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INTERIOR STRUCTURE OF THE SUN

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This paper describes a computational method of estimating physical and chemical properties within the solar interior without employing calculations involving opacities. Instead of using opacities to help determine how interior solar temperatures vary with the radial distance between the centre of the sun and its 'surface', an iterative technique employing empirical adiabatic 'cooling' and a fusion energy production rate expression were employed for this purpose. Other iterative calculations were also made (nearly simultaneously) to ensure that all known solar constraint conditions were precisely satisfied (except for the photospheric 'surface' temperature) during this computational process. In addition, all calculations could be performed using a conventional PC employing an Intel Pentium CPU and a computer program coded in ANSI C. Due to the simplifications that were possible using the techniques employed during this study, as well as the advantages associated with using a programming language that produces machine code when compiled, all solar structural details could be generated very rapidly using an ordinary computer. The results of this study were compared with more conventional results obtained by others. This comparison indicated that the methods employed within this paper produced interior intensive solar properties that were in reasonable agreement with similar properties obtained by employing more sophisticated computational approaches. Although it is not claimed that the results generated during this study are any better than more conventionally obtained findings, it is thought that these results, as well as our computational methods, are interesting and potentially useful to others. In particular, it is thought that the techniques outlined in this paper may provide a useful introduction to more complicated techniques of solar modelling.

KEY WORDS   Solar, interior, structure, model, fusion, hydrogen, PC

1   INTRODUCTION

Even the most recent theoretical solar structure calculations only permit estimates of the sun's interior physical properties; and, probably the greatest difficulty in this kind of effort, as well as a cause of the greatest uncertainties in the results, is associated with the use of opacities (i.e., the resistance of solar matter to the transport of radiation) in making the calculations. There are at least two significant uncertainties in solar structure calculations which are due to opacities. One of

these uncertainties involves the fact that opacity factors are typically calculated at extremely high temperatures and pressures. Therefore, experimentally corroborating these calculated results is difficult, if not impossible, using the same conditions that may occur within the solar interior. However, the main problem with opacities is connected to the fact that opacity factors are highly dependent upon the heavy metal content (sometimes abbreviated as HM which, in this context, means all elements heavier than helium) of any particular mass element that solar radiation may be passing through. And, since there is still a considerable uncertainty in how heavy metal concentrations vary with radial distances from the sun's centre, there is a corresponding uncertainty in the accuracy of the interrelated opacity calculations. In further support of these comments, the reader may wish to refer to some of the comments made by others with regard to opacities (i.e., see Gough, et al., 1966; Guenther and Demarque, 1996; and Swartzschild, 1958). In any case, the main point here is that there are at least two kinds of errors in opacities. One of these errors involves uncertainty in the assumptions that one uses to calculate opacities in the first place and the other error involves uncertainty in the chemical composition within the solar interior. So, even if the calculations themselves are flawless, for a given (assumed) chemical composition in a specific region within the sun, the overall errors can still be quite large given the uncertainty in the true chemical composition within the solar interior.

In the light of opacity related complications and uncertainties associated with the overall problem of finding reasonable solutions for interior solar structures, we wondered if it would be possible to somehow eliminate opacity calculations altogether and still come up with reasonable theoretical solutions for interior solar structures. The techniques employed in, and results related to, solving this problem are the main subjects of this paper.

2   TERMINOLOGY AND STRUCTURAL CONSIDERATIONS

2.1   Standard Solar Model

A standard solar model is a theoretically constructed representation of the interior physical, chemical, and structural properties of the sun, and there are numerous and lengthy descriptions (as well as criticisms) of standard solar models, derived using slightly varying assumptions and slightly differing computational approaches, in the previously published literature covering this subject (e.g., see Bahcall and Ulrich, 1988; Guenther and Demarque, 1996; and Guenther et al., 1992).

2.2   Chemical and Structural Properties

Many, but not all, current theories regarding the interior structure of the sun, divide it into three principal regions. The innermost region, sometimes referred to as the core, is thought to consist primarily of a mixture of completely ionized hydrogen and helium with much smaller concentrations of partially ionized heavy

'metal' elements. This is the only region, within the solar interior, where pressures and temperatures are high enough to sustain fusion reactions at appreciable rates. This central spherical region of the sun is typically considered to have a radius of approximately $0.25R_S$, where R_S is the overall radius of the sun which is about 6.960×10^{10} cm (Ulrich and Rhodes, 1983).

Surrounding the core region is a second zone of very high pressure and temperature solar material. However, in this region, the pressures and temperatures are not high enough to promote the fusion process at appreciable rates. This internal region of the sun is normally referred to as the radiative zone. Its boundaries lie between the core and base of the solar convection zone (to be discussed in more detail below).

Apparently, the two innermost solar regions share a common physical property. This property is stagnicity. In other words, solar material within any particular 'thin' spherical shell of matter (anywhere within these two inner parts of the sun) does not seem mix very well with material enclosed by adjacent thin spherical shells of solar matter.

The third principal solar region is the outer convection zone. We can see the outermost turbulent 'surface' of this solar region using appropriate telescopic instruments. This observational evidence indicates that solar matter continually rises from deep within this relatively thick shell of material up to the solar surface. Recent helioseismic measurements indicate that the base of the solar convection zone is located at $0.713R_S$ (Christensen-Dalsgaard et al., 1991).

2.3 Modelling Constraints

Some of the similarities between a standard solar model and the solar model to be described later in this paper involve obvious constraint conditions that must be satisfied no matter how a particular solar model is arrived at. Two of these constraint conditions involve the sun's mass, and size. The actual values we attempted to match here were: 1.989×10^{33} g (Cohen and Taylor, 1986, 1987) enclosed by a perfect sphere having a radius of 6.960×10^{10} cm (Ulrich and Rhodes, 1983).

A third constraint condition involves the sun's energy emission rate (often referred to as its luminosity) from the solar 'surface' or the photosphere. The value we used was 3.85×10^{33} erg sec^{-1} (Hickey and Alton, 1983).

A forth constraint condition that must be met by all solar models involves a near-surface chemical composition constancy constraint related to the turbulent mixing that occurs within the convection zone. In other words, the mass percentages of all elements must be nearly uniform between the base and 'surface' of the convection zone. In any case, the values we used there were: $X = 0.740$ (estimated), $Y = 0.240$ (Hernandez and Christensen-Dalsgaard, 1994), and $Z = 0.020$ (estimate based on Bahcall and Pinsonneault, 1992).

A fifth constraint condition is related to the temperature of solar matter at the base of the convection zone. This temperature has been estimated to be in the vicinity of 2.0×10^6 K.

A sixth constraint condition is related to the conversion of some of the sun's initial quantity of hydrogen into helium since the onset of the fusion process about 4.52×10^9 yr ago (Guenther and Demarque, 1996). Using this time period, as well as other technical data, we estimated that approximately 3.6% of the original solar mass has been converted from hydrogen into helium since the onset of the solar fusion process. The result of this calculation means that if the sun's overall primordial concentration of hydrogen was 74.0% (by mass), then the average concentration of hydrogen (throughout the entire sun) must now equal 70.4% (by mass). This is a subtle but extremely important consideration that must be used to find the variation in the mass concentration of hydrogen, with respect to solar radius, that exists at the present time within the sun.

Two other possible constraints involve the near surface solar pressure and temperature conditions. Atmospheric pressures near the solar 'surface' must approach zero while the near surface (photospheric) temperature condition is typically assumed to be approximately 6000 K. But these conditions were not given a great deal of weight within our calculations because, as long as the calculated values are very low (relative to the near-subsurface values), they have practically no effect on the other more important constraint conditions mentioned in the preceding paragraphs.

3 THE FUSION PROCESS

At present, there are only two fusion processes that are believed to be active within the core region of the sun. The primary solar energy generating fusion process, already mentioned above, involves a relatively direct sequence of nuclear reactions which converts hydrogen into helium (often referred to as the proton–proton or pp fusion cycle). An additional, but secondary fusion process (often referred to as the carbon–nitrogen–oxygen or CNO fusion cycle) is believed to account for a much smaller fraction of the total energy generated within the sun. According to some estimates, about 1.2% of the total power generated within the sun is due to the CNO fusion cycle. In this process, different mechanisms, which have the same net effect of converting hydrogen nuclei into helium, are thought to be at work. For additional details regarding these nuclear processes, the reader may wish to refer to other more detailed sources of information covering these subjects (e.g., see Brodenheimer, 1989; Kuchowicz, 1976; Hansen and Kawaler, 1995; Parker, 1986; Phillips, 1996; and Schwarzschild, 1958).

The two fusion cycles mentioned above are believed to account for all of the radiant energy (as well as other forms) that is emitted from stars that are similar to the sun in mass, size, and elemental composition. In addition, about 2–6% of the observable energy emission rate of the sun is believed to be carried away by neutrinos (e.g., see Bahcall, 1989). Since the energy carried away from the sun by neutrinos can only be estimated, and not precisely measured, the total solar energy emission rate must also be estimated.

Subsequent solar modelling calculations, discussed in more detail below, assume that the solar energy emission rate, due to the hydrogen pp fusion process, is nearly equal to the directly observable solar luminosity of 3.85×10^{33} erg sec^{-1} at the sun's surface (Hickey and Alton, 1983). It has been further assumed that the CNO fusion cycle contributes the remaining quantity of energy emitted from the sun and that this extra quantity of energy is carried away by neutrinos and by the 'solar wind' even if some of the energy associated with these effects is actually a result of the pp fusion cycle.

Considering the hydrogen fusion process as the only source of total solar energy output throughout the past lifetime of the sun, it is possible to estimate the total mass of hydrogen that has been converted into helium since the onset of the solar fusion process. This was accomplished by multiplying the solar luminosity of 3.85×10^{33} erg sec^{-1} by a correction factor of 0.84 (estimated using previously published data in Bahcall and Pinsonneault, 1992). Then, it was possible to calculate that about 7.19×10^{31} g of hydrogen must have been converted into nearly 7.14×10^{31} g of helium within the sun's interior since the onset of the fusion process. This conversion of hydrogen into helium represents a total hydrogen mass loss, and thus helium mass gain, equal to about 3.6% of the present solar mass of 1.9891×10^{33} g (Cohen and Taylor, 1986). An earlier average luminosity correction factor of roughly 0.81, estimated from data published by Schwarzschild (1958), pp. 206–207, was not used in the calculation above because it was thought that the more recent estimate of 0.84, noted above, was probably more accurate. However, the point here is to emphasize that there is some uncertainty in our use of the 0.84 correction factor value.

The slight difference between the hydrogen mass loss and helium mass gain (ca. 5.12×10^{29} g, from the data listed in the preceding paragraph) represents the amount of solar matter that has been converted into energy and thus dispersed into space (primarily in the form of electromagnetic radiation) since the onset of the fusion process within the sun. This mass 'defect' represents about 0.026% of the present solar mass. This actual loss in solar mass is so small (relative to the total mass of the sun) that it is nearly the same as the uncertainty in the mass of the sun which is about ±0.02% of the present solar mass. Additional mass losses due to the 'solar wind' have been estimated to be in the vicinity of 10^{-14} solar masses per year (Cassinelli and MacGregor, 1986; Ostlie and Carroll, 1996). Therefore, both of these mass losses have been ignored in our calculations because, for all practical purposes, the overall mass of the sun has not changed significantly since the onset of the fusion process within the solar interior.

The calculations noted above may seem trivial but they have important consequences regarding the determination of potentially accurate models of the solar interior. In fact, assuming that the primordial mass percentages of hydrogen, helium, and heavy metal elements within the sun were exactly 74.0, 24.0, and 2.0% (respectively) prior to the onset of the fusion process, then the integrated mass percentages of hydrogen, helium, and heavy metal elements (within all successive thin shells of matter from the centre of the present-day sun up to its surface) must add up to nearly 70.4, 27.6, and 2.0% (respectively). And, the integrated overall

mass of the sun must add up to its present-day mass with due allowances made for appropriate uncertainties in all of these quantities.

4 COMPUTATIONAL CONSIDERATIONS

4.1 Elimination of Opacity Calculations

The main difference between the model determined as a result of this study and more conventional (and modern) standard solar models involves the computational methods that were employed in finding temperature variations between the solar centre and surface. More specifically, a general practice is to embed opacity related interpolation tables (typically generated beforehand using a separate computer program as well as modem supercomputers) within the computer code that is designed to iteratively find out how internal solar temperatures change with the sun's radius. Additional computational details, as well as classical approaches, are described more comprehensively elsewhere (e.g., see Lane, 1870; Eddington, 1926; Chandrasekhar, 1939; Schwarzschild, 1958; Sears, 1964; Kippenhahn et al., 1967; Clayton, 1968; Cox and Giuli, 1968; Hanson and Kawaler, 1995; Kippenhahn and Weigert, 1990). However, the main problem with opacity dominated calculations is that the exact chemical composition, within the interior of the sun, is not very well known. So, no matter how accurate the opacity related calculations are, the final results are still subject to question (Guenther et al., 1989; Iglesias and Rogers, 1991; Guenther and Demarque, 1996; Bahcall et al., 1997).

In developing the alternative solar model described within this paper, no opacity constants or opacity calculations were employed at all. Instead, changes in interior solar temperatures were calculated iteratively, between successive 'thin' spherical shells of solar matter, by assuming that there was an 'adiabatic-like' temperature drop related to the pressure drop behaviour between these shells of solar matter. The same equation relating the before and after pressure/temperature behaviour of an ideal gas, undergoing an adiabatic expansion, was employed during this procedure. However, instead of employing the usual adiabatic exponent that applies in the case of an ideal gas (i.e., 0.4), empirical exponents were employed. Although highly ionized solar matter (within stars similar to the sun) behaves in many respects like an ideal gas, it does not behave like an ideal gas in all respects because each successive thin shell of solar matter, between the sun's centre and its surface, experiences a net outward flux of radiant energy (generated by fusion reactions within all of the thin shells of solar material that happen to be enclosed by the particular thin shell in question). This radiant energy may be scattered or absorbed and then re-emitted within each matter shell it passes through. So, this process tends to keep all successive shells of solar matter much hotter than would otherwise be the case if the enclosed shells of matter contained only ideal gas particles and no radiation effects at all were present. Therefore, one cannot expect to find ideal gas adiabatic exponents that will be capable of accurately describing the real pressure/temperature behaviour within any star.

Using this approach, only two different empirical exponents were required in order to produce a reasonably realistic model of the interior structure of the sun. One exponent was needed for the region between the centre of the sun and the base of the convection zone. A second exponent was needed between the base of the convection zone and the solar surface. Both of these exponents were found by trial by repeating all of the interrelated calculations until all of the necessary boundary conditions were satisfied. The simplicity of this approach (i.e., the use of only one exponent in the radiative zone and one in the convection zone) is in stark contrast to some earlier methods that have employed at least 40 different 'polytropic' exponents, as well as interrelated opacity calculations, in making interior solar structure calculations (e.g., see Schwartzschild, 1958, p. 259). More modern techniques, involving opacity tables calculated using supercomputers, are even more complicated.

4.2 Nearsurface Chemical Composition

In some solar modelling calculations, the material in the exterior regions of the sun (i.e., in the convection zone) is considered to be primordial. In other solar modelling approaches, the present external chemical composition of the sun is believed to have changed slightly from its original composition due to gravitationally induced inward diffusion of elements heavier than hydrogen (e.g., see Bahcall et al., 1995; Bahcall and Loeb, 1990; Christensen-Dalsgaard et al., 1993; Richard et al., 1996); and, in still other theoretical treatments, changes in the surface primordial composition are thought to be due to the influence of relatively large outward radiation forces upon partially ionized heavy metal elements (Bahcall and Pinsonneault, 1992; Proffitt, 1994; Unglaub and Bues, 1996). However, at present, this effect is not considered to be significant within the sun. In finding our solar model, only the first condition noted in this paragraph was assumed to hold.

4.3 Near-surface Lithium Concentration

Another interesting chemical feature in the near surface regions of the sun is the solar lithium concentration. Although the overall lithium concentration is only a minor chemical component in the convection zone, it is much lower than one might ordinarily expect. In fact, one estimate of the solar lithium abundance is that it is about 160 times smaller than that found in meteoritic material (Bahcall et al., 1995). Although other sources of information are not in exact agreement with this estimate of the near-surface solar lithium abundance (e.g., see Guenther et al., 1992; Pinsonneault et al., 1989; Press, 1986), there is a consensus of opinion, based upon experimental measurements, that there has been a substantial depletion in the original solar lithium concentration, and this fact has an important correlation with the temperatures that must exist at the base of the solar convection zone.

The temperature at or near the base of the solar convective zone (sometimes designated as T_{bcz}) is usually assumed to be in the vicinity of 2.0×10^6 K. If this temperature is much higher than 2.0×10^6 K, the 'observed' depletion in the

lithium concentration would have been greater than current estimates since lithium 'burning'(i.e., $Li^7 + H^1 = 2He^4$) would have been much more efficient. If this temperature is much lower than 2.0×10^6 K, then the lithium depletion would have been less than current estimates. Some authors also argue that even the temperature of 2.0×10^6 K, at the base of the solar convection zone, is too low to have caused the observed depletion in the lithium abundance. However, lithium burning has been estimated to be reasonably efficient at temperatures in the range of $2.4-2.8 \times 10^6$ K (Bahcall et al., 1995; Brodenheimer, 1989; Press, 1986; Schatzman and Mader, 1981; Turck-Chieze and Lopes, 1993). Therefore, even if the base of the solar convective zone is really at an average temperature near 2.0×10^6 K, much higher temperatures in this region are also possible because of the Maxwellian temperature distribution. In any case, these facts provide a reasonable temperature constraint condition of approximately 2.0×10^6 K at the base of the solar convection zone.

4.4   Base of Convection Zone

Prior to accurate helioseismic measurements, the exact position of the transitional location separating the inner stagnant region of the sun from the outer convective region was not very well known. However, according to relatively recent experimental helioseismic results (Christensen-Dalsgaard et al., 1991), the base of the solar convection zone (R_{bcz}) has been determined. Its location has been estimated to lie at a radial distance (measured from the solar centre of $0.713R_S$ ($\pm 0.003R_S$). It may also be interesting to note here that at least one earlier estimate regarding convection zone base locations, for certain red dwarf stars having some characteristics in common with the sun, placed the inward distance at ca. 30% of the stellar radius. Temperatures at the bases of these convection zones were estimated to be in the vicinity of $2.0-2.5 \times 10^6$ K (Osterbrook, 1953). These findings had nothing to do with helioseismic measurements but are in remarkably good agreement with the helioseismic results.

4.5   Chemical Constraints

The elemental surface composition of any present-day solar model must be identical to that of the present-day sun. However, there is still some doubt regarding the exact chemical composition of the sun, not only in its interior but in its surface regions as well. There is, however, not much doubt that the sun is composed primarily of hydrogen, with smaller concentrations of helium, and still smaller concentrations of heavier elements (often referred to collectively as heavy 'metal' elements). In the near surface regions of the sun, these elements are usually estimated to be present in the following approximate percentage ranges, by mass: 70–78% hydrogen, 20–28% helium, and about 1.8–2.0% heavy metals. Typically, these estimated values are derived from spectroscopic and/or helioseismic measurements and different authors cite slightly differing compositions based upon their own experimental findings or their evaluation of relevant data published by others (e.g., see Antia and Basu, 1994;

Basu and Antia, 1995; Dar and Shaviv, 1996; Dziembowski *et al.*, 1991; Kosovichev, 1995; Kosovichev *et al.*, 1992; Richard *et al.*, 1996; Rolfs and Rodney, 1988).

5 CALCULATION DETAILS

5.1 Non-Evolutionary Approach

Aside from some of the differences already noted, the solar model described in this paper was not generated using evolutionary calculations, as in some of the earliest, as well as more recent models (e.g., see Bahcall and Pinsonneault, 1992; Guenther *et al.*, 1992 (and references therein); Henyey *et al.*, 1964; Kippenhahn and Weigert, 1967; Sears, 1964). Instead, the present known (or, to be more precise, estimated) solar conditions and constraints were employed without any stepwise approach toward these conditions from some presumed zero age model. In addition, no information regarding (or calculations employing) opacities were used by us. These omissions of standard evolutionary calculations, as well as opacity related effects, are what really distinguish this study from other modem interior solar structure calculations.

5.2 Iterative Procedure

Iterative calculations were performed starting near the centre of the sun, then integrating outward to the solar surface, through a succession of 'thin' shells until all of the known (or estimated) solar constraints were satisfied. This integration procedure began with a small sphere, located at the sun's centre and progressed outward through relatively thin successive mass shells. The initial radius of the innermost solar sphere was taken to equal $R_S/1000$ and the first thin shell of solar matter, surrounding this initial small sphere, was taken to be exactly $1/100$ of that inner sphere's radius in thickness. Then, the next inner sphere of solar matter was taken to have a new radius equal to the previous inner sphere's radius plus the thickness of the previous shell of solar matter. Then, the next shell of solar matter was taken to have a thickness equal to $1/100$ of the new super-enclosed spherical volume element's radius. In this way, each new shell of solar matter had the same thickness, relative to all internal spherical volume elements, as every other shell of solar matter. Using this computational approach we could more precisely account for changes in all of the internal solar properties and, at the same time, not have to deal with an excessive number of calculations far from the core (because each 'thin' shell of solar matter became thicker than the preceding one), wherein changes in the solar properties (with respect to radius) were not as drastic.

We want to emphasize here that our procedure produced successive shells of solar matter that were all progressively thicker than the preceding one. But, they all were exactly $1/100$ of the radius of each 'new' spherical volume element enclosed by the newest shell in question. Furthermore, in making these kinds of calculations it is tempting to make each shell of matter very thin, especially if one is using a powerful

computer and a very fast (or efficient) computer code. This temptation must be resisted in order to avoid serious 'round off' errors associated with the diminishing number of significant figures that this kind of procedure can lead to. In other words, less (rather than more) is sometimes better in this kind of calculation. Additional details regarding computational errors of this type are discussed by Clayton (1968).

Calculating the pressure within each new and successive spherical shell of mass was relatively straightforward provided that accurate values for the 'new' densities, temperatures, and molecular weights within each mass shell were well known. However, these parameters were not well known at all. So, the mass within the first thin mass shell, surrounding the initial small sphere at the 'centre' of the sun, was estimated by assuming that the average density of solar matter in this shell was the same as in the super-enclosed spherical volume element (in the case of subsequent shell volume elements, initial densities were taken to be equal to that within the preceding shell volume element). Then, the change in pressure between the bottom and top layer of this matter shell was determined using a conventional hydrodynamic pressure drop calculation. Next, an ideal gas type adiabatic pressure/temperature relationship was used to estimate the temperature within this matter shell by guessing an initial value of an appropriate 'adiabatic-like' exponent. Then, based upon these calculated values, a new mass density was re-calculated for the matter only within that thin mass shell using the ideal gas law as well as the average pressure and temperature within that mass shell. Using the newly calculated mass density value, the initial calculation process was repeated until successively calculated values of pressure, temperature, and density were in good agreement (i.e., when differences between these successively calculated variables were in the vicinity of the sixth significant figure). These results could be obtained within ten, or fewer, iterations.

Following, this procedure, the same process was repeated by incrementally stepping through each successive solar mass shell until we reached a radius equal to $0.713R_S$ (i.e., the base of the convection zone). At this radius, if the temperature was not very near 2.0×10^6 K, new initial condition values were chosen (usually one at a time) and the entire process was repeated.

The iteration process described in the preceding paragraph also demanded a value for the molecular weight of the solar material within each specific mass shell in question. These molecular weight values were all calculated during program execution using an empirical mathematical relationship that allowed the mass fraction of hydrogen to rapidly increase from an initial condition core value estimate up to its surface concentration value. The estimated core concentration of hydrogen and one constant term within this equation could be adjusted repeatedly until the overall hydrogen mass percentage within the sun matched the estimated constraint condition value of 70.4% at the end of the entire sequence of calculations.

Initial condition temperatures (i.e., the solar core temperature) had to be adjusted to get the right integrated quantity of solar energy (within the core and near-core solar regions as well as in each successive matter shell further from the near-core regions) at the solar surface; and, the X, Y, and Z values, at the base of the convection zone or beneath it, had to equal the presumed surface values.

These were additional constraint conditions that had to be satisfied. When all of these parameters were in good agreement, we supposed that a reasonably accurate approximation to the actual solar structure had been found for interior regions of the sun beneath the base of the convection zone.

The same computational procedure, between the base of the convection zone and the solar surface, was also performed. However, within this region of the sun, changes in the molecular weight of the solar matter were presumed to be negligible; and a different adiabatic-like exponent was chosen to account for temperature changes between the base of the convection zone and the solar surface. The actual value of this exponent also had to be found by trial. However, it was not possible to find a single adiabatic exponent that would precisely satisfy both of the temperature boundary conditions at the base and surface of the convection zone and at the same time satisfy all other solar constraints. So, in order to avoid complicating this approach with additional corrections in order to artificially force a surface temperature condition in the vicinity of 6000 K, a 'best fit' exponent was selected.

The computational approach described above has been referred to in the past as a 'shooting' technique because one guesses at initial condition values, then 'shoots' to the surface (through the calculations) to see how well the constraints (or boundary conditions) are satisfied. If good agreement is not achieved, the initial condition values are changed somewhat and the process is repeated until all of the calculated variables are in good agreement with the constraint conditions. When that has been achieved, one assumes that a good model of the interior solar structure has been generated. We have mentioned this terminology to indicate that we are aware of this kind of computational procedure. However, to the best of our knowledge, our specific approach to this problem (which avoids opacity calculations altogether but does not ignore energy considerations) is unique. We are also aware that other approaches to this problem are possible (e.g.. Lane's method as well as certain 'polytropic' methods which also ignore opacities) but, to the best of our knowledge, all of these methods also ignore detailed balancing between fusion energy production rates within the solar interior and the solar energy emission rate at the sun's surface (e.g., see Cox and Giuli, 1968, Vol. 2, p. 580–583).

5.3 Initial Conditions

Initial density and temperature conditions were arbitrarily chosen for the relatively small inner sphere that was concentric with the solar centre. An initial mass fraction of hydrogen, within this small sphere, was also chosen to reflect a depletion in the original primordial mass fraction of hydrogen that was assumed to be present just prior to the onset of the fusion process. Assuming that the mass fraction of heavy metals is constant throughout the solar interior (at $Z = 0.02$), the mass fraction of helium could be found by difference because $(Y = 1.0 - X - Z)$. To account for changes in the mass fraction of hydrogen with respect to radius, a mathematical model for the change in the mass fraction of hydrogen, as a function of solar radius, was selected. Then, at each new radius, it was possible to calculate a new mass fraction for hydrogen. The mathematical relationship chosen to represent

the change in the mass fraction of hydrogen with radius expressed the fact that the hydrogen mass fraction at the sun's centre must be relatively low but that it rapidly and smoothly increases, approaching its primordial value, at radial distances that are somewhat beneath the base of the convection zone. The primordial surface conditions were assumed to be as follows: $X = 0.740$, $Y = 0.240$, and $Z = 0.020$ (references have been previously noted herein and above).

5.4   Ideal Gas Law

The ideal gas law was employed in all calculations to find internal solar pressures from the estimated or calculated values of the localized average molecular weight, density, and temperature. Corrections for the local mass concentration of hydrogen as well as other non-ideal effects were ignored although we were aware of the fact that more complicated expressions for these types of corrections do exist (e.g., see Guenther et al., 1992; Stolzmann and Blocker, 1996). As noted previously, uncertainties in the chemical composition within the solar interior make the applicability of more precise equations here doubtful, at best. In any case, this approach seems to be reasonable since data from at least one of the most recently published standard solar models indicates that more accurately calculated solar core pressures only differ from the ideal gas results by about 0.6% (e.g., see Table VII in Bahcall et al., 1995).

5.5   Radiation Pressures

Cox and Giuli (1968, Vol. 1, p. 256) concluded that radiation pressures, even at stellar centres, are only significant in stars having masses that are greater than 2–3 solar masses. Therefore, corrections for radiation pressures were ignored in our calculations.

5.6   Fusion Energy

The total fusion power (i.e., energy production rate) generated within the core region, and each successive shell of solar matter, was computed once we had obtained the localized: mass fraction of hydrogen, density, and temperature conditions. Each of these energy production rate contributions were totaled together for the entire sun and the final sum was compared with the solar luminosity at the end of each trial run. This approach assumes that all of the fusion power (in the form of radiation) generated within the solar core, and in all successive shells of solar matter, must leave these regions by travelling outward, eventually passing into the convection zone, and then (by convection) to the solar surface where it is radiated into space. If this did not happen each interior shell region of solar matter would keep getting hotter due to 'optical' absorption and temperature enhanced fusion, thus causing the internal temperatures to continue to increase, and so on. So, if one picks the right 'adiabatic' exponents, all energy transport resistance effects are taken care

of automatically. Of course, the question is: How does one know that the 'right' exponents have been chosen? The answer is that when the right exponents have been chosen (and used along with all other appropriate adjustable variables), all of the solar constraint conditions will be satisfied. This can be a tedious exercise but it is also a good example of the kind of problem that is ideally suited to be solved, iteratively, using a digital computer.

5.7 Basic Equations

Whenever possible, simplified or basic equations for all of the calculations that were required within this study were employed. It is understood that more refined versions of some of the equations employed herein and discussed in more detail below (e.g., the ideal gas law and the energy production equation associated with the hydrogen fusion process) could have been used instead of the ones we selected. These more refined equations were not employed because even today no one can be certain of the exact chemical composition within the solar interior, and this fact also applies at the surface of the present-day sun. In addition, future improvements in our understanding of the basic chemistry, physics, and nuclear processes taking place within the sun are likely to occur. Therefore, it is reasonable to assume that there will be some future advancements in the presently accepted theories of solar evolution, structure, and composition as well as improvements in some of the more refined equations in use today. But, the more basic equations employed herein, although they may only be approximations, will not change significantly over time.

All of the basic equations, as well as special constants employed in making the calculations described above, are listed and briefly discussed below.

The ideal gas law, in the form of:

$$P = \frac{\sigma R_G T}{M_W}, \tag{1}$$

was employed to make all internal solar pressure calculations. In this equation, P is the absolute pressure, σ represents mass density, T is the absolute temperature, M_W is the molecular weight of the solar matter in whatever specific mass shell (or in the small central sphere) that was under consideration, and, R_G is the universal gas constant. Initially, σ, T, and M_W were estimated at the centre of the sun in order to calculate a value for the pressure there.

The molecular weight of solar matter, as a function of solar radius (i.e., M_W), was calculated using the following approximation:

$$M_W = \frac{4}{6X_R + Y_R + 2}, \tag{2}$$

wherein X_R and Y_R (previously defined using X and Y) are the local mass fractions of hydrogen and helium, respectively. The subscripts employed here emphasize that both of these quantities are functions of the particular solar radius in question. There are numerous references that describe the derivation of this equation (e.g., see Cox and Giuli, 1968; Tayler, 1994).

The mass fraction of hydrogen within the sun, as a function of solar radius (i.e., X_R), was estimated using the following equation:

$$X_R = X_{\text{max}} - (X_{\text{max}} - X_c) \exp\left(-\frac{C_0 R}{R_S}\right),$$

$$(3)$$

where X_{max} is the mass fraction of hydrogen at the sun's surface, X_c is the mass fraction of hydrogen at the centre of the sun, C_0 is a constant, and R/R_S is the fractional solar radius (varying between 0.0, at the sun's centre, and 1.0 at the solar surface). Using this equation, it was possible to selectively change all of the 'X' parameters, in addition to the constant term, in order to find a reasonable solution to the variation in hydrogen mass fraction with respect to solar radius. Other equations could have been employed instead of this one. However, this equation was chosen because it was one of the simplest equations that we could think of that seemed to apply in this case. It also produced hydrogen versus solar radius profiles that were similar to profiles, described within another publication (see Kippenhahn and Weigert, 1990, p. 277), that applied to stars comparable to the sun at the present time. As noted previously, values of X_c, X_{max}, and C_0 that we selected for final use within this equation were: 0.490, 0.740, and 9.722, respectively.

The mass fraction of helium within the sun, as a function of solar radius (i.e., Y_R), was estimated using the following equation:

$$Y_R = 1.0 - X_R - Z,$$

$$(4)$$

where X_R is defined above and Z (the mass fraction of heavy elements) was taken as a constant, throughout the entire sun, equal to 0.020.

Pressure changes between successive 'thin' shells of solar matter were estimated using the conventional condition for hydrostatic equilibrium, namely:

$$dP = -\sigma \frac{G M_R}{R^2}\, dR,$$

$$(5)$$

where dP is the pressure change that occurs between the bottom and top of any 'thin' shell of solar matter within the solar interior, dR is the thin shell thickness, σ is the local mass density of matter within that mass shell, G is the universal gravitational constant, and M_R and R are the mass and spherical radius of all solar matter circumscribed by the particular mass shell in question. dP is negative because interior solar pressures decrease with increasing solar radius. Additional details regarding stipulations and the applicability of this equation may be referred to elsewhere (e.g., see Ostlie and Carroll, 1996, p. 318).

The form of the equation employed within our calculations to find the quantity of solar matter in each new volumetric mass shell was as follows:

$$dM = \sigma \frac{4}{3}\pi(R^3 - R_0^3),$$

$$(6)$$

where dM is the mass within the particular mass shell in question and R and R_0 are (respectively) the outer and inner radius of that same mass shell. All other symbols

used here are either well known or have been previously defined. Of course, the mass contained within our initial very small spherical volume element (at the very centre of the sun) was calculated by multiplying the local estimated (by testing various trial values) mass density by the volume of that sphere.

The form of the pressure/temperature equation used within our calculations was as follows:

$$T = T_0 \left(\frac{P}{P_0}\right)^{\delta}, \tag{7}$$

where T and P represent the absolute temperature and pressure at the outer surface of any specific shell of solar matter, T_0 and P_0 represent the absolute temperature and pressure at the outer surface of the previous shell of solar matter, and δ is an empirical exponent for either the inner stagnant region of the solar interior or for the outer convective region of solar matter. Within this study, the inner and outer values of δ were estimated (also by testing various trial values) to be equal to 0.2218 and 0.3230, respectively. The number of significant figures listed here, as well as elsewhere, provide an indication of the sensitivity that these numbers had on changing the overall calculated results. It may not be obvious, but this equation can also be expressed in another equivalent form that may be referred to as a 'polytropic' relation. In this case, δ is related to (but not equal to) the polytropic index (Arnett, 1966; Eddington, 1926; Chandrasekhar, 1939; Clayton, 1968; Cox and Giuli, 1968; Kippenhahn and Weigert, 1990; Schwarzschild, 1958).

The equation used to calculate the amount of power per unit mass generated by the solar fusion process, within any specific shell of matter within the solar interior, was:

$$E_R = 0.0680\sigma X^2 (T_7)^{4.6}, \tag{8}$$

where E_R is the power per unit mass in ergs g^{-1} sec^{-1}, σ is the local average mass density of solar matter, X is the local average mass fraction of hydrogen, and T_7 is the local average absolute temperature divided by 10^7. In order to find the fusion power generated within any specific shell of solar matter, values of E_R were multiplied by the mass of solar matter contained within that volume element. This equation was taken from Arnett (1996), p. 188. There are many other more complicated expressions that could have been employed here instead of this equation (e.g., see Hansen and Kawaler, 1995; Schwarzschild, 1968). However, this expression was the most recently cited and simplest equation that we could find that related the local fusion power per unit mass to the local mass density, mass fraction of hydrogen, and temperature.

The mass constraint condition can be expressed using the following equation:

$$\sum dM = M_S, \tag{9}$$

where $\sum dM$ is the summation of all individually calculated solar mass elements (contained within 'exactly' one solar radius) and M_S is the experimentally determined mass of the sun (i.e., 1.989×10^{33} g). If the sum of all solar mass elements did not add up to (or very near to) M_S, one or more of the adjustable variables had to be changed to satisfy this condition.

The hydrogen depletion constraint condition can be expressed using the equation:

$$\sum X_R \frac{dM}{M_S} = 0.704, \tag{10}$$

where $\sum X_R \, dM$ gives the total quantity of hydrogen within the present-day sun, M_S is as defined above, and 0.704 is the present-day overall mass fraction of hydrogen (estimated as indicated previously) within the entire sun.

The luminosity or energy constraint condition can be expressed by the following equation:

$$\sum E_R \, dM = L_S, \tag{11}$$

where $\sum E_R \, dM$ is the summation of all fusion energy generated within each solar mass element per unit time and L_S is the observed solar energy emission rate or luminosity (namely: 3.85×10^{33} erg sec^{-1}.

5.8 Solving the Equations

In order to solve or satisfy all of the equations listed in the preceding paragraph, there were three solar core condition variables that had to be empirically determined. They were the solar centre conditions of temperature, density, and mass fraction of hydrogen, at the present time. In addition, four other variables had to be empirically determined. These four variables were the maximum mass fraction of hydrogen within the sun just prior to the onset of the fusion process (designated by the symbol X_{max} in equation (3) and presumed here to also equal the present-day mass fraction of hydrogen within the solar convection zone), the constant term (C_0) within equation (3) and two separate values for δ in equation (7) (one for the inner core and radiant solar regions and one for the solar convection zone). Since there are seven independent equations, relating these variables, listed above, namely equations (3), (5), (7), (8), (9), (10) and (11), all of the necessary variables could be solved for using the iteration and successive approximation process described earlier. However, in order to simplify our iteration process, the maximum mass fraction of hydrogen, within the sun just prior to the onset of the fusion process, was assumed to be exactly equal to 0.740. All other variables were determined iteratively. As indicated earlier, this problem would have been nearly impossible to solve (within any reasonable time period) without the aid of a modem digital computer).

5.9 Summary of Variable Values

For the sake of convenience, all variable values (estimated or determined as a result of our iteration process and noted within the text above) have been grouped together and listed in Table 2 below.
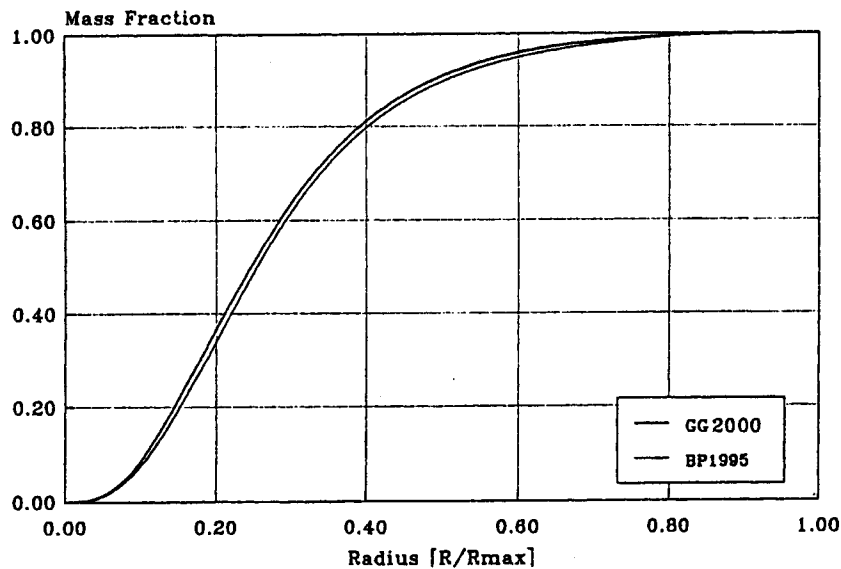
Figure 1 Cummulative solar mass vs. fractional radius.

6   RESULTS AND DISCUSSION

6.1   General Comments

The main results of our computer calculations have been compared with one of the most recently published solar models that also contained a detailed list of tabulated numerical results (Bahcall *et al.*, 1995). The authors of that study (hereinafter, for the sake of brevity, referred to as BP1995) have a long history of involvement in creating very detailed and progressively more sophisticated standard solar models (*inter alia*), as well as solar models incorporating the effects of helium and heavy element diffusion. Since our study deliberately did not take into account the effects of diffusion, no comparisons involving it have been made herein. Instead, we have only compared the results obtained with regard to our non-diffusional model and theirs. Our objective in making this comparison was to show that our simplified method of computing interior solar structures produced results that were in reasonably good agreement with one of the best currently available standard models that did not disregard opacity calculations within the solar interior.

In order to simplify our comparisons we have plotted several graphs detailing our results (labelled using GG2000) and the results in BP1995. These graphs involve the cumulative mass fraction of the sun as a function of fractional solar radius (Figure 1) the interior solar mass fraction of hydrogen as a function of fractional radius (Figure 2), interior solar temperatures as a function of fractional radius (Figure 3), interior solar pressures as a function of fractional radius (Figure 4),
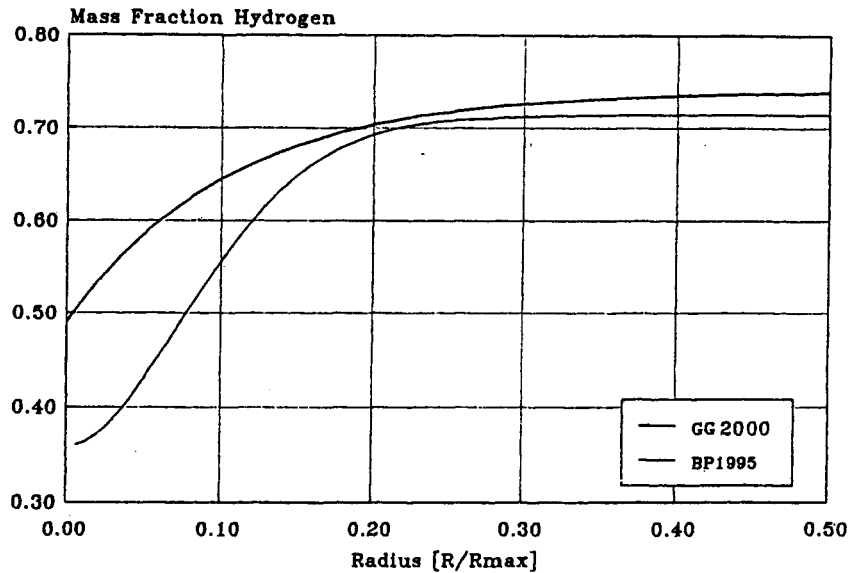
Figure 2   Mass fraction of hydrogen vs. fractional radius.

interior solar mass densities as a function of fractional radius (Figure 5), and the cumulative fractional solar power generated within the sun as a function of fractional radius (Figure 6).

6.2   Cumulative Solar Mass

Cumulative solar masses as a function of fractional solar radius, obtained as a result of our calculations and those of BP1995, have been plotted in Figure 1. It is clear that the agreement between our data set and theirs is very good. It is also interesting to note that about one-half of the sun's mass is enclosed within one-forth of one solar radius. Or, in other words, about 1/64 of the sun's volume contains one-half of the entire mass of the sun.

6.3   Interior Mass Fraction of Hydrogen

Mass fractions of hydrogen, as a function of fractional radius, within the solar interior, have been plotted in Figure 2. It is clear that there are significant differences between our estimates in the behaviour of this variable and those of BP1995. However, as noted earlier, our hydrogen concentration profile is similar to solar hydrogen profiles described by Kippenhahn and Weigert (1990), p. 277. This discrepancy only emphasizes that there is still considerable uncertainty regarding the true chemical composition within the solar interior as well as at the solar surface. In connection
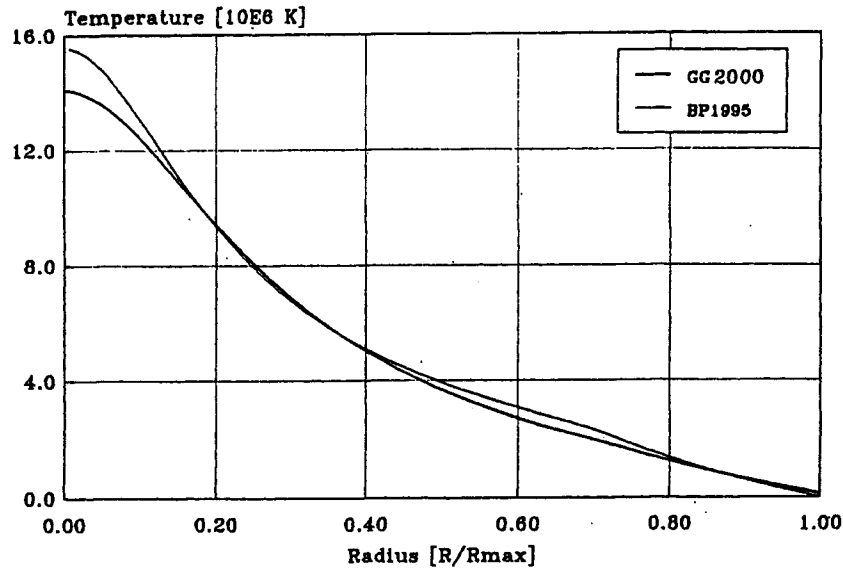
Figure 3 Interior solar temperatures vs. fractional radius.

with this comment, the reader may note that hydrogen mass fractions of about 0.3613 and 0.7146 were employed near the centre of the sun and at the surface, respectively, in BP1995. Within the present study, hydrogen mass fractions of 0.490 and 0.740 were employed at these locations. We tried using several other hydrogen centre and surface mass fraction values as well as different hydrogen concentration (vs. solar radius) profile equations, in place of equation (3), in order to solve the overall problem of determining the interior structure of the sun. However, we could not satisfy all of the solar constraint conditions until equation (3) was employed along with hydrogen mass fractions of 0.490 and 0.740 at the centre and surface, respectively, of the sun. This result in no way ensures that the mass fraction of hydrogen at the centre of the sun, or at the solar surface, is exactly 0.490 or 0.740, or that equation (3) is absolutely accurate, but only that these assumptions (along with our other methodology) produce one possible solution to the solar structure problem.

6.4 Interior Solar Temperatures

Interior solar temperatures obtained as a result of our calculations and those of BP1995 are plotted in Figure 3. The agreement between our temperatures, as a function of solar radius, and theirs is reasonably good everywhere except in the central core region of the sun. The exact central temperature that we obtained was very near 14.1×10^6 K versus a value of about 15.5×10^6 K in BP1995. Aside from our neglect of oapcities, there are at least two significant reasons for these core
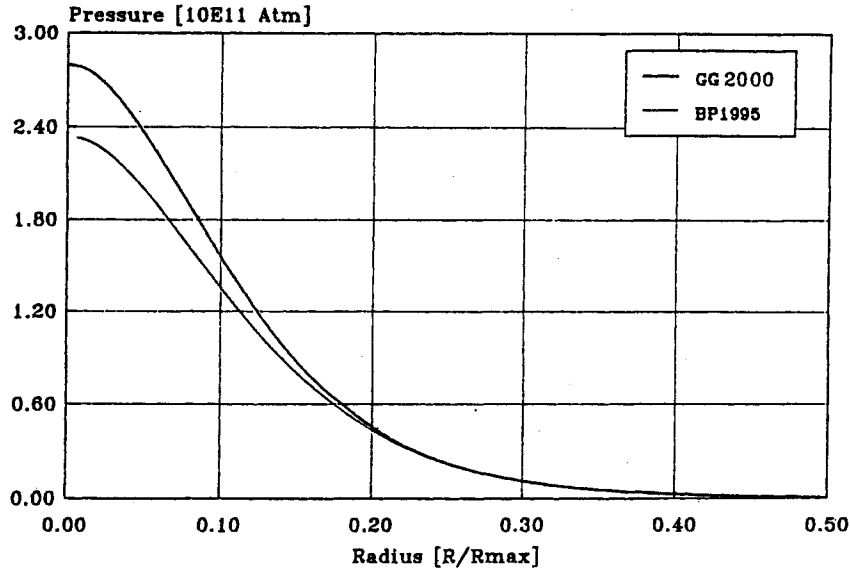
Figure 4   Interior solar pressures vs. fractional radius.

temperature differences. One of these reasons is related to the fact that we estimated that the solar interior contains a higher mass fraction of hydrogen than that used in BP1995. This higher mass fraction of hydrogen, determined by iteration, was a result of our attempt to satisfy several of the constraint conditions. The primary constraints, in this case, were the overall mass concentration of hydrogen throughout the sun and the total solar energy generation rate. A second possible reason for the difference between our core temperature results and those in BP1995 (which is related to the immediately preceding case) involves our use of a relatively simple equation (i.e., equation 8) to calculate solar fusion power within the solar core and in all successive matter shells throughout the sun.

Lower central temperature conditions, within the sun, have been mentioned in the past as a possible explanation of the solar neutrino problem (Bahcall, 1989 and Kuchowicz, 1976) although some form of mixing behaviour within the solar interior is typically invoked in order to account for this behaviour (Bahcall et al., 1997). What our calculations indicate is that much higher (and stagnant) mass fractions of hydrogen within the deep interior of the sun, instead of mixing behaviour, can be used alone to validate lower central temperatures. This is an obvious conclusion for anyone with significant experience in making interior solar structure calculations. In any case, we mention this point because it is interesting, not because we are claiming that our results have anything to do with the solar neutrino problem. We don't know enough about that problem to make any relevant comments or predictions.

Regarding the behaviour of the interior solar temperature data plotted at some distance from the solar core, the agreement between our calculated temperatures
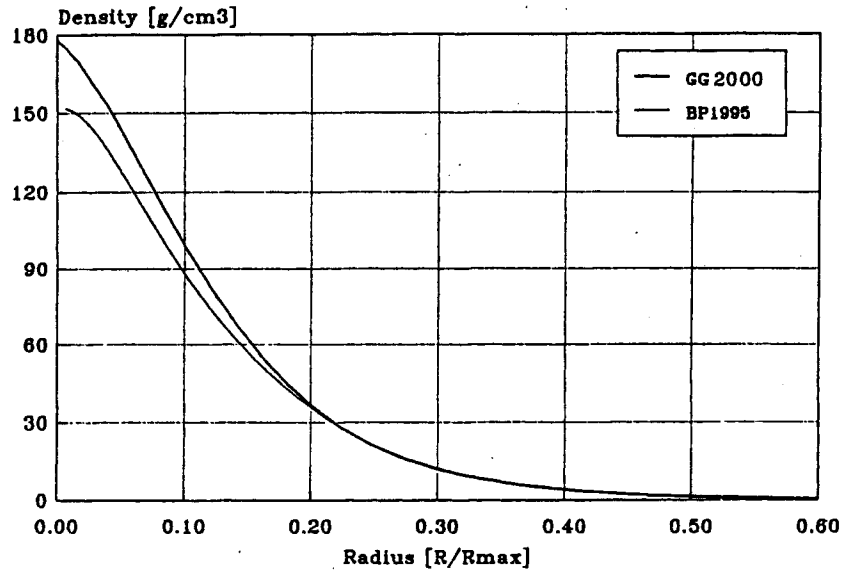
Figure 5 Interior solar densities vs. fractional radius.

and the corresponding temperatures in BP1995 is pretty good. In addition, both curves exhibit a slight discontinuity in their slopes in the vicinity of $0.713R_S$. In the case of our curve, this behaviour was produced because of our deliberate change in the value of the adiabatic exponent that was used to relate the pressure/temperature behaviour from this interior location out to the solar 'surface'. As noted earlier, we also came up with a much higher solar surface temperature than we wanted but that condition is not too noticeable due to the overall range of temperature values plotted in Figure 3. It is believed that this temperature error is due, primarily, to our unmodified use (for the sake of simplicity) of equation (2) all the way up to the solar surface. The form of this equation must begin to change markedly in the vicinity of (and below) 10^5 K due to significantly less complete ionization of all elements within the near-surface solar atmosphere (Schwarzschild, 1958). In any case, as mentioned previously, this surface temperature result had practically no effect on any of the other primary constraint conditions we were attempting to satisfy.

6.5   Interior Solar Pressures

Interior solar pressures obtained as a result of this work and the corresponding pressures listed in BP1995 are plotted in Figure 4. The agreement between these pressure curves, as a function of solar radius, is also reasonably good everywhere except in the central core regions of the sun. The exact central pressure that we obtained was 2.794×10^{11} atm versus a value of about 2.364×10^{11} atm in
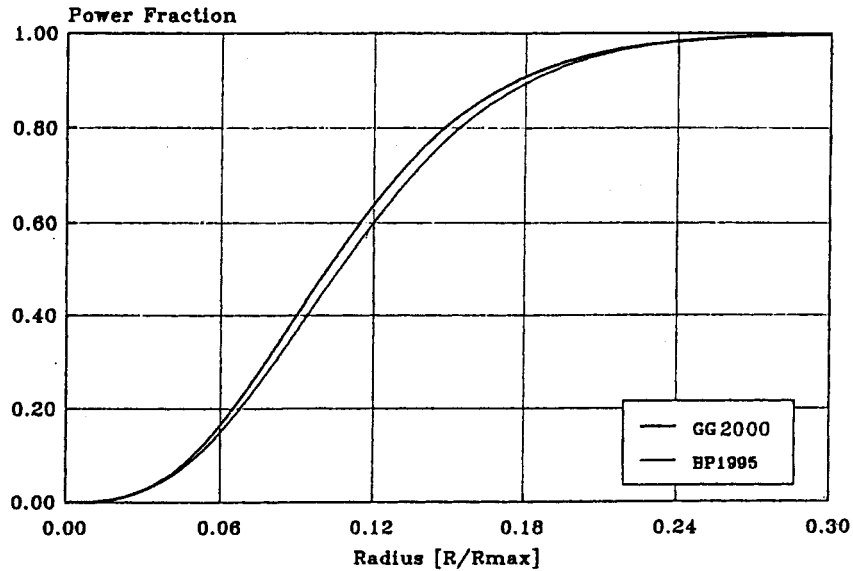
Figure 6   Cummulative solar power vs. fractional radius.

BP1995 (expressed as 2.363×10^{17} dyne cm^{-2} at $R = 0.00651 R_S$ in BP1995). At all solar radii greater than about $0.15 R_S$, our calculated pressures were in very good agreement with those listed in BP1995.

6.6 Interior Density of Solar Matter

Calculated values of interior solar densities, as a function of solar radius, obtained from this study and from BP1995, have been plotted in Figure 5. Again, the agreement is pretty good everywhere except at the centre of the sun. Our calculations produced a central density of 177.9 g cm^{-3} versus the value of about 151.9 g cm^{-3} in BP1995. This result of our iteration and successive approximation process, as well as our relatively high internal mass fractions of hydrogen, are in line with the lower value of the central solar temperature that we determined.

6.7 Cumulative Solar Power

The cumulative fraction of solar power generated within the sun and the corresponding fractional powers reported in BP1995 have been plotted in Figure 6. The agreement between our data set and theirs appears to be very good almost everywhere. However, we did make a small correction to our data before plotting. This procedure was followed because we had difficulty in adjusting all of our independent variables in order to end up with exactly 100% of the actual solar luminosity at the solar surface. This may have been possible but, after numerous adjustments

Table 1. Summary of solar properties.

Property	Best estimate	Reference
Age	$(4.52 \pm 0.04) \times 10^9$ yr	1
Mass (M_S)	$(1.9891 \pm 0.0004) \times 10^{33} g$	2
Radius (R_S)	$(6.960 \pm 0.001) \times 10^{10}$ cm	3
Luminosity (L_S)	$(3.85 \pm 0.01) \times 10^{33}$ erg sec^{-1}	4
R_{bcz}	$(0.713 \pm 0.003) R_{max}$	5
T_{bcz}	2.0×10^6 K	6
X at Surface	0.739 ± 0.031	7
Y at Surface	0.242 ± 0.003	8
Y at Surface	0.242 ± 0.030	9
Z at Surface	0.019 ± 0.001	10
X at Surface	0.740	6
Y at Surface	0.240	6
Z at Surface	0.020	6

1, Guenther and Demarque (1996);
2, Cohen and Taylor (1986, 1987);
3, Ulrich and Rhodes (1983);
4, Hickey and Alton (1983);
5, Christensen-Dalsgaard et al. (1991);
6, Estimate employed in this paper;
7, Calculated from: $X = 1.0 - Y - Z$ and values from references 9 and 10;
8, Hernandez and Christensen-Dalsgaard (1994);
9, Dar and Shaviv (1996);
10, Estimate based on Bahcall and Pinsonneault (1992).

in our initial conditions, we decided to accept a value equal to $0.9922L_S$. Our acceptance of this result, along with our initial condition estimates (see Tables 1 and 2) produced a cumulative solar mass fraction of 1.0000, a near-surface temperature of 1.420×10^5 K, and average hydrogen, helium, and heavy metal mass fractions of 0.7040, 0.2760, and 0.0200 (respectively); all at $R = 0.9978R_S$. Since some small percentage of the observed (i.e., measured) solar luminosity is believed to come from the CNO fusion cycle anyway, this cutoff at $0.9922L_S$ is justifiable because no quantitative account of the CNO fusion cycle was made within our power calculations. So, it is reasonable to assume that the 'missing' power of $0.0078L_S$, as well any additional power losses due to the energy carried away from the sun by neutrinos and the solar wind, is compensated for by the CNO fusion process.

The actual correction made in our cumulative power fractions, as a function of solar radius, involved dividing all of our calculated values by $0.9922L_S$. Using this procedure, we were able to normalize our maximum fractional power output value to 1.0000. This modification in our original data automatically corrected for the fact that the data in BP1995 did (presumably) include some power generation term related to the CNO fusion cycle.

Table 2. Summary of estimated/calculated variable values.

Present-day conditions at the centre of the sun		
Variable	*Value*	*Type*[*]
Pressure (P_c)	2.794×10^{11} atm	D
Temperature (T_c)	14.10×10^6 K	I
Density (σ_c)	177.9 g cm^{-3}	I
Mass Fraction H (X_c)	0.490	I
Mass Fraction He (Y_c)	0.490	D
Mass Fraction HM (Z_c)	0.020	C
Molecular Weight (M_c)	0.737 g mol^{-1}	D
Other variables		
Variable	*Value*	*Type*[*]
Initial Mass Fract. H (X_{max})	0.740	I
Initial Mass Fract. He (Y_{min})	0.240	D
Initial Mass Fract. HM (Z)	0.020	C
Term in equation (3) (C_0)	9.722	I
Inner Adia. Exponent (δ_0)	0.2218	I
Outer Adia. Exponent (δ)	0.3230	I

[*]Independent variables are denoted with 'I' (there are seven of these), dependent variables are denoted with 'D', and the constant terms (i.e., Z and Z_c) are denoted with 'C'.

7 CONCLUSION

This paper describes a computational method of estimating physical and chemical properties within the solar interior without employing calculations involving opacities. Instead of using opacities to help determine how interior solar temperatures vary with the radial distance between the centre of the sun and its 'surface' an iterative technique employing empirical adiabatic 'cooling' and a fusion energy production rate expression were employed for this purpose. Other iterative calculations were also made (nearly simultaneously) to ensure that all known solar constraint conditions were also precisely satisfied (except for the photospheric 'surface' temperature) during this process. The results of this study have been compared with more conventional results obtained by others (Table 3). This comparison indicates that the methods employed within this paper produced interior intensive solar properties that were in good agreement with more conventionally obtained findings. Although it is not claimed that these results are any better than more conventionally obtained findings, it is thought that these results, as well as our computational methods, are interesting and potentially useful to others. In particular, it is thought that the

Table 3. Comparison of constraint conditions and results obtained using iteration process.

	Actual or Estimated Value	Iteration Value*
Solar Mass (M/M_S)	1.0000	1.0000
Luminosity (L/L_S)	1.0000	0.9922
Average H (X_{avg})	0.7040	0.7040
Average He (Y_{avg})	0.2760	0.2760
Average HM (Z_{avg})	0.0200	0.0200
$T_{bcz}/10^6$	2.0000	1.9306†
Surface temperature ($T_S/10^3$)	6.0000	142**

*Most of the calculated values in this column were rounded off at the fourth place to the right of the decimal point. In many cases, agreement between the calculated and actual (or estimated) values was very close, but not as exact as some of these numbers may imply (due to rounding off).

† This temperature value was obtained at $R = 0.7114R_S$, not at $R = 0.713R_S$. A temperature value at exactly $0.713R_S$ was not calculated because it was not necessary. We only needed to reach a temperature near 2.0×10^6 K in the vicinity of $0.713R_S$.

**This value was actually calculated at $0.9978R_S$ and although the poorest agreement was obtained here, this 'error' had very little effect on any of the important constraint conditions. As noted in the text, we could have 'fixed' this but any effort in that regard was deemed unnecessary.

techniques outlined in this paper may provide a useful introduction to more complicated techniques of solar modelling.

The main things we see as primary problems with our methodology are the approximations (i.e., the equations) used to calculate the pp fusion energy production rates and in the expression used to estimate how the sun's internal elemental composition changes with respect to solar radius. Both of these expressions, but not the calculated results, were unaffected by our iteration process. If the form of these expressions had been exact (i.e., without error or uncertainty), and if we knew how to precisely account for other energy generating/loss mechanisms within the sun (e.g., the CNO fusion cycle and the exact energy carried away from the sun by neutrinos), our method would be capable of producing very accurate internal solar structures, regardless of what results others might get using their techniques. We can be confident in this belief because the total energy production rate within the sun must equal (for all practical purposes) the solar luminosity and it doesn't matter what mechanism (or mechanisms) are operable in transporting this energy from the solar interior to the 'surface' of the sun. However, the main problem here is to accurately estimate temperatures within each successive 'shell' of solar matter within the sun's core and radiant zone so that the calculated energy production rates equal (or nearly equal) the observed solar luminosity. The temperature constraints, at the base of the convective zone and at the solar 'surface', are also

important to aid in estimating the structural characteristics that exist within the convective zone because fusion energy production rates in that region are negligible. In more conventional approaches, this problem is solved (in part) by using opacities. In this paper, this was accomplished by using a simple 'adiabatic' type of pressure/temperature relation, only one adjustable 'adiabatic' exponent within the solar core and radiant zone and one in the convection zone, and a process of successive approximations within each shell of solar matter, as outlined in this paper, that permitted satisfying (within reasonable limits) all other solar constraint conditions. These are the reasons that this method 'works' as well as it does, even though opacities have been ignored.

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