan (1986) used stochastic velocity fields to integrate a large number

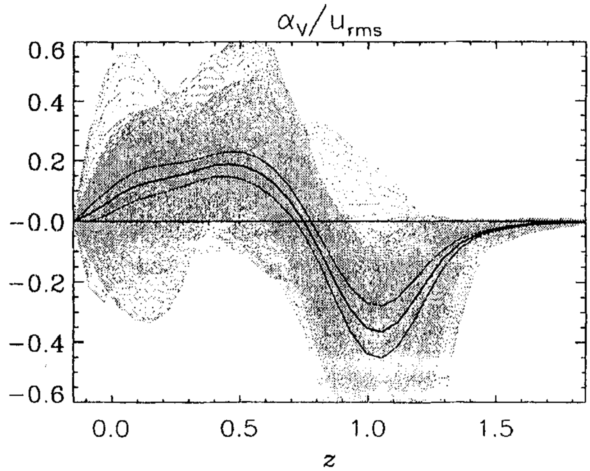

Figure 3 Vertical α effect, α_V, as a function of depth, averaged horizontally (grey curves). The thick drawn curve is the time average; the surrounding thin curves give the error of the mean, determined from the number of coherence times covered by the simulation. From Ossendrijver *et al.* (2000).

of paths of fluid parcels and, with a Lagrangian solution of the induction equation (e.g., Moffatt, 1978), calculated the dynamo coefficients. They could confirm the predictions of first-order smoothing in the appropriate limits, but otherwise found that α depends very sensitively on the diffusivity η, as well as on the correlation time of the assumed stochastic velocity.

A more direct approach is numerical simulation of three-dimensional magneto-convection in a compressible rotating fluid, that is, numerical integration of the hydrodynamic equations (including the Lorentz force) together with the induction equation for the magnetic field. But the problems are obvious. First, there is a wide range of length scales in the solar convection zone, which would require a large number of grid points, not tractable even on the most capable computers. Second, the Reynolds number, the Rayleigh number, and the magnetic Reynolds number all are much larger on the Sun than can be dealt with in a numerical simulation; other parameters, such as the Prandtl number and the Mach number, are very small and for this reason pose problems. As a consequence of the first difficulty the simulation resolves only the largest eddies, especially in global calculations (Gilman 1983, Glatzmaier 1985), but also if one restricts the calculation to a small box that can be placed at various depths and latitudes in the convection zone. In both cases the second difficulty is most severe: the parameter ranges are rather restricted, although values far beyond the limits of first-order smoothing have been reached. The hope is that, by variation of the parameters, a certain behavior is found that can be extrapolated to very large values, and perhaps be compared to the results of asymptotic theories.

Numerical magneto-convection in a box has been described by Brandenburg *et al.* (1990, 1996). Depending on the choice of parameters and on the boundary conditions, spontaneous field amplification may occur in such a simulation (Nordlund *et al.*, 1992). However, instead of such local dynamo action we are here more interested in the Sun's global dynamo. For this reason one chooses subcritical (for the local instability) conditions, calculates $\boldsymbol{u}$ and $\boldsymbol{b}$ and therefrom the components of $\langle \boldsymbol{u} \times \boldsymbol{b} \rangle$ in the direction of $\langle \boldsymbol{B} \rangle$. The components of the α tensor can thus be determined; Figure 3 shows an example of α_{V}, for the case where $\langle \boldsymbol{B} \rangle$ is vertical.

It is clear from simulations such as shown in Figure 3 that the dynamo coefficients are averages over extremely fluctuating quantities. Extensive simulations are therefore necessary to obtain a significant result. Of course the reason for the large fluctuations is the large value of the Reynolds numbers; in the shown example they were still quite moderate. Nevertheless, it has been possible to demonstrate important effects such as the quenching of the α effect when $\langle \boldsymbol{B} \rangle$ becomes large, or when the rotation becomes fast, at small Rossby number (e.g., Tao *et al.*, 1993; Ossendrijver *et al.*, 2000). In addition, some of the quenching predictions of first-order smoothing (Rüdiger and Kitchatinov, 1993) were confirmed.

2.3 Kinematic or dynamic α effect?

The recipes for calculating α described so far aim at the *kinematic* α effect. However, there are reasons (Sect. 4) for a field strength of order 10 T of the toroidal field at the base of the convection zone. Such a field strength exceeds the equipartition value, and α quenching must be expected. Therefore, the instability of the field itself, either in the form of a magnetic Rayleigh–Taylor instability (Brandenburg and Schmitt, 1998; Thelen, 2000) or in the form of a flux-tube instability (Ferriz-Mas *et al.*, 1994) has been suggested as a source of the α effect. Only preliminary results exist, partly based on first-order smoothing.

3 SUFFICIENT SHEAR?

Although a quantitative determination of the α effect appears impossible at present, there are indications that near the base of the convection zone (especially in the layer of convective overshooting) α should be negative in the northern and positive in the southern hemisphere. With this sign of α the $\alpha\Omega$ dynamo requires positive shear $\partial\Omega/\partial r$ for equatorward migration of the mean field (e.g., Stix 1976b); this is indeed the case on the Sun, as helioseismology has shown (Figure 4). At low latitude, we take from the figure $r\partial\Omega/\partial r \approx 2.5 \times 10^{-6}\,\mathrm{s}^{-1}$. Hence an amplification factor of 10^3 may be reached within 10 years.

However, in an $\alpha\Omega$ dynamo the poloidal field from which the shear generates the toroidal field must be generated from the toroidal field of the previous cycle. As a consequence, the ratio of the toroidal and poloidal field components is of order $(r^2(\partial\Omega/\partial r)/\alpha)^{1/2}$, which is only 10–100 for $\alpha = 10\,\mathrm{m/s}$, a typical value obtained for

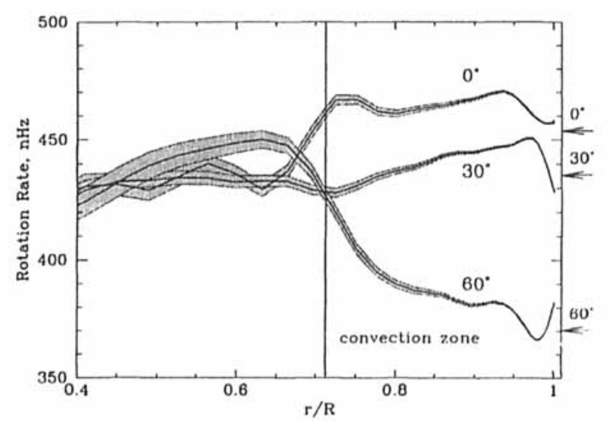

Figure 4 Solar rotation rate, as a function of r, for three heliographic latitudes; an inversion of data obtained with the Michelson Doppler Imager on SOHO. The vertical line marks the base of the convection zone; the arrows indicate the rate measured spectroscopically at the surface. From Kosovichev et al. (1997).

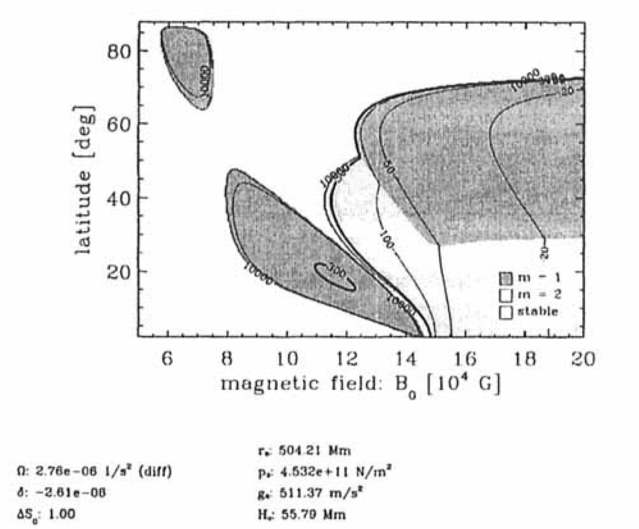

Figure 5 Instability of toroidal flux tubes, as a function of magnetic field strength and heliographic latitude, for $\nabla - \nabla_{\mathrm{ad}} = -2.6 \times 10^{-6}$. The labels on the contours are growth times in days. Courtesy P. Caligari.

the deep part of the convection zone. In view of the rather weak mean poloidal field observed at the solar surface this ratio appears small. Hence we may ask whether the shear revealed by helioseismology is indeed sufficient for an $\alpha\Omega$ dynamo.

4 HOW MUCH OVERSHOOTING?

Within the super-adiabatic stratification of the convection zone, magnetic flux tubes experience a buoyancy force (Parker 1955a) and rise to the surface. On the other hand, in the sub-adiabatic environment of a layer with convective overshooting, stable flux tubes with a field strength of up to $\approx 10\,\mathrm{T}$ may exist (Spruit and van Ballegooijen, 1982; Caligari *et al.*, 1995). Beyond this value the tubes become unstable; the instability depends on latitude, see Figure 5. Except for a very slow, and therefore insignificant, instability at high latitude the tubes start to rise at low latitudes. If the field is not in the form of narrow tubes, instabilities also arise at a comparable field strength, e.g., as a magnetic Rayleigh-Taylor instability that leads to magnetostrophic waves (Schmitt, 1987), or as a joint instability of the field and the latitudinal differential rotation (Gilman and Fox, 1997, 1999; Gilman and Dikpati, 2000). A further argument in favor of a strong toroidal field is that rising tubes with initially $\approx 10\,\mathrm{T}$ emerge at the right latitude (the sunspot zones), and exhibit the right asymmetry and tilt of the emergent bipolar active regions. Weaker tubes would rise parallel to the axis of rotation and emerge at high latitude, due to the rotational constraint (Choudhuri and Gilman, 1987).

Theoretical solar models based on a non-local form of the mixing-length theory of convection include a layer of convective overshooting, with a sub-adiabaticity that is suitable for the storage of a 10-T field (Pidatella and Stix, 1986; Skaley and Stix, 1991; Kiefer *et al.*, 2000). The thickness of this layer is somewhat larger than one-tenth of a pressure scale height H_{P}; the temperature gradient in the layer is nearly adiabatic, with a rather sharp transition to the radiative gradient at the depth where convection ceases.

Helioseismology has established the temperature profile in the solar interior with high precision. A difficulty in the present context is: there is no indication for a sharp transition of the temperature gradient such as predicted by those theoretical models. Upper limits for the extent of such overshooting are $0.07H_{\mathrm{P}}$ (Monteiro *et al.*, 1994), and $0.1H_{\mathrm{P}}$ (Basu *et al.*, 1994). Perhaps the solution lies in a more gradual transition layer, but models of convective overshooting with this property have yet to be built. Another possible difficulty has to do with lithium: a turbulent flow at and below the base of the convection zone that is sufficiently mild to allow for the survival of some lithium may be insufficient for the α effect and turbulent diffusivity as required by the dynamo (e.g., Rüdiger and Pipin, 2000).

5 THE PHASE DILEMMA

The mean toroidal magnetic field, $\langle B_\phi \rangle$, of the Sun can be inferred from the polarities of bipolar spot groups, while the mean radial field, $\langle B_r \rangle$, can be obtained from averaged solar magnetograms. There appears to be a phase shift of approximately 180° between the two field components (Stix, 1976a).

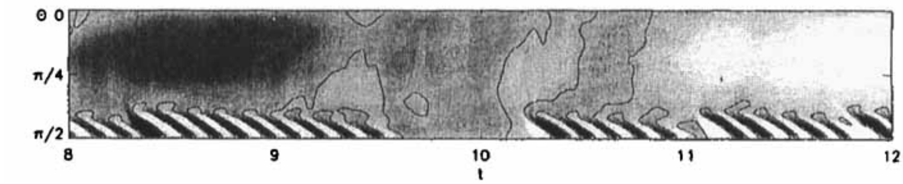

Figure 6 Butterfly diagram of a dynamo that includes α fluctuations. The contours mark equal field strength at a fixed depth. The cycle fades around time $t = 9.5$ and resumes near $t = 10.2$ (magnetic diffusion time as unit). Courtesy M. Ossendrijver.

The dynamo models that incorporate the shear near the base of the convection zone, and an α effect as described above, predict the correct field migration from higher to lower latitude, but $\langle B_r \rangle$ and $\langle B_\phi \rangle$ are approximately *in phase*, in apparent contradiction to observations (Parker, 1987). This is the phase dilemma.

Perhaps the dilemma is resolved in such a way that $\langle B_r \rangle$ and $\langle B_\phi \rangle$ vary in phase within the dynamo layer but out of phase at the solar surface (Schlichenmaier and Stix, 1995). Dynamos with proper variation of the α effect and turbulent diffusivity might do it, for example the interface-wave dynamo of Parker (1993; see also Charbonneau and MacGregor, 1997), or dynamos that include a suitable meridional circulation (Choudhuri *et al.*, 1995; Dikpati and Charbonneau, 1999). Gilman and Charbonneau (1999) test diverse models and conclude that phases such as those observed can indeed be produced. They also believe that the magnetograms are dominated by decaying active regions, and that, e.g., polar faculae (Sheeley, 1991) are better indicators of the mean poloidal field. The phase difference is then (at high latitude) ca. 90° rather than 180°. The model of Dikpati and Charbonneau (1999) yields such a phase difference.

6 GRAND MINIMA

The solar cycle exhibits long-term variation; most prominent was the Maunder minimum in the 17th century. There are essentially two ideas how to account for such grand minima and related variations within the framework of the $\alpha\Omega$ dynamo.

6.1 Fluctuating α effect

We have seen that the α effect is a highly fluctuating quantity (Figure 3). Fluctuations of α have been incorporated into dynamos (Schmitt *et al.*, 1996; Ossendrijver *et al.*, 1996), and grand minima have been modeled in this way. Other variations of the cycle, e.g., such that shorter cycles have higher amplitudes, have been obtained as well (Ossendrijver and Hoyng, 1996). Figure 6 shows an example (Ossendrijver, 2000b) where the dynamic α effect (Sect. 2.3) in the layer of strong toroidal field drives the cycle, but is supplemented by a fluctuating kinematic α within the convection zone. Such an additional kinematic effect is necessary because the dynamic

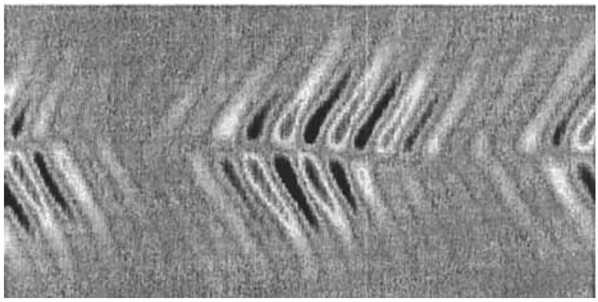

Figure 7 Butterfly diagram of an $\alpha\Omega$ dynamo with an additional equation that describes the variation of the rotational shearing. The long-term variation is a consequence of the non-linear dynamics. Courtesy S.M. Tobias.

α exists only beyond the instability threshold of the toroidal field (Ossendrijver, 2000a). With such an α alone the cycle could not resume after a grand minimum.

6.2 On the path to chaos

The second model of long-term variation rests on the non-linear character of the dynamo equations. The simplest model, with α quenching explicitly taken into account, yields limit cycles in the form of non-linear dynamo waves (Stix, 1972; Kuzanyan and Sokoloff, 1995; Bassom *et al.*, 1999), which reproduce several features of the solar cycle. However, with an additional equation (as in real magnetohydrodynamics) describing the dynamics of either the shear (Weiss *et al.*, 1984) or α effect (Schmalz and Stix, 1991), multiply-periodic and chaotic behavior has been generated. The bifurcations of the diverse solutions have been investigated in detail (e.g., Tobias *et al.*, 1995). Grand minima occur in particular when the magnetic Prandtl number is small (Tobias, 1996). The dynamic system also shows transitions between various symmetries (Jennings, 1991). The butterfly diagram of Figure 7 was obtained from such a dynamic system.

7 CONCLUDING REMARK

In spite of the difficulties described in this contribution I am convinced that mean-field theory is the appropriate approach to the solar dynamo. However, the problems reviewed here are by no means an exhaustive list, but a rather personal selection, complementing my earlier review (Stix, 1991). Other reviewers have discussed different points. Hoyng (2000) puts the emphasis on the fundamental concepts of mean-field theory, and especially on its relation to stochastic differential equations. Parker (1996) points out that the small-scale magnetic fluctuations have large field strength and hence should essentially be Alfvén waves, which makes it difficult to understand reconnection and dissipation on a small scale.

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