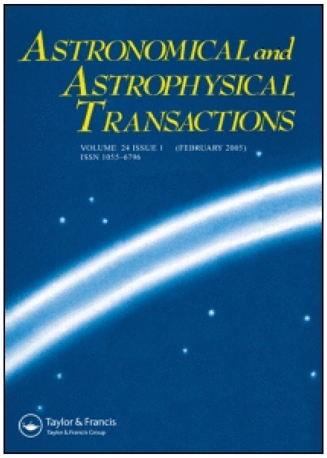

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DYNAMICS AND STABILITY OF RESONANCE RINGS IN GALAXIES

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A stellar-dynamical model of a one-fold resonance ring galaxy is analyzed and its stability status is determined. The model is applied to the ring galaxy NGC 7020.

KEY WORDS Dynamics, rings, resonance, stability

1 INTRODUCTION

The problem investigated in this paper deals with rings in galaxies. NGC 7702 is a typical example of a galaxy exhibiting in pure form a 'nucleus-ring' pattern, which is somewhat reminiscent of Saturnian rings. According to Buta (1991), this galaxy even possesses two well separated rings. Antonov and Nuritdinov (1983) constructed a simple stellar-dynamical model of a ring with a special emphasis on collective effects. Here we analyze a stellar-dynamical model of a ring galaxy focusing on the 1:1 resonance motion of individual stars, and determine its stability status. We apply our model to the ring galaxy NGC 7020 whose structure was carefully investigated by Buta (1990).

2 THE MOTION OF A STAR IN AN ELLIPTICAL RING

Consider an axisymmetric galaxy possessing a homogeneous stellar ring with an elliptic cross section

$$(x_3^2/a_3^2) + (\xi^2/a_1^2) = 1, \quad a_1 \geqslant a_3, \tag{1}$$

where $\xi = R - R_0$. Its internal potential is

$$\varphi_r = \text{const} - \alpha_1\xi^2 - \alpha_3 x_3^2, \tag{2}$$

where the coefficients $\alpha_1 = 2\pi G\rho a_3/(a_1 + a_3)$; $\alpha_3 = 2\pi G\rho a_1/(a_1 + a_3)$. Stars of the ring are also affected by the gravitational field of the galaxy. The galactic potential can be expanded into a Taylor series: to within the second-order terms, we have

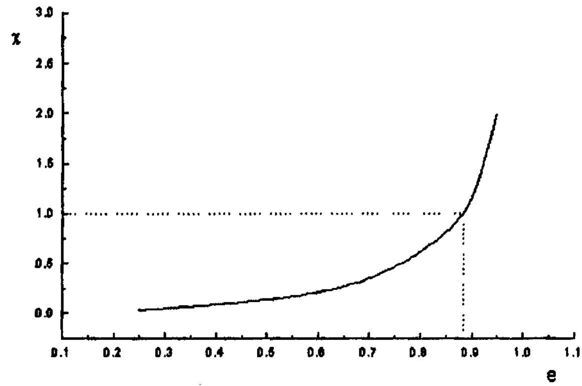

Figure 1 An interesting regularity is immediately apparent: resonance ring cross sections in weakly oblate galaxies should be prolate in the radial direction and those in galaxies with oblateness values exceeding the critical threshold ($e > e_{cr}$) should also be prolate in the $0x_3$ axis direction. The critical eccentricity e_{cr} for our χ is determined from equation (12). For NGC 7020 we have $e_{cr} \approx 0.88$, which is very close to the average e for this galaxy. Therefore, within the framework of the given resonance model the ring in NGC 7020 has an approximately circular cross section.

$$\varphi_g = \text{const} - \Omega_c^2 R_0 \xi - A_1 \xi^2 - A_3 x_3^2, \tag{3}$$

where $\Omega_c^2(R_0) = -(1/R)(\partial\varphi_g)/(\partial R)_{R=R_0}$ is the circular angular velocity squared,

$$A_1 = -\frac{1}{2}\left(\frac{\partial^2 \varphi_g}{\partial R^2}\right)_{\substack{R=R_0\\x_3=0}},$$
$$A_3 = -\frac{1}{2}\left(\frac{\partial^2 \varphi_g}{\partial x_3^2}\right)_{\substack{R=R_0\\x_3=0}}.$$

The total internal potential then is

$$\varphi = \varphi_r + \varphi_g = \text{const} - \Omega_c^2 R_0 \xi - (\alpha_1 + A_1)\xi^2 - (\alpha_3 + A_3)x_3^2. \tag{4}$$

In cylindrical coordinates (R, θ, x_3) the equations of motion of a star are

$$\ddot{R} = \frac{\partial\varphi}{\partial R} + R\dot{\theta}^2, \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(R^2\dot{\theta}\right) = \frac{\partial\varphi}{\partial\theta} = 0, \quad \ddot{x}_3 = \frac{\partial\varphi}{\partial x_3}. \tag{5}$$

In a reference frame rotating with circular angular velocity Ω_c

$$\theta(t) = \tilde{\theta}(t) + \Omega_c t \tag{6}$$

equations (5) can be rewritten in the linear approximation with respect to ξ and $\dot{\tilde{\theta}}$ as follows

$$\begin{aligned} \ddot{\xi} &= -\omega_1^2 \xi, & \omega_1^2 &= 3\Omega_c^2 + 2(\alpha_1 + A_1), \\ \ddot{x}_3 &= -\omega_3^2 x_3, & \omega_3^2 &= 2(\alpha_3 + A_3). \end{aligned} \tag{7}$$

Table. Solutions of equation (14) for some e.

e	χ	ω_1^2	ω_2^2	ω_3^2
0.25	0.029431	5.341189	5.133680	−0.176781
0.50	0.139000	5.442480	5.005362	−0.140321
0.75	0.461141	5.439000	4.615930	−0.078983
0.85	0.810901	5.224364	4.292386	−0.060398
0.88	0.998620	5.091055	4.151278	−0.060223
0.90	1.169900	4.970489	4.037862	−0.063190
0.95	1.995270	4.467905	3.633548	−0.093558

3 MOTION OF STARS AT A FREQUENCY RESONANCE

The complete solution to equations (7) is

$$\xi = C_1 \cos(\omega_1 t + k_1), \quad x_3 = C_3 \sin(\omega_3 t + k_3). \tag{8}$$

Here we consider a particular elementary case of the 1:1 resonance

$$\omega_1 = \omega_3, \quad k_1 = k_3. \tag{9}$$

If $C_1/C_3 = a_1/a_3$ stars move in ellipses that are homothetic to the boundary ellipse and the ring consists of concentric tori with homothetic elliptical cross sections. The condition of conservation of boundary (1) is satisfied.

It follows from equalities (9) that the ring axial ratio $\chi = a_3/a_1$ is related to the parameters of the galaxy model as follows

$$\Psi = 3\Omega_c^2 + 2(A_1 - A_3) = 4\pi G\rho(1-\chi)(1+\chi)^{-1}. \tag{10}$$

4 THE GALAXY MODEL

Consider a simple galaxy model in the form of a stratified inhomogeneous spheroid with normalized density distribution

$$\rho(m^2) = (1 + \beta m^2)^{-3/2}, \tag{11}$$

where

$$m^2 = (R^2/\tilde{a}_1^2) + (x_3^2/\tilde{a}_3^2),$$

$0 \leqslant m \leqslant 1$, and $\tilde{a}_1$ and $\tilde{a}_3$ are the semiaxes of the galaxy. It then follows from equation (10) that

$$\Psi(e) = \frac{4\pi G}{e^2 + \beta R_0^2}\left(2\sqrt{\frac{1-e^2}{e^2+\beta R_0^2}}\left(\frac{e^2(F-E)}{\beta R_0^2} + E\right) - \frac{2 - e^2 + \beta R_0^2}{(1+\beta R_0^2)^{3/2}}\right). \tag{12}$$

Here e is the eccentricity of the galaxy; β,a constant estimated by smoothing the observed data; R_0, the central radius of the ring normalized to $\tilde{a}_1$, and F and E are elliptic integrals. Formula (12) defines, through parameters R_0 and β, the $\chi = a_3/a_1$ ratio for the ring as a function of the eccentricity e of the galaxy. Our calculations showed χ ***to depend rather weakly*** on β and $R/\tilde{a}_1$ over a wide range of parameter values. The results can therefore be applied, with no significant errors involved, to many ring galaxies with azimuthal symmetry. This condition is satisfied, for example, by the ring galaxy NGC 7020. The corresponding plot is shown in Figure 1.

5 STABILITY

Let the elliptic cross section at each instant be slightly deformed into another ellipse through a linear affine transformation

$$\delta\xi = a\xi + bx_3, \quad \delta x_3 = c\xi + dx_3, \tag{13}$$

where the functions of time a, b, c, d are small allowing higher than second-oder terms to be neglected. We obtain, after a number of transformations, the main following bicubic dispersion equation of the problem

$$\omega^6 + K_1\omega^4 + K_2\omega^2 + K_0 = 0 \tag{14}$$

with known factors K_1, K_2 and K_0. The solutions of this equation for some eccentricities e are given in the table.

In the case of slow evolution isoentropic invariants should be conserved. Since we have a harmonic oscillator, the products of squared amplitudes with the frequency are conserved

$$a_1^2\sqrt{3\Omega_c^2 + 2(\alpha_1 + A_1)} = a_3^2\sqrt{2(\alpha_3 + A_3)}. \tag{15}$$

Additionally, the mass per unit ring length should be conserved. The set of equations (12), (15) and $\rho a_1 a_2 = \text{const}$ describes the secular evolution of the parameters of this ring.

6 CONCLUSIONS

The most interesting corollary is the dependence of the shape of the ring cross section on the oblateness of a spheroidal galaxy. In particular, NGC 7020 should have an almost circular ring. It follows from the solution of the dispersion equation for the same galaxy that stellar orbits in its ring should be weakly unstable. Stellar orbits in the ring of this galaxy will drift from the 1:1 resonance and the ring as a whole will slowly evolve. See Kondratyev (2000) for a more detailed discussion.

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