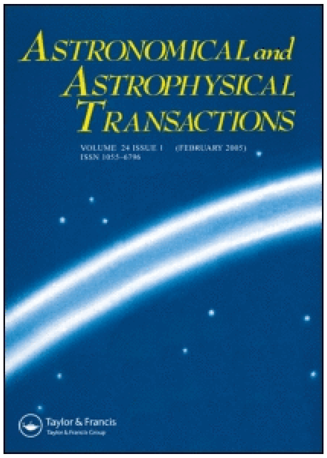

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DYNAMICS OF A TIDALLY DISRUPTED STAR

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A new semi-analytical model of a star evolving in a tidal field is proposed. The model is a generalization of the so-called 'affine' stellar model. In our model the star is composed of elliptical shells with different parameters and different orientations, depending on time and on the radial Lagrangian coordinate of the shell. The evolution equations of this model are described. The comparison of results of solution of these equations with 3D numerical computations shows very good agreement between the main 'integral' characteristics describing the tidal interaction event in our model and in the 3D computations. Our model can be evolved numerically 10^2–10^3 times faster than the 3D approach allows.

KEY WORDS Black hole physics-celestial mechanics, stellar dynamics-hydrodynamics

1 INTRODUCTION

Starting from the paper by Roche the problem of the tidal influence of a gravitating source on a satellite has been addressed by numerous researchers. More recently, interest in this problem has been raised by a paper by Hills (1975), who proposed tidal disruption processes as the main processes of fueling QSOs and AGNs. For modeling OSOs and AGNs it is very important to understand quantitatively the main characteristics of the tidal encounter itself, and a lot of of work has been devoted to the physical processes occurring in a star during a fly-by of a black hole. A significant step forward in this problem was made by Lattimer and Schramm (Lattimer and Schramm, 1976) and by Carter and Luminet (Carter and Luminet, 1983, 1985) who proposed the so-called affine model of the tidally disrupted star. In this model the law of time evolution of different elements of the star is defined in terms of some spatially uniform 3×3 matrix $\boldsymbol{Q}(t)$:

$$x^i = Q^i_j(t) x^j_0, \tag{1}$$

where x^i are the components of the position vector of a gas element, x_0^j are the components of the position vector in some reference state (say, before the tidal field 'is switched on'), and summation over repeated indices is assumed. Then one can find the evolution equations for the matrix elements from the so-called virial relations written for the whole star. Unfortunately, the affine model is too idealized. In the last decade, progress in numerical simulations has allowed researchers to perform direct 3D simulations of the tidal interaction and tidal disruption events. However, the 3D simulations are still very time consuming. On the other hand, the astrophysics of AGNs and QSOs requires a rather rough description of a single tidal encounter, and only a few 'averaged' quantities are of interest from the astrophysical viewpoint.

In this contribution we briefly introduce a new, semi-analytical model of the tidally interacting or tidally disrupted star which combines the simplicity of the affine model and advantages of the 3D hydrosimulations, and could be used for intensive calculations covering the whole parameter space of the problem (see Ivanov and Novikov, 2000 for more details). Our model is a straightforward generalization of the affine model. However, in contrast to the affine model, the different layers of the star evolve differently in our model, and are connected to each other by a force determined by pressure. Instead of the position matrix $\boldsymbol{Q}(t)$ of the affine model, we use the position matrix

$$\boldsymbol{T}(t, r_0), \tag{2}$$

which depends not only on time, but in addition on the value r_0 of the 'reference' vector x_0^i (obviously the radius r_0 plays the role of a Lagrangian coordinate, so we will later call it the Lagrangian radius). Thus, in our model the star consists of elliptical shells which are composed of all elements of the star with a given Lagrangian radius r_0. We compare results of calculations of the tidal encounters in our model with 3D finite difference simulations for the same problem and the same parameters, and find very good agreement.

We assume that summation is performed over all indices appearing in our expressions more than once, but summation is not performed if the indices are enclosed in brackets. Bold letters represent matrices in abstract form.

2 EVOLUTION EQUATIONS AND THEIR SOLUTION

As was mentioned in the Introduction, we divide the star into a set of elliptical shells. Each shell consists of gas elements which had the same distance from the center of the star in the unperturbed spherical state. We assume that the star layers corresponding to the same Lagrangian radius $r_0 = \sqrt{x_{0i} x_0^i}$ always keep an elliptical form. Let us consider the Eulerian coordinates of the gas elements x^i with respect to some inertial reference system centered at the star's geometrical center. The law of transformation between the Lagrangian and the Eulerian coordinates is

$$x^i = T_j^i(t, r_0) e_0^j, \tag{3}$$

where $e_0^i = x_0^i/r_0$. Let the matrix $\boldsymbol{S}$ be the inverse of the matrix $\boldsymbol{T}$. We have

$$T_j^i = A_l^i B_m^l E_j^m = a_l A_l^i E_j^l, \quad S_j^i = a_l^{-1} A_l^j E_i^l, \tag{4}$$

where $B_m^l = a_{(l)}\delta_m^{(l)}$, a_l are the principal axes of the elliptical shell. Two rotational matrices $\boldsymbol{A}$ and $\boldsymbol{E}$ describe rotation of the elliptical shell with respect to the Eulerian and Lagrangian reference frames. Let us define the determinant g of the position matrix:

$$g = |\boldsymbol{T}| = a_1 a_2 a_3, \tag{5}$$

the mass $M = 4\pi \int_0^{r_0} \rho_0(r_1)\,\mathrm{d}r_1$ of the gas inside the shell of the radius r_0, and the dimensionless quantities D_j

$$D_j = g \int_0^{\infty} \frac{\mathrm{d}u}{\Delta(a_j^2 + u)}, \tag{6}$$

where

$$\Delta = \sqrt{(a_1^2 + u)(a_2^2 + u)(a_3^2 + u)}.$$

The evolution equations of our model can be deduced from the virial relations written for a particular shell. They have the form

$$\ddot{T}_n^i = -4\pi S_i^n g \frac{\mathrm{d}\bar{p}}{\mathrm{d}M} - \frac{3}{2} A_j^i a_j D_j E_n^j \frac{GM}{g} + C_j^i T_n^j, \tag{7}$$

where the matrix $\boldsymbol{C}$ represents the tidal tensor. It is symmetric and traceless. $\bar{p}$ is an 'averaged' pressure. The evolution of the 'averaged' pressure follows from the energy conservation law. In the adiabatic approximation we have

$$\mathrm{d}\left(\frac{\bar{\epsilon}}{\bar{\rho}}\right) + \bar{p}\,\mathrm{d}\left(\frac{1}{\bar{\rho}}\right) = 0, \tag{8}$$

where the 'averaged' density

$$\bar{\rho} = \frac{3}{4\pi}\frac{\mathrm{d}M}{\mathrm{d}g}, \tag{9}$$

and the 'averaged' energy density $\bar{\epsilon}$ is related to $\bar{\rho}$ and $\bar{p}$ by the equation of state.

For our numerical work we choose a simple problem, the tidal encounter of a polytropic star moving around a source of Newtonian gravity (referred to as a black hole) on a parabolic orbit. We assume that the star consists of an ideal gas with constant specific heat ratio $\gamma = 5/3$. In this case our problem can be described by two parameters: (a) the polytropic index n, and (b) the parameter η reflecting the strength of the tidal encounter:

$$\eta = \sqrt{\frac{M_*}{M_h}\frac{R_p^3}{R_*^3}} = \left(\frac{R_p}{R_T}\right)^{3/2}, \tag{10}$$

where M_h is mass of the black hole, M_* and R_* are the mass and radius of the star, respectively, and R_p is the value of the pericentric separation distance between the star and the black hole. The comparison of the main characteristics of the star (the energy deposited in the star by the tidal field, the central density, the angular momentum of the star, etc.) after a tidal encounter with the same characteristics, but calculated in the 3D hydrosimulations by Khokhlov *et al.* (1993a, b) shows a typical disagreement of the order of 10–20 percent. This disagreement is of the order of the disagreement among the 3D methods themselves (say, among finite difference and SPH methods). We estimated the amount of mass lost by the star after a fly-by of the black hole as a function of η. For the $n = 3$ polytrope the star loses its mass if $\eta < \eta_{\rm strip} \approx 1.5$, and the star is completely disrupted if $\eta < \eta_{\rm crit} \approx 0.4$. For the $n = 2$ polytrope we have $\eta_{\rm strip} \approx 2$ and $\eta_{\rm crit} \approx 0.9$ and for the $n = 1.5$ polytrope we have $\eta_{\rm strip} \approx 2.5$ and $\eta_{\rm crit} \approx 1.14$. The values of $\eta_{\rm strip}$ and $\eta_{\rm crit}$ are in excellent agreement with estimates based on the 3D simulations (see Khokhlov *et al.*, 1993b).

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