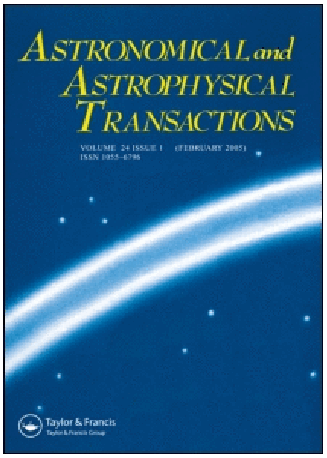

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J. Heyvaerts [a]; C. Norman [b]
[a] Observatoire Astronomique, Université de Strasbourg, Strasbourg, F
[b] Johns Hopkins University and Space Telescope Science Institute, Baltimore, MD, USA

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ASYMPTOTIC STRUCTURE OF MHD WINDS AND JETS

J. HEYVAERTS[1] and C. NORMAN[2]

[1] *Observatoire Astronomique, Université de Strasbourg, Strasbourg, F.*
[2] *Johns Hopkins University and Space Telescope Science Institute, Homewood Campus, Baltimore MD, USA.*

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We describe asymptotic solutions for stationary, axisymmetric, perfect MHD, polytropic winds, both classical and relativistic. They are expressed as field-region solutions and current-carrying boundary layer solutions smoothly joined by asymptotic matching. The vicinity of the polar axis is one of these boundary layers. In general, the boundary layers are null surfaces. It is argued that the boundary layer regions, in particular the axial one, should stand out observationally because of their larger density and activity. We associate the axial boundary layer with a jet. Current closure is self-consistently achieved in these solutions, which we obtain both in the case of vanishing or non-vanishing circumpolar asymptotic current. It is shown that the total current about the polar axis is simply related to the set of the five first integrals which characterize the flow and that non-vanishing values of this quantity are not available to all winds, but only to a restricted class which we present here. We show that winds of this class separate clearly into an axial jet and a circum-equatorial conical wind.

KEY WORDS Jets, winds, MHD

1 INTRODUCTION

Rotating MHD winds are of interest in accretion disk and outflow physics because of the role played by Lorentz stresses in the transport of angular momentum and in plasma acceleration and jet axial focusing. This communication describes in some detail the asymptotic structure of such flows.

Actual jets are certainly of a finite extent, non-stationary, non-axisymmetric and dissipative (due to MHD instabilities that are likely to develop in them). Our study nevertheless considers perfect MHD, stationary, axisymmetric polytropic flows. We expect this idealized model to share with actual MHD winds some global properties of its gross structure, such as the large scale properties of its electric current circulation and the existence of associated boundary layers in the asymptotic region which we discuss below. Details of the magnetic flux distribution certainly differ both in reality and in theoretical models. The study of the latter is justified by their relative simplicity and the expected robustness regarding global structures.

2 NOTATION AND BASIC PRINCIPLES

Axisymmetric MHD structures are best described in cylindrical coordinates (r, θ, z). Magnetic field lines as well as flow lines are drawn on a set of common surfaces, on which a flux function $a(r, z)$ assumes a constant value, which can be used to label them. Perfect, axisymmetric, stationary, polytropic flows conserve five so-called first integrals following the motion. These quantities are then functions of the flux variable a only. We note them as $Q(a)$, $\alpha(a)$, $\Omega(a)$, $L(a)$, $E(a)$. They are defined by:

$$P = Q(a)\rho^{\gamma}, \tag{1.1}$$

$$\hat{\gamma}\rho \boldsymbol{v}_P = \alpha(a)\boldsymbol{B}_P, \tag{1.2}$$

$$\hat{\gamma}\rho(v_\theta - r\Omega(a)) = \alpha(a)B_\theta, \tag{1.3}$$

$$\hat{\gamma}\xi r v_\theta - \frac{rB_\theta}{\mu_0\alpha(a)} = L(a), \tag{1.4}$$

$$\hat{\gamma}\xi\left(c^2 - \frac{GM_*}{\sqrt{r^2+z^2}}\right) - \frac{r\Omega(a)B_\theta}{\mu_0\alpha(a)} = E(a). \tag{1.5}$$

Here the subscript P denotes the poloidal part of a vector, μ_0 is the permeability of free space, G Newton's constant, γ the polytropic index, ρ is the proper mass density, P the associated pressure, $\boldsymbol{v}$ is the plasma velocity, $\boldsymbol{B}$ the magnetic field, $\hat{\gamma}$ the Lorentz factor and M_* the mass of the wind-emitting point source. The function ξ is defined by $c^2(\gamma-1)(\xi-1) = \gamma Q\rho^{\gamma-1}$. For relativistic flows, unlike for classical ones, E, as defined by eq.(1.5), includes the rest mass energy. The Alfvén radius is given by $r_A(a) = (L/\Omega)^{1/2}$ and the Alfvén density is given by $\rho_A(a) = \mu_0\alpha^2$. The component of the equation of motion normal to magnetic surfaces, the transfield equation, does not integrate in the form of a first-integral. It determines the shape of magnetic surfaces. The five first integrals, supposedly known here, are determined from boundary conditions and from the dynamics of the flow in an extended, but finite, region about the wind source.

3 GENERAL FOCUSING PROPERTIES OF POLYTROPIC MHD WINDS

It has been shown that axial asymptotic focusing is a general property of such polytropic flows, both classical (Heyvaerts and Norman, 1989) and relativistic (Chiueh, Li and Begelman, 1991). Intense heating, or isothermality, may lead to different results. These conclusions rest on consideration of the Bernoulli Eq. (1.5) and result from the obvious statement that neither the Poynting flux carried by the wind, nor its kinetic energy flux, can exceed the total energy flux. These conditions can be used to constrain the asymptotic shape of magnetic surfaces, though not to explicitly calculate them, which we do here. Our general results (Heyvaerts and Norman, 1989) indicated that, in the classical regime,

(1) No magnetic surface can asymptotically bend towards the equator.
(2) Flaring magnetic surfaces carry no electric current to infinity between them.
(3a) If $\lim(\rho r^2) \neq 0$ on them, they enclose a finite total current.
(3b) If not, this current vanishes.
It results that two different types of winds may be distinguished, based on their asymptotic properties.
I. Winds which carry a finite electric current and a finite total Poynting flux to infinity in a circumpolar region consist of cylindrical magnetic surfaces possibly nested in flaring magnetic surfaces.
II. Winds which carry a vanishing electric current to infinity in any space bounded by flaring magnetic surfaces. In this case the energy flow at infinity is all in kinetic energy form. It has been shown that then all magnetic surfaces are asymptotic to a set of nested paraboloids.

4 PENDING QUESTIONS

Many questions are left unanswered by these general results. In particular it could be asked whether the asymptotics really are of type I or II above. If it is of type I what is the actual value of the total asymptotic current? How does the current system close in the asymptotic domain, and what exactly is the shape of magnetic surfaces far from the wind source? Although our previous work (Heyvaerts and Norman, 1989) gave partial answers and locally valid solutions, a complete self-consistent and space-filling solution was still to be obtained. We present here answers to a number of the above questions, both for classical and relativistic winds.

5 ASYMPTOTIC TRANSFIELD EQUATION AND FLOW STRUCTURE

Examination of the dominating terms in the transfield equation show that, for classical winds, the hoop stress force density $\boldsymbol{j}_P \times \boldsymbol{B}_\theta$ and the gradient of the gas pressure force dominate over the centrifugal force, the poloidal magnetic pressure, the curvature inertia force and gravity on any flaring magnetic surfaces, and more generally on any asymptotically cylindrical surface with a radius $r_\infty(a)$ much larger than its Alfvén radius. A very small, or vanishing, polytropic entropy $Q(a)$ may upset this ordering, in which case the poloidal magnetic pressure would replace the gas pressure as the second major term. For relativistic flows, the electrical force remains a major part of the mechanical equilibrium. Keeping gas pressure as one of the two dominant contributions, the asymptotic form of the transfield equation becomes, for classical as well as for relativistic winds:

$$\frac{1}{\rho}(\boldsymbol{n}\cdot\nabla)Q\rho^\gamma + \frac{\Omega}{\alpha}(\boldsymbol{n}\cdot\nabla)\frac{\rho r^2\Omega}{\mu_0\alpha} = 0, \tag{2}$$

where n is the unit vector normal to the local magnetic surface. Further attention shows that the second term, which in the classical case represents hoop stress, dominates over pressure everywhere except in a small vicinity of magnetic surfaces where it vanishes, i.e. near the polar axis, where $r = 0$, and near null magnetic surfaces, where α becomes infinite. The polar axis and null surfaces are places where the hoop stress force density vanishes, since B_θ vanishes. Actually no azimuthal field is built up by rotation on such surfaces. Note that this implies that the electric current closes exactly in cells bordered by null magnetic surfaces or the polar axis. If the general field structure is endowed with a dipolar type of symmetry, the equatorial plane is a null surface, and, in this particular case, the only one. We restrict ourselves to this situation in the following for simplicity. Therefore, the asymptotic domain consists of large 'field-regions' in which the transfield equation simply reduces to

$$(\boldsymbol{n} \cdot \boldsymbol{\nabla}) \frac{\rho r^2 \Omega}{\mu_0 \alpha} = 0, \tag{3}$$

bordered by a circumpolar region and regions about the null surfaces where gas pressure must be taken into account. If for some reason the latter is too small or vanishes, or if the ratio r_∞ / r_A is not large enough, other forces might have to be considered too. Equation (3) expresses the vector relation $\boldsymbol{n} \cdot (\boldsymbol{j} \times \boldsymbol{B})$ in the classical case and $\boldsymbol{n} \cdot (\rho_e \boldsymbol{E} + \boldsymbol{j} \times \boldsymbol{B})$ in the relativistic case, ρ_e being the electric charge density.

6 SOLUTION IN THE FIELD-REGION OF CURRENT CARRYING WINDS

Wherever valid, Eq. (3) expresses constancy on an orthogonal trajectory to the magnetic surfaces of $(\rho r^2 \Omega / \mu_0 \alpha)$. This quantity represents the poloidal current enclosed in magnetic surface a, $I_\infty(a)$, in the classical case and $I_\infty(a)/\hat{\gamma}_\infty(a)$ in the relativistic case, $\hat{\gamma}_\infty$ being the asymptotic value of the Lorentz factor on this surface. Labeling these orthogonal trajectories by a variable b, Eq. (3) can thus be generally written as $I_\infty(a)/\hat{\gamma}_\infty(a) = K(b)$. If $K(b)$ is to approach a non-vanishing limit, K_∞, this equation reduces, for a certain function $S(a)$, to an Hamilton-Jacobi equation of the form $|\boldsymbol{\nabla} S| = 1/r$. This is the eikonal equation for the propagation of waves in a medium with a refractive index $N = 1/r$. The orthogonal trajectories to lines of constant S, that is of constant a, can then be simply found by ray-tracing methods, starting perpendicular to the polar axis and ending perpendicular to the equator. Detailed analysis shows that these lines are circles centered on the axis. Imposing the boundary condition at the equator shows that they are in fact centered at the origin, which shows that magnetic surfaces flaring out to infinitely large radial cylindrical coordinates are in this case conical.

7 WKB ANALYSIS OF PARABOLIC WINDS

If the integration constant $K(b)$ approaches zero as the radius of the orthogonal trajectory grows large, explicit integration of the asymptotic transfield equation is not possible in a general form. However if $K(b)$ declines to zero only very slowly, which can be shown *a posteriori* to be so, it is possible to work out a WKB type of approximation to the solution. Trajectories orthogonal to magnetic surfaces are regarded as being locally circles centered at the origin, the radius R of which can be taken as the as yet unspecified variable b. The distribution of flux along such a circle, represented by the angle $\psi(a, R)$ at which the flux variable takes the value a, slowly changes as R grows larger. This distribution is found to be given by:

$$(\cos(\psi(a,R)))^{-1} = \cosh\left(\int_a^A \frac{\Omega(a')\ da'\ (c^2 + K(R)\Omega(a')/\alpha(a'))}{\mu_0 c K(R)\ \sqrt{E^2(a') - (c^2 + K(R)\Omega(a')/\alpha(a'))^2}}\right). \tag{4}$$

A similar result is readily obtained in the classical case by taking the appropriate limit, taking care of the different definition of $E(a)$. Cylindrical magnetic surfaces exist, nested in flaring ones, if $K(R)$ approaches the non-vanishing constant K_∞. Their radius $r_\infty(a)$, supposedly large compared to the Alfvén radius, is given by

$$r_\infty(a) = r_0 \exp\left(\int_0^a \frac{\Omega(a')\ da'\ (c^2 + K_\infty\Omega(a')/\alpha(a'))}{\mu_0 c K_\infty\ \sqrt{E^2(a') - (c^2 + K_\infty\Omega(a')/\alpha(a'))^2}}\right) \tag{5}$$

A similar result applies to the classical case. This however can be solved for the asymptotic structure only in the field-regions and leaves open the determination of $K(R)$ and r_0. We outline below solutions in the polar and equatorial boundary layers and will match them to the field-regions, resolving these indeterminacies. But we need to discuss first the asymptotic value of $K(R)$.

8 THE VALUE OF THE ASYMPTOTIC CURRENT

If $K(R)$ approaches a finite limit for large R's, a cylindrical region has to be nested inside an asymptotically conical one. These two regions of different geometry have to join smoothly at some value a_* of the flux variable in such a way that the solution globally fills all space. This implies that $r_\infty(a)$ diverges as a approaches a_* from below, as $\tan(\psi(a))$ should also behave as a approaches a_* from above. Both conditions are granted if the integrals which appear on the right hand side of Eqs. (4) and (5) diverge for a approaching a_*. This requires that, for such a solution to exist at all, the square root denominator has a double zero at a_*, implying the exact vanishing of the asymptotic wind velocity on this particular flux surface, obviously because of the extreme divergence of the cross section of any flux tube containing it.

This shows that such a solution exhibits two well separated sub-structures: a polar jet of cylindrical structure, which is current carrying and thus hoop-stress focused, and an equatorial wind of asymptotically conical geometry. These two rather distinctive flows are separated by a region of very small, and in fact vanishing, wind speed.

Since the argument of this square root denominator in (4) and (5) has to remain positive, it is concluded that the actual value of the constant K_∞, if not zero, must be the absolute minimum of the function $\alpha(E - c^2)/\Omega$:

$$K_\infty = \min\left(\frac{\alpha(a)(E(a) - c^2)}{\Omega(a)}\right) \tag{6}$$

In the classical case, this should be read as giving the total electric current carried to infinity, I_∞, being the minimum value of $\alpha E/\Omega$. Since in general the first integrals vary linearly with a near the polar axis, this minimum should not be located at $a = 0$.

9 THE POLAR BOUNDARY LAYER

Regions where gas pressure has a marked influence on the mechanical equilibrium must be geometrically thin in the case of polytropic winds, since any wind expansion reduces the pressure. This is the reason why they can be treated as boundary layers. They must be located wherever the hoop stress force is due to vanish, i.e. near the polar axis and null surfaces. The structure of the polar boundary layer is then that of a pressure-supported plasma pinch column, described by equation (2). To solve for it we further assume that the five first integrals are almost constant over this region. Noting quantities evaluated at the polar axis by a subscript 0, equation (2) can then be reduced to

$$(\boldsymbol{n}\cdot\boldsymbol{\nabla})\left(\frac{\gamma}{\gamma-1}Q_0\rho^{\gamma-1} + \frac{\Omega_0^2}{\mu_0\alpha_0^2}\,\rho r^2\right) = 0 \tag{7}$$

which can be solved, coupled to the mass conservation Eq. (1.2) and the asymptotic form of the Bernoulli Eq. (1.5), in terms of the parameter x defined by $\rho = \rho_0(R)\,x$. This gives:

$$a = \frac{\gamma Q_0\rho_0^{\gamma-1}\mu_0\alpha_0}{\sqrt{2}(\gamma-1)\Omega_0^2}\sqrt{E_0 - \frac{\gamma}{\gamma-1}Q_0\rho_0^{\gamma-1}\left(\ln\frac{1}{x} - \frac{2-\gamma}{\gamma-1}(1 - x^{\gamma-1})\right)}, \tag{8}$$

$$r^2 = \frac{\gamma}{\gamma-1}\frac{Q_0\mu_0\alpha_0^2}{\Omega_0^2\rho_0^{2-\gamma}}\left(\frac{1}{x} - \frac{1}{x^{2-\gamma}}\right). \tag{9}$$

The axial density, a constant for cylindrical jets, depends on the distance R to the wind source for parabolically focused ones.

10 THE BENNET PINCH RELATION AND THE DECLINE OF THE AXIAL DENSITY

The solution in the boundary layer expressed by (8)–(9) can be asymptotically matched to the solution (5) or (4) in the field-region. This gives rise to two relations, one of them, common to both asymptotic regimes, being that

$$\frac{K(R)\Omega_0}{\alpha_0} = \frac{\gamma}{\gamma-1} Q_0 \rho_0^{\gamma-1}(R). \tag{10}$$

For classical winds $K(R)$ is just the axial current, and this then reduces to a relation between the gas pressure at the center of the axial pinch column and the total current in it, usually known as a Bennet pinch relation. The matching also gives the scale r_0 of the axial pinch (Eq. (5)), which need not been reproduced here. For parabolic asymptotics (case II of section (3)), this same matching allows us to determine the variation with distance R of the as yet undetermined quantity $K(R)$, or equivalently of the axial density $\rho_0(R)$. It is found that

$$K(R) \sim \frac{1}{\ln(R/\ell)}, \qquad \rho_0(R) \sim \left(\frac{1}{\ln(R/\ell)}\right)^{1/(\gamma-1)}. \tag{11}$$

11 THE EQUATORIAL BOUNDARY LAYER

At a null surface, the first integral $\alpha(a)$ diverges and again gas pressure cannot be ignored in the transfield balance. This region has the structure of a sheet pinch. Assuming the first integrals to be constant in it, except of course $\alpha(a)$, a solution can similarly be found in parametric form, and matched to the field-region solution. This yields a relation between $K(R)$ and the equatorial density, $\rho_{eq}(R)$ as

$$\mu_0 K(R) = \sqrt{2\mu_0 Q_{\mathrm{eq}} R^2 \rho_{\mathrm{eq}}^{\gamma}(R)}. \tag{12}$$

This is a sheet Bennet pinch relation which, in the classical case with dipolar-type symmetry, expresses the balance between gas pressure on the equatorial plane and magnetic pressure at the outer boundary of the sheet pinch boundary layer.

12 CONCLUSIONS

Within the framework of our approximations, which are easily met, we have obtained a complete solution of the wind and jet asymptotic structure in terms of the five first-integral functions. It consists of an axial 'needle-shaped' boundary layer region which has the structure of a pressure supported pinch column and a central density which declines only very slowly with distance, if at all. Electric current returns at null magnetic surfaces, which constitute boundary layers having the structure of pressure supported sheet pinches.

The density contrast of these boundary layers, at the axis or at null surfaces, as well as their activity, associated with the electric current flowing in them, makes them stand out quite distinctly on the background of the diffuse and low density field-region. Observable jets might then just be the polar boundary-layer parts of more extended wind structures.

Solutions with a cylindrically focused core and a net Poynting flux output are not available to all winds, but only to those for which the function $\alpha(E - c^2)/\Omega$ (or $\alpha E/\Omega$ in the classical case) has a flat absolute minimum at a non-zero flux value. In this case the wind asymptotically separates into a jet and an equatorial conical wind. From our solutions, the precise shape of magnetic surfaces can be found in all different regions of the asymptotic domain. For lack of space, we do not describe here these results.

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