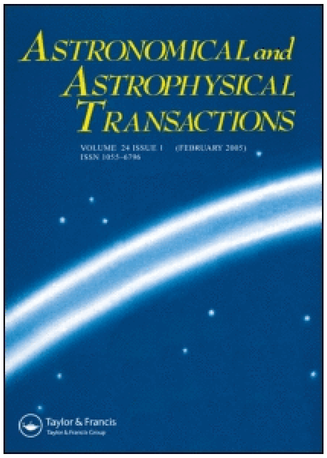

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DYNAMICS OF STAR CLUSTERS AND DENSE NUCLEI

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The study of the dynamical evolution of thermally relaxing star clusters has made enormous progress during the last decades, starting from the pioneering works by Antonov and Lynden-Bell to the present-day complex modelling including astrophysical effects such as stellar evolution, tidal fields and gravitational shocking, and binaries. Nevertheless there remain a couple of unsolved questions for surprisingly simple systems, such as initially rotating star clusters consisting of equal point masses only. 'Thermally relaxing' here means that the two-body relaxation time scale is shorter than the lifetime of the system, as opposed to collisionless systems. An overview of our present understanding is given and recent studies in the field of rotating star clusters and those containing many primordial binaries are given in somewhat more detail.

KEY WORDS Methods: numerical, galaxies: star clusters

1 INTRODUCTION

The dynamics of gravothermal star clusters pose very different challenges to physical and numerical modelling than do collisionless systems. Gravothermal means here that the standard two-body relaxation time scale

$$T = \frac{9}{16\sqrt{\pi}} \frac{\sigma^3}{G^2 m \rho \log(\gamma N)} \tag{1}$$

is smaller than the lifetime of the system. We give T here in our preferred notation of Larson (1970), but originally it was derived by Chandrasekhar (1942). σ, m, ρ, N denote the local one-dimensional r.m.s. velocity dispersion of the stars, stellar mass, mass density, and total particle number, respectively. For γ we use here the empirical value of $\gamma = 0.11$ (Giersz and Heggie, 1994a), but the exact value of the Coulomb logarithm may depend on parameters such as the structure and boundary conditions of the system and is a matter of debate. G is the gravitational constant. In virial equilibrium one can derive (e.g. Spitzer, 1987) that

$$T \propto \frac{N}{\log(\gamma N)} T_{\rm dyn}, \tag{2}$$

where $T_{\rm dyn}$ is a local dynamical time, in the case of a spherical system at radius R given by the local crossing time $T_{\rm cr} = 2R/\sigma$. If we determine these time scales using the local parameters at the half-mass radius of a globular cluster typical values could be (although individual variations by more than an order of magnitude occur) $T_{\rm cr} = 10^6$ yrs and $T = 10^8$ yrs, while the lifetime at least of the globular clusters of our Milky Way is of the order of 10^{10} yrs. Therefore, and because they are at present practically gas-free, such clusters are ideal laboratories to study the gravothermal evolution of self-gravitating N-body systems. In contrast to that collisionless stellar systems are distinguished from gravothermal ones (sometimes the term 'collisional' is used here, but it is confusing with respect to real physical collisions between stars, so here the term 'gravothermal' is preferred) by their very large relaxation time (longer than the age of the objects or the entire universe). Typical collisionless systems are galaxies and the N-body configurations used to study gravitational clustering and structure formation in the universe. Numerical simulation of such systems very often has to discuss unwanted effects of two-body relaxation which are in the models due to small particle numbers (as compared to the real system, see e.g. Steinmetz and White, 1997). Open star clusters and clusters of galaxies typically have dynamical and relaxation times which are not many orders of magnitude apart, so they are neither clearly gravothermal nor collisionless systems. Dense galactic nuclei are gravothermal.

They evolve through sequences of dynamical equilibria (reached on a dynamical time scale) via small perturbations from the relaxation effect (on the much longer relaxation time scale). At least in spherical stellar systems it has been proven by direct comparison with N-body simulation (Giersz and Heggie, 1994a,b; Giersz and Spurzem, 1994; Spurzem and Aarseth, 1996) that the cumulative effect of small angle deflections by distant gravitative encounters between stars indeed is the dominating relaxation effect, so Chandrasekhar's or Larson's time scales are useful to determine the evolutionary time scale of the system. Moreover, the cited papers have also shown that the idea of heat conduction, using a conductivity proportional to ρ/σ, which is equivalent to assuming that the local Jeans length is an analogue to the mean free path, described very accurately the N-body evolution. This is nothing other than the confirmation of a proposition by Lynden-Bell and Eggleton (1980). In this approximation the dynamical evolution of a star cluster can be analysed by solving quasi-hydrodynamic equations (moment equations of the Fokker–Planck equation) with a heat flux closure equation (Bettwieser, 1983; Heggie, 1984; Louis and Spurzem, 1991). Such a model has been successfully used to detect gravothermal oscillations (Bettwieser and Sugimoto, 1984), and it is called in its latest species as described by Spurzem (1996, 1999) an anisotropic gaseous model. In an alternative approach the model equations are solved semi-numerically, by searching for a self-similar solution (reducing the time-dependent partial differential equations to a set of ordinary ones, see e.g. Lynden-Bell and Eggleton, 1980; Inagaki and Lynden-Bell, 1983; Louis and Spurzem, 1991, and most recently for rotating clusters Lynden-Bell, 2001).

Gaseous models use further approximations, not just the Fokker–Planck approximation (dominance of small angle encounters), such as the local approximation (derivation of collisional terms as a function of local quantities, no orbit average) and the already mentioned phenomenological heat flux closure equation. Thus there are other kinds of useful physical models of gravothermal star clusters, such as the direct solution of the orbit-averaged Fokker–Planck equation (Cohn, 1980; Chernoff and Weinberg, 1990; Lee, Fahlman and Richer, 1991; Takahashi, 1995, 1996, 1997; Takahashi, Lee and Inagaki, 1997, and this is only a small and by no means exhaustive list of important papers here), and Monte-Carlo methods (Hénon, 1975; Spitzer, 1975; Stodolkiewicz, 1982, 1986), recently improved and upgraded by Giersz (1998, 2000) and Watters, Joshi and Rasio (2000), Joshi, Rasio and Portegies Zwart (2000), Joshi, Nave and Rasio (2000). Both Monte-Carlo and Fokker–Planck models (henceforth MC and FP) are so far limited to spherical symmetry and the Fokker–Planck limit, i.e. they cannot follow dynamical adjustments (while gaseous models can) and are not *a priori* applicable if close binaries (correlations of two or more bodies) or large angle encounters (e.g. in the star cluster surrounding a massive black hole) play a role. First orbit-averaged FP models of rotating, axisymmetric clusters have been published (Einsel and Spurzem 1999, see more detailed discussion below), and the effects of binaries are included by suitable additional terms (such as a heating due to close binaries).

At the other extreme, regarding the number of approximations contained in the model, we find the method of direct N-body simulation, solving the Newtonian equations of motion for N bodies with the optimal accuracy required for the study of relaxation; see for reviews Aarseth (1999), Spurzem (1999), and also with a broader scope Makino and Taiji (1998). Very often direct N-body models are used to 'prove' something predicted by other models (such as gravothermal oscillations by Makino, 1996), but what is generally less appreciated is that there are also fundamental questions of the validity of results of N-body simulations (Miller, 1964). They are connected to the exponential divergence of trajectories in chaotic N-body systems which occurs as soon as after about a crossing time (Goodman, Heggie and Hut, 1993). Generally it is believed, however, that the careful use of the results of N-body models (e.g. by averaging either locally or by ensemble averages, see Giersz and Heggie, 1994 a,b) does yield meaningful results, and also some theoretical studies support this view (Quinlan and Tremaine, 1992).

2 ASTROPHYSICS

The confirmation by direct solutions of the orbit-averaged Fokker–Planck equation of gravothermal collapse induced by two-body relaxation, which leads to infinite central density in a finite time (Cohn, 1980) led to the question of why there are so many galactic globular clusters which have obviously not yet collapsed into singular central density profiles. Analytic approximations and the direct numerical simulations of close three-body encounters (Heggie, 1975; Hut, 1983; Hut and Bahcall, 1983; Hut, 1985; Hut, 1993; Goodman and Hut, 1993) introduced the idea of formation of close binaries by three-body encounters in high-density phases and showed that such binaries, once formed with a suprathermal binding energy ('hard binaries') get even harder by the averaged statistic effect of superelastic three-body encounters. Such three-body encounters contradict the Fokker–Planck approximation; the heating effect of a few binaries in a large cluster could, however, successfully be included by a phenomenological heating term (Bettwieser and Sugimoto, 1984; Goodman, 1987; Cohn *et al.*, 1989). For large star clusters it turned out that such heating causes quasiperiodic oscillations in the post-collapse phase. Therefore post-collapse clusters can have a similar, non-singular structure as in pre-collapse.

There are many effects in real globular star clusters, which are 'hostile' for the occurrence of gravothermal oscillations. Mass loss by stellar evolution (Chernoff and Weinberg, 1990), the presence of a mass spectrum (Murphy *et al.*, 1990; Kim, Lee and Goodman, 1998) with very heavy stars, mass loss due to a galactic tidal field (Takahashi, Lee and Inagaki, 1997; Takahashi and Lee, 2000) and disk shocking (Gnedin, Lee and Ostriker, 1999) are some of the effects which shorten lifetimes of clusters, and decrease their mass. The smaller the mass, the smaller the possibility for the occurrence of gravothermal oscillations.

Furthermore it is now generally believed that star clusters form with a large fraction of initial, so-called primordial binaries. They can also prolong the initial phase of the first collapse of a star cluster by a factor of three to four due to close encounters between binaries ('binary burning') (Gao *et al.*, 1991; Hut *et al.*, 1992; Kroupa, 1995).

3 ROTATION

Observations show that flattening is a common feature of globular clusters, which has been known since the early work done by Pease and Shapley (1917). Measuring projected ellipticities $e = 1 - b/a$ of large globular cluster samples White and Shawl (1987) derive a mean $\bar{e} = 0.07 \pm 0.01$ for 99 clusters in the Milky Way, and Staneva *et al.* (1996) find $\bar{e} = 0.086 \pm 0.038$ for 173 clusters in M31, with maximum values 0.27 and 0.24 of individual globulars, respectively. Kinematical data, i.e. radial velocities of large numbers of cluster members, reveal that this flattening may indeed be explained in terms of rotation, and that the minor axes are nearly coincident with the determined rotation axes (Meylan and Mayor, 1986). Dust obscuration, anisotropy or tidal distortion are able to explain individual cases of flattening, but can statistically be ruled out as the main mechanism (White and Shawl, 1987). Significant ellipticity variations are found within globular clusters (e.g. Geyer *et al.*, 1983), and these also partly coincide with the rotation curves obtained by fits to the radial velocity data with some parametrization specified for the velocity field (Meylan and Mayor, 1986). Moreover, Kontizas *et al.* (1990) show that the outer parts of globulars in the Small Magellanic Cloud are obviously rounder than the parts inside the half mass radius and it is likely that their structure differs from that of the galactic globular clusters because they are younger (in general) and subject to different tidal forces. The importance of age for the interpretation of observed ellipticities has already been emphasized by Frenk and Fall (1982), who undertook eye-estimates of cluster ellipticities in the Milky Way and the Magellanic

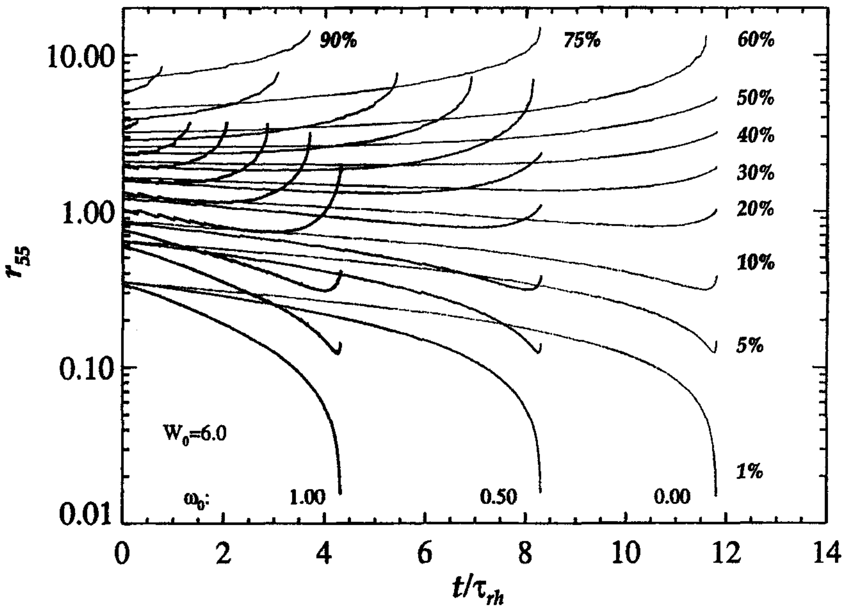

Figure 1 Evolution of mass shells (Lagrange radii) for the model $(W_{0,i}, \Omega_{0,i}) = (6.0, 0.60)$. Shown are the radii for mass columns containing the indicated percentage of total mass in the direction of the $\theta = 54.74°$-angle, the tidal radius $r_{\rm tid}$ determined from $(\phi(R, z) = E_{\rm tid})$, and the core radius $r_c = (9\sigma^2/(4\pi G\rho)^{1/2}$.

Clouds, the latter being slightly larger than the former, which is explained again in terms of internal globular cluster evolution. This view is supported by studies relating Milky Way globular cluster ellipticities to the cluster concentration parameter $c = \log(r_t/r_c)$ (White and Shawl, 1987), where r_t is the tidal radius and r_c is the core radius, or to the half mass relaxation time t_{r_h} (Davoust and Prugniel, 1990), both representing the evolutionary status of the respective globular cluster. In these two investigations, the average flattening of the dynamically younger systems is significantly larger as well, indicating that loss of angular momentum originating from diffusion past the escape velocity on relaxation time scales decreases the ellipticity of a cluster.

Indeed, Agekian (1958) suggested a model in which specific angular momentum is lost due to a relatively large fraction of escaping stars residing in the tail of a Maxwell velocity distribution shifted towards the direction of rotation as compared to the fraction residing in the opposite direction having less angular momentum. Considering this effect for every volume element of rotating ellipsoids he obtained a critical ellipticity of $e \leq 0.74$ below which the systems become rounder with time.

In a study by Einsel and Spurzem (1999) the evolution of the distribution function f as a function of the energy E and the z-component of the angular momentum

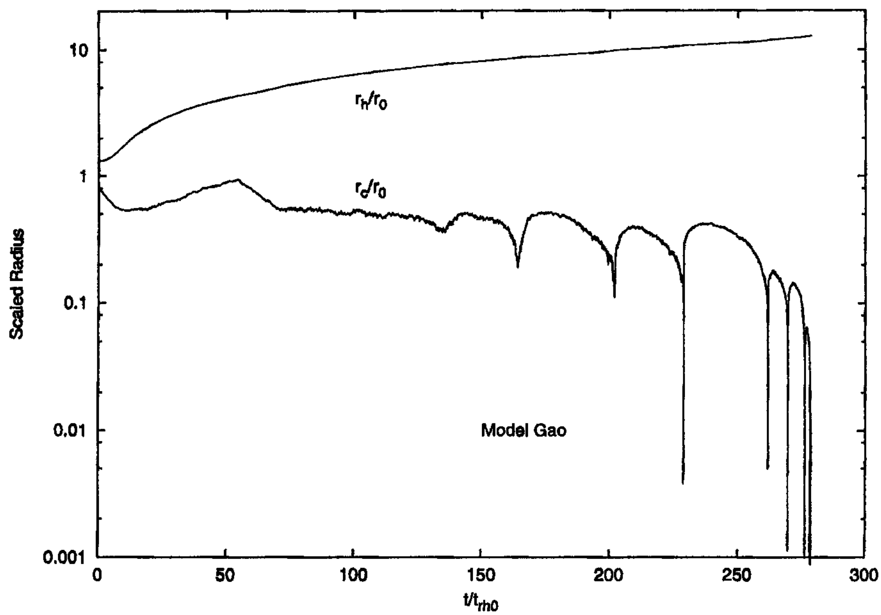

Figure 2 Evolution of the core and half-mass radii (scaled by the scale length of the Plummer model) as a function of time in units of initial half-mass relaxation time; this time unit is used in all following figures.

J_z, both representing velocity variables, with time t was followed. E and J_z were considered as the only isolating integrals and any possible non-ergodicity on the hypersurface in phase space given by E and J_z due to any third integral was neglected.

As a result we find that the evolution of the star cluster to core collapse is accelerated by a factor of up to three due to the initial rotation of the cluster.

We find that large amounts of initial rotation drive the system into a phase of strong mass loss while it contracts slightly. The core rotates even faster than before although the angular momentum is transported outwards. At the same time the core heats. Given these features we associate this phase with the gravo-gyro phase found by Hachisu (1979). The total collapse time is shortened appreciably by this effect, but the increase in central angular momentum levels off after about $2t_{r_h,i}$ indicating that the source of this 'catastrophe' is depleted. The final state is an increase in central angular velocity again, but with a rather small power of the central density – the same power as for the central velocity dispersion during self-similar contraction.

The rotation curves decrease with time, as do the ellipticity profiles, but the relative shapes do not vary much. The maximum values of rotational velocity and ellipticity occur at about the half mass radius, respectively.

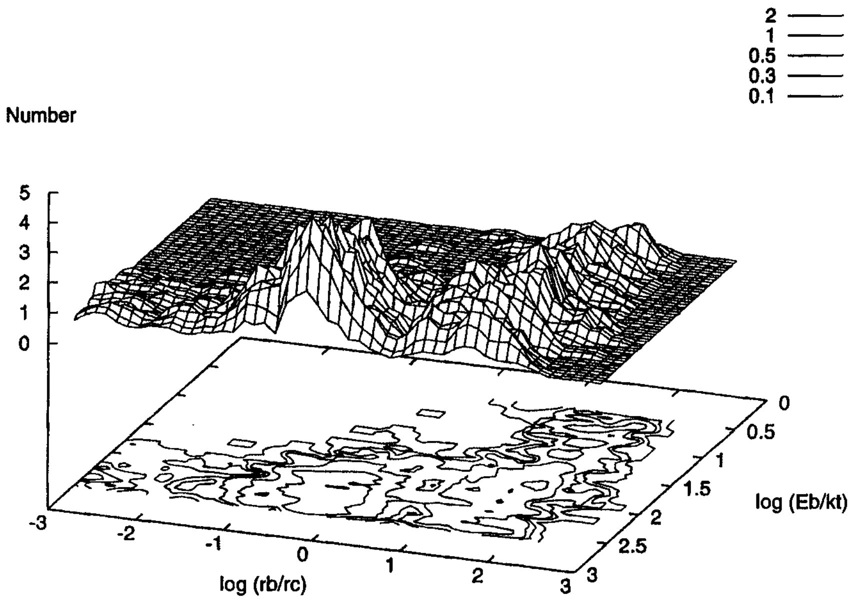

Figure 3 Snapshot plots of 3D distributions of bound binaries. Plotted is the frequency of binaries in logarithmically equidistant bins of binding energy in kT (initial) and position in units of initial core radius.

4 BINARIES

Finally we report that a new approach outlined in Spurzem and Giersz (1996) to follow the individual formation and evolution of binaries in an evolving, equal point-mass star cluster is extended for the self-consistent treatment of relaxation and close three- and four-body encounters for many binaries (typically a few percent of the initial number of stars in the cluster mass initially). The distribution of single stars is treated as a conducting gas sphere with a standard anisotropic gaseous model. The Monte Carlo technique is used to model the motion of binaries, their formation and subsequent hardening by close encounters, and their relaxation (dynamical friction) with single stars and other binaries. The results are a further approach towards a realistic model of globular clusters with primordial binaries without using special hardware. We present, as our main result, the self-consistent evolution of a cluster consisting of 300000 equal point-mass stars, plus 30000 equal mass binaries over several hundred half-mass relaxation times, well into the phase where most of the binaries have been dissolved and evacuated from the core (Giersz and Spurzem, 2000). The cluster evolution is about three times slower than found by Gao *et al.* (1991). Other features are rather comparable. At every moment we are able to show the individual distribution of binaries in the cluster. The figures display the time evolution of core and half-mass radii (showing the delay of core collapse due to 'binary burning') and the distribution of binaries in a binding energy position plane. The selected time for this plot is a maximum of density during gravothermal oscillations. Note how some binaries are drawn into the centre of the cluster. The corresponding distributions of binaries for other evolutionary phases can be found in Giersz and Spurzem (2000).

5 SUMMARY AND OUTLOOK

The study of the interplay between relaxation and rotation in star clusters will be very important in the understanding of the dynamical evolution of galactic nuclei. While models of non-rotating star clusters with a central massive star accreting black hole exist for various methods (Frank and Rees, 1976; Marchant and Shapiro, 1980; Quinlan and Shapiro, 1990) the effect of rotation has not yet been studied self-consistently.

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