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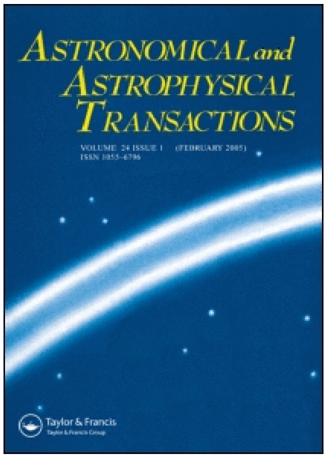

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ON AN ORBITER CROSSING AN ACCRETION DISC

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Considering the application to galactic nuclei, we further investigate the long-term evolution of the trajectory of a stellar-mass orbiter which is gravitationally bound to a massive central core. We employ a simple toy model of the orbital evolution of a body which passes on an inclined trajectory through the rarefied gaseous environment of a stationary disc. We demonstrate that the drag on the satellite should be taken into account in calculations of the stellar distribution near a super-massive black hole in galactic centers. The drag does not impose serious restrictions on the results of gravitational wave experiments if the disc is diluted, but it may be important for environments with rather high density.

KEY WORDS Accretion discs – Galaxies: nuclei

1 INTRODUCTION

In this note we extend a toy-model description of the star-disc system which suffers from hydrodynamical losses during repetitive interactions (Šubr and Karas, 1999). Here we compare the energy radiated from the system by means of gravitational waves with energy losses due to direct collisions. The aim of this calculation is to evaluate the mutual importance of the two effects depending on the surface density profile of the disc and on orbital parameters (eccentricity, semimajor axis) of the satellite. The orbiter is assumed to cross the disc with supersonic velocity at the moment of their interaction. Passages last a small fraction of the orbital period at corresponding radius, and can be treated as an instantaneous event during which the passing satellite expels out of the disc some amount of material that lies along its trajectory. For a geometrically thin disc, the influence upon stellar orbits can be treated as tiny kicks (impulsive changes of their momenta) at points of intersection with the plane of the disc. Furthermore, we assume that this interaction does not affect the structure of the satellites. This is a plausible assumption for stars with surface densities much larger than that of the disc, while it is inadequate for giants which must quickly lose their rarefied atmospheres (Armitage *et al.*, 1996).

1.1 Energy Losses Due to Collisions with the Disc

The collisions lead to a change of the orbiter's velocity $\boldsymbol{v} \rightarrow \boldsymbol{v}' = (A+1)^{-1}[v_r\, \boldsymbol{e}_r + v_\vartheta\, \boldsymbol{e}_\vartheta + (v_\varphi + A v_{\mathrm{K}})\boldsymbol{e}_\phi]$, where $A(r) \equiv \Sigma_{\mathrm{d}} v_{\mathrm{rel}} \Sigma_*^{-1} v_\vartheta^{-1}$, v_{rel} is the relative speed between the orbiter and the disc matter, Σ_* is surface density ascribed to the orbiter and defined by $\Sigma_* = M_*/(\pi R_*^2)$ (quantities denoted by an asterisk refer to the orbiter), and $\Sigma_{\mathrm{d}}(r)$ is the disc surface density. Let us assume rotation of the disc to be Keplerian for the moment, $v_\phi \equiv v_{\mathrm{K}} = \sqrt{GM/r}$, and consider a star on an orbit with semi-major axis a, eccentricity e, inclination i, and longitude ω of the ascending node. The above given relation $\boldsymbol{v} \rightarrow \boldsymbol{v}'$ implies the change of orbital parameters of the orbiter ($x = \cos i$, $y = 1 - e^2$, and $z = 1 + e\cos\omega$ are convenient variables expressed in terms of usual osculating elements i, e, ω). The temporal evolution can be also examined by introducing the orbital period, $\mathrm{d}t = 2\pi a^{3/2}/\sqrt{GM}$. For example, with a power-law surface-density profile of the disc, $\Sigma_{\mathrm{d}} = K(r/r_{\mathrm{g}})^s \Sigma_*$, and for a special orientation of the orbit, $z = 1$, we can solve the orbital evolution analytically with respect to $x(a, e)$, and explore how the orbits become circular and inclined into the disc plane (Šubr and Karas, 1999). The temporal evolution is then obtained in the form

$$t = \frac{2\pi}{K\sqrt{GM}} \int_{x_0}^{x} \frac{a(\bar{x})^{3/2}[a(\bar{x})y(\bar{x})/r_{\mathrm{g}}]^{-s}\mathrm{d}\bar{x}}{\sqrt{(3 - y(\bar{x}) - 2\bar{x})(1 - \bar{x}^2)}}. \tag{1}$$

The above presented solution also offers a good approximation for orbits with arbitrary orientation, and its accuracy was calculated in (Šubr and Karas, 1999) where we plot the relative difference Δa_{f} between the final radius a_{f} of an orbit (when it is completely circularized and brought into the plane of the disc) and the corresponding approximate value. Hence, the energy losses due to star-disc collisions are obtained per revolution:

$$\left[\frac{\mathrm{d}E}{\mathrm{d}t}\right]_{\mathrm{coll}} = \frac{GMM_*}{2a^2}\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{K}{\pi}\frac{\sqrt{GM}M_* c^2}{a^{3/2}}\left[\frac{a(1-e^2)}{r_{\mathrm{g}}}\right]^{s-1}\sqrt{\frac{2+e^2-2x}{1-x^2}}(1+e^2-x). \tag{2}$$

1.2 Energy Losses Due to Gravitational Radiation

The rate of energy loss via gravitational radiation can be averaged over one period and expressed in the form (Peters and Mathews, 1963)

$$\left[\frac{\mathrm{d}E}{\mathrm{d}t}\right]_{\mathrm{gw}} = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3 M_*^2}{a^5(1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right). \tag{3}$$

A similar formula holds for angular momentum losses, too (Peters and Mathews, 1963). Here it is assumed that the star ($M_* \ll M$) follows an eccentric orbit in Schwarzschild geometry. Gravitational losses compete with energy losses caused by star-disc encounters but both effects tend to bind the orbit more tightly to the centre. The relative importance of these two effects is given by the ratio $R = [\mathrm{d}E/\mathrm{d}t]_{\mathrm{coll}}/[\mathrm{d}E/\mathrm{d}t]_{\mathrm{gw}}$,

$$R = \frac{5K}{32\pi}\frac{M}{M_*}\left(\frac{a}{r_\mathrm{g}}\right)^{s+5/2}(1-e^2)^{s+5/2}\frac{1+e^2-x}{1+(73/24)e^2+(37/96)e^4}\sqrt{\frac{2+e^2-2x}{1-x^2}}. \tag{4}$$

For the standard model of a gas-dominated thin disc, R is equal to

$$10^5\left[\frac{M}{M_8}\right]^{1/2}\left[\frac{M_*}{M_\odot}\right]^{-1}\left[\frac{\alpha}{0.01}\right]^{-4/5}\left[\frac{\ell_\mathrm{E}}{0.1}\right]^{7/10}\left[\frac{\epsilon}{0.1}\right]^{-7/10}\left[\frac{\Sigma_*}{\Sigma_\odot}\right]^{-1}\left[\frac{a}{r_\mathrm{g}}\right]^{7/4} f(x,e)\,\mathcal{L}, \tag{5}$$

where α is the viscosity parameter, ℓ_E is the accretion rate (units of the Eddington accretion rate), ϵ is the mass-to-luminosity conversion efficiency, index $s = -3/4$, and $\mathcal{L}$ is a general-relativity correction factor. We explored the functional dependence of R in terms of the factor $f(x,e)$ (Karas and Šubr, 2001). For a solar-type star it is only with highly eccentric orbits that f becomes small enough to bring R close to unity, and the required eccentricity is so high that the star would be trapped by the central hole directly. Otherwise, $R \gg 1$ for for standard values of the disc parameters ($\alpha \lesssim 1$, $\ell_\mathrm{E} \lesssim 1$, $\epsilon \approx 0.1$), which means that direct hydrodynamical collisions play a dominant role in the orbital evolution of satellites crossing a standard disc.

2 CONCLUSIONS

In this note the expected value of energy loss of an orbiting satellite was examined in terms of simple estimates. We verified with the standard thin disc model that hydrodynamic drag is, typically, more important for the long-term orbital evolution than gravitational radiation losses. However, this conclusion must be revised if the disc is of low density and/or the orbiter is of high compactness (a neutron star or a black hole), which is the relevant situation for gravitational wave experiments.

In the latter case, the orbiter's surface density Σ_* is orders of magnitude larger than that of a solar-type star. For a compact object, the influence of gravitational radiation is comparable to effects of star-disc collisions even for orbits with semi-major axes $a \approx 10^2 r_\mathrm{g}$ and eccentricities $e \approx 0.9$. Hence, gravitational radiation energy losses dominate the orbital evolution if the satellite is compact and the gaseous environment has a much lower density than the standard thin disc. This result agrees with a similar discussion of late stages of motion of a body embedded within a dilute gaseous medium (Narayan, 2000): for such an orbiter with $a \leq 10 r_\mathrm{g}$ the gravitational radiation losses dominate over the hydrodynamic drag.

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