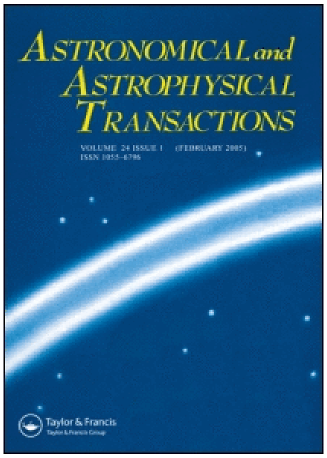

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PHOTON SPLITTING IN THE STRONG MAGNETIC FIELD OF A NEUTRON STAR

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We show that the radiation transfer equation for the photon splitting cascade has a one-parameter set of self-similar solutions and show that they are very useful for the proper treatment of a general solution. The main advantage is that any initial spectrum converges quickly to the self-similar spectrum provided most of the initial energy is injected at the hard spectral edge.

When turning to the astrophysical consequences of photon splitting we focus mainly on its general qualitative features and spectral imprints which are not sensitive to the specific emission mechanism and details of magnetosphere structure. The analysis includes possible polarization and effective softening of the post-cascade spectrum as well as remarkable formation of a spectral break and condensation of all hard-energy radiation in the vicinity of that break.

KEY WORDS Neutron stars, magnetic field, photon splitting

1 INTRODUCTION

Very strong magnetic fields of neutron stars favour exotic QED effects in vacuum which have never been observed in laboratory experiments. These effects can dramatically change the γ- and X-ray spectrum expected from an electron–positron plasma surrounding a neutron star. The existence of magnetic fields as strong as 10^{13}–10^{14} Gauss, at least in several cases of so-called magnetars, was supported by measuring pulsars' slowing down interpreted as the result of magnetodipole radiation (Duncan and Thomson, 1992). There also exist objects with smaller fields, $B \sim 10^{12}$–10^{13} Gauss, which were measured by means of identification of cyclotron lines in their spectra (Murakami, 1991).

Two QED processes forbidden in free space are the most important as far as spectrum formation is concerned. One is the well-known process of one-quantum pair creation and annihilation, which becomes allowed in magnetic fields since only the projection of the momentum on the field direction has to be conserved. Another is so-called photon splitting, $\gamma \to \gamma + \gamma$, which is mediated by virtual electron–positron pairs and is forbidden in free space due to charge conjugation symmetry of

QED (Furry's theorem). This process had attracted little or no attention until the corresponding cross-section was first calculated in 1970–1971 (Adler, 1971). More detailed work appeared later (Stoneham, 1979).

The astrophysical importance of photon splitting was discussed by Mitrofanov *et al.* (1986). In particular, this process was supposed to be responsible for the unusually soft spectra of SGRs. Though photon splitting is a third-order process, it is the only allowed mechanism of spectral evolution in a vacuum magnetic field below the pair production threshold $\hbar\omega < 2mc^2/\sin\theta$. Here $\hbar\omega$ is the photon energy, θ is the absolute value of the angle between $\boldsymbol{B}$ and $\boldsymbol{k}$ ($\hbar\boldsymbol{k}$ is the photon momentum) and m is the electron mass. In addition, the probability of photon splitting has a power-law dependence on the magnetic field strength B, while the probability of one-photon pair creation has an exponential cutoff and cannot compete with the former at low magnetic fields.

Nevertheless, photon splitting for high-energy quanta $\hbar\omega > 2mc^2/\sin\theta$ is usually negligible in comparison with true absorption, $\gamma \to e^+ + e^-$ for magnetic fields of interest $B \gtrsim 0.1B_{\rm cr}$. The reference magnetic field here is the so-called critical field $B_{\rm cr} = m^2c^3/e\hbar \simeq 4.4 \times 10^{13}$ Gauss, where e is the absolute value of the electron electric charge. At the same time, photons of energies $\ll 0.1mc^2/\sin\theta$ do not practically split on the neutron stars' scale even in a field of the order of $B_{\rm cr}$. Thus, throughout the paper we keep in mind the sub-MeV energy range, 50 keV–1 MeV.

We focus mainly on general qualitative features of photon splitting and its possible imprint on the observed spectrum which is not sensitive to the specific magnetosphere structure and emission mechanism. Therefore, we assume a simple model of a neutron star with dipole magnetic field and a magnetospheric plasma uniformly filling a spherical shell of ~ 10 km width. We do not expect that photons propagating at small angles to the magnetic field dominate spectrum formation, and make all estimates for a general case taking $\sin\theta = 1$.

It is important that in a strong magnetic field the vacuum is a birefrigent and dispersive medium: the refractive indexes $n_{\parallel}$ and $n_{\perp}$, for photons having an electric vector parallel and perpendicular the external field respectively, are different and depend on the frequency ω (Berestetscii *et al.*, 1971). This fact leads to the noteworthy conclusion that not all polarization modes are allowed in the photon splitting process, and the detected radiation may be essentially polarized (see Section 2). However, the splitting cascade can develop thanks to multiple (Thomson or cyclotron) scattering if the magnetospheric plasma is dense enough (see Section 3). But if the density is too high, the splitting cascade spectrum produced in the inner part of magnetosphere will be masked by the comptonization process and cannot be observed.

2 PURE PHOTON SPLITTING MECHANISM OF SPECTRAL FORMATION

The initial spectrum emerging from a neutron star surface is reprocessed by photon splitting. One of the main features of photon splitting is strong polarization dependence, which is due to the difference of refraction indexes for two principal polarization modes (Berestetskii *et al.*, 1971):

$$n_{\parallel} = 1 + \frac{7\alpha}{90} b^2 \sin^2\theta,$$
$$n_{\perp} = 1 + \frac{2\alpha}{45} b^2 \sin^2\theta. \tag{1}$$

Here b is in units of $B_{\rm cr}$, $\alpha = e^2/\hbar c \simeq 1/137$ is the fine structure constant. Since photons are chargeless particles, in a uniform magnetic field both energy conservation and momentum conservation laws should be observed in the photon splitting process. This leads to the following condition:

$$\boldsymbol{k}_{\sigma}(\omega) = \boldsymbol{k}'_{\sigma'}(\omega') + \boldsymbol{k}''_{\sigma''}(\omega''), \quad \omega = \omega' + \omega'', \tag{2}$$

where σ denotes one of two principal modes. Together with (1) this condition forbids the splitting of a $\parallel$-polarised photon into $\perp$- and $\parallel$-polarized photons or into $\perp$- and $\perp$-polarized photons. With small dispersive effects included, both $n_{\parallel}$ and $n_{\perp}$ prove to be increasing functions of ω, so that the processes $\parallel \to \parallel + \parallel$ and $\perp \to \perp + \perp$ are also kinematically forbidden. Finally, one of the two processes left, $\perp \to \perp + \parallel$, has zero rate in a first approximation (more precisely, its relative rate is of order $\alpha b^2 \ll 1$). The small difference between the refraction indexes for two principal modes makes a collinear approximation a natural choice when photon splitting is considered.

As a result, only $\perp$-polarized photons undergo splitting $\perp \to \parallel + \parallel$, and, to a first approximation, only once. The cross-section for photon splitting is given by Stoneham (1979)

$$\sigma_{\rm sp}(\varepsilon) = \frac{\alpha^3}{30\pi^2}\left(\frac{13}{315}\right)^2 \frac{1}{\lambda_{\rm c}}(b\sin\theta)^6\varepsilon^5, \tag{3}$$

where ε is the photon energy in units of mc^2 and $\lambda_c = \hbar/mc \simeq 3.9 \times 10^{-11}$ cm is the electron's Compton wavelength. Expression (3) represents low-field low-energy limit, and for $B \sim B_{\rm cr}$ should contain a modification factor $M(B)$ (see e.g. Mentzel *et al.*, 1994). But it is still exact in the photon frequency dependence if only Feynman diagrams with six vertices are taken into account (which is correct for $\varepsilon b \sin\theta \ll 1$, (Adler, 1971)). We should note that in very strong fields $b \gg 1$ the splitting cross-section ceases growing (Stoneham, 1979) and the outer parts of the magnetoshpere (where $b < 1$) play a more important role than the inner parts (since the outer magnetoshpere is still not transparent for splitting when the condition $\varepsilon b \ll 1$ is already satisfied, we can use (3) as a general expression).

The usage of a polarization-averaged cross-section (3) in the radiation transfer equation below implies some efficient depolarizing mechanism, for example Thomson scattering. The magnetosphere of a neutron star remains transparent until the electron and positron density exceeds

$$n_{\rm tr} = \frac{1}{\sigma_{\rm T} l_{\rm msp}} \simeq 10^{18}\ {\rm cm}^{-3}. \tag{4}$$

Here σ_{T} is the Thomson cross-section and $l_{\mathrm{msp}} \simeq 10$ km is the width of the magnetosphere, of the order of the neutron star's radius. If the opacity is pair-dominated, the annihilation rate is equal to (plasma assumed to be not highly relativistic)

$$R_{\mathrm{ann}} = \frac{3}{32}\sigma_{\mathrm{T}} n^2 c, \tag{5}$$

where n is the total electron and positron density. Due to one-photon pair production, it limits the γ-ray luminosity of a neutron star at $\hbar\omega > 2mc^2$:

$$L_{\mathrm{pd}} \simeq 10^{35}\ \mathrm{erg\ s^{-1}}. \tag{6}$$

The limit (6) is even smaller in fields of $B > 10^{13}$ Gauss, since the one-photon annihilation probability becomes greater than the two-photon annihilation probability and multiple reactions of the type $\gamma \to e^+ + e^- \to \gamma'$ are possible.

If the conditions of transparency $n < n_{\mathrm{tr}}$ and $L < L_{\mathrm{pd}}$ (see Eqs. (4) and (6)) are satisfied, photon splitting acts in its pure form. In this case of a transparent magnetized vacuum, as already noted, splitting occurs once, no matter how large the optical depth is. Due to the polarization selectivity of photon splitting, a single decay has three main imprints on the initial spectrum. First of all, it can produce an observable excess of soft photons, for which the optical depth is less than unity; next, it results in a smoother spectrum, making all spectral features less distinguishable; and the most prominent of these three, strong polarization of the outgoing spectrum with the electric vector parallel to the magnetic field. The degree of polarization approaches 100% for hard photons and continuously decreases towards smaller energies. The characteristic energy at which the degree of polarization reaches 50% approximately corresponds to that for which the optical depth for splitting is equal to unity. Unfortunately, more definite estimation seems to be impossible without knowing the initial spectrum more or less precisely. To make matters worse, the optical depth is different for different sites, so that the uncertainty depends on how uniformly sources are distributed over the neutron star's surface.

3 SELF-SIMILAR SOLUTION OF THE RADIATION TRANSFER EQUATION

Given the differential splitting rate for a photon with energy ε to photons with energies ω and $\varepsilon - \omega$ (in units mc^2):

$$\sigma_{\mathrm{sp}}(\varepsilon, \omega) = 30\frac{\omega^2(\varepsilon-\omega)^2}{\varepsilon^5}\sigma_{\mathrm{sp}}(\varepsilon), \tag{7}$$

one can easily obtain the radiation transfer equation for a spectral density of photons F in the collinear approximation:

$$\frac{\partial}{\partial l}F(\omega) = -\sigma_{\mathrm{sp}}(\omega)F(\omega) + 2\int_{\omega}^{\infty} F(\omega')\sigma_{\mathrm{sp}}(\omega', \omega)\ \mathrm{d}\omega'. \tag{8}$$

In this equation l is the coordinate along the line of sight, $F(\omega)$ is the number of photons per energy unit, and $\sigma_{\rm sp}$ is the polarization-averaged cross-section for photon splitting. The first term in Eq. (8) describes photon loss due to their decay, the second one includes newly generated photons with smaller energies. Equation (8) makes no difference between two principal polarization modes that implies depolarization via Thomson scattering. Factor 2 here is due to equal probabilities of different processes: $\omega' \to \omega + (\omega' - \omega)$ and $\omega' \to (\omega' - \omega) + \omega$. Angular variables are omitted and no other processes are included. Though photons change their propagation angles chaotically from one scattering to another, we can still use the collinear approximation (i.e. do not include the angular dependence in (8)), but the exact physical meaning of $F(\omega)$ and l will change. In this case, $F(\omega)$ should be considered as a probability for a photon to have energy ω, and l is the coordinate along the actual path of the observed photon prolonged through all splitting events.

When the optical depth is small and each photon undergoes splitting at most once, the resulting spectrum, symmetric and bell-shaped, could be directly obtained from (7). In the opposite case, equation (8) should be solved. To understand the physics of photon splitting better one needs those kinds of solutions which reflect the essential features of this process. When found, these solutions open the possibility to analyze an arbitrary spectrum by decomposing it into a set of natural solutions (in a way similar to Fourier expansion). There is a one-parameter set of self-similar solutions for Equation (8) that meets these requirements.

First of all, let us use a normalized frequency, $x = \omega/\omega_0$, where the parameter ω_0 depends on l, and rewrite the expression (3) in the following form:

$$\sigma_{\rm sp} = \mu(b,\theta)\varepsilon^5, \quad \mu(b,\theta) \equiv \frac{\alpha^3}{30\pi^2}\left(\frac{13}{315}\right)^2 \frac{1}{\lambda_c}(b\sin\theta)^6. \tag{9}$$

Then suppose that the function $F(\omega)$ has the form

$$F(\omega) = A(l)f(x) \equiv A(l)f\left(\frac{\omega}{\omega_0(l)}\right). \tag{10}$$

Using the expressions (10), (9), (7) and the Equation (8) one comes to the following equation (prime denotes $\mathrm{d}/\,\mathrm{d}l$):

$$A'f(x) - A\frac{\omega_0'}{\omega_0}x\frac{\mathrm{d}f}{\mathrm{d}x}(x) = -A\mu\omega_0^5 x^5 f(x) + 60\int_x^\infty \mu\omega_0^5 x^2 (x'-x)^2 A f(x')\,\mathrm{d}x'. \tag{11}$$

There is one more condition left, namely, the conservation of the total energy flux, which gives:

$$\int_0^\infty \omega F(\omega)\,\mathrm{d}\omega = A\omega_0^2\int_0^\infty x f(x)\,\mathrm{d}x = \mathrm{const.} \tag{12}$$

Since $\int_0^\infty x f(x)\,\mathrm{d}x$ does not depend on l, Eq. (12) establishes a relation between ω_0' and A':

$$\frac{2\omega_0'}{w_0} = -\frac{A'}{A}. \tag{13}$$

Substituting A' with ω_0' in (11) and making some transformations one finally separates variables and gets two equations for $\omega_0(l)$ and $f(x)$:

$$-\frac{1}{\mu}\frac{\omega_0'}{\omega_0^6} = \frac{1}{n^5}, \tag{14}$$

$$\left(-x^5 f(x) + 60\int\limits_x^\infty x^2(x'-x)^2 f(x')\,\mathrm{d}x'\right)\left(2f(x) + x\frac{\mathrm{d}f}{\mathrm{d}x}\right)^{-1} = \frac{1}{n^5}. \tag{15}$$

The physical meaning of the constant $1/n^5$ will be discussed later. Equation (14) has a simple solution:

$$\tilde{\omega}_0(l) = \tilde{\omega}_0(0)\left(1 + 5\int\limits_0^l \mu\,\mathrm{d}l\tilde{\omega}_0^5(0)\right)^{-1/5}. \tag{16}$$

With the optical depth $\int_0^l \mu\,\mathrm{d}l$ growing, $\tilde{\omega}_0(l)$ becomes almost independent of its initial value. Equation (15) has a set of solutions $f_n(x)$ which depend on the constant n. The equation for $f_1(x)$ allows no further simplifications and looks as follows:

$$2f_1 + xf_1' = -x^5 f_1 + 60x^2\int\limits_x^\infty (x'-x)^2 f_1(x')\,\mathrm{d}x'. \tag{17}$$

By substitution of the variable $y = nx$ in (17) one can prove that

$$f_n(x) \equiv f_1(nx). \tag{18}$$

Relation (18) means that the actual choice of the constant n in (14) has no physical meaning since the solution we get, $f_n(\omega/\omega_0)$, can be presented in the form $f_1(n\omega/\omega_0) \equiv f_1(\omega/\tilde{\omega}_0)$. We then restrict ourselves to consideration of $f_1(x)$ only, eliminating the freedom in choosing the parameter ω_0. The latter is completely defined by a value of the frequency corresponding to the maximum of the initial spectrum, ω_{max}. The solution of Equation (17), $f_1(x)$, is obtained by numerical integration and is presented in Derishev *et al.* (1997). It reaches a maximum at $x \simeq 0.7$ and, consequently,

$$\omega_0 \simeq 1.4\omega_{\mathrm{max}}. \tag{19}$$

The asymptotic behaviour of $f_1(x)$ at $x \to 0$ and $x \to \infty$ are investigated analytically. The asymptotics for $x \to \infty$ are just a solution of (17) with the integral term ignored. This solution has the form $(C_1/x^2)\exp(-x^5/5)$, which gives an estimate for the integral term $(120/x^{10})f_1(x)$, justifying very quick asymptotic convergence (the value $C_1 \simeq 1.5$ was derived numerically). To get an asymptotic at $x \to 0$ we replace $f_1(x)$ by its Laurent expansion $\sum_{n=-\infty}^{+\infty} a_n x^n$ in Equation (17) and find that $a_n \equiv 0$ for $n < -2$ and $-1 \le n \le 1$. Numerical computations then give $a_{-2} = 0$ and $a_2 \simeq 10$. This result follows directly from the radiation transfer Equation (8), because its solution is just a superposition of bell-shaped spectra (7) with different values of ε, and each of them has a quadratic asymptotic near zero frequency.

4 EVOLUTION OF POWER SPECTRA. METHOD OF DECOMPOSITION

It follows from Section 3 that there is a set of self-similar solutions of the radiation transfer equation for photon splitting:

$$F(\omega) = A f_1 \left(\frac{\omega}{\omega_0(l)}\right) \left(\frac{\omega_0(0)}{\omega_0(l)}\right)^2. \tag{20}$$

Here the parameter ω_0 changes with l in accordance with (16), so that the spectrum (20) shrinks and grows in magnitude but preserves its shape. This is true for any geometry of magnetic field. However, in real astrophysical sources one can hardly expect the spectrum to be of the particular form (20). In the general case, one can use the following decomposition of radiation intensity per unit spectral interval:

$$I(\omega) = \omega \int_0^\infty A(\omega_0) f_1 \left(\frac{\omega}{\omega_0}\right) \mathrm{d}\omega_0. \tag{21}$$

This is a type-one Fredholm integral equation. Replacing $A(\omega_0)$ by $B(\omega_0)/\omega_0$, ω_0 by e^u and ω by e^v one obtains from (21) a convolution-type integral equation:

$$\tilde{I}(v)e^{-v} = \int_{-\infty}^{\infty} \tilde{B}(u)\tilde{f}_1(u-v)\,\mathrm{d}u. \tag{22}$$

The latter can formally be solved by Fourier transformation, which reduces equations of this type to algebraic ones provided the Fourier transformation of $\tilde{I}(v)e^{-v}$ exists (the asymptotic behaviour of $\tilde{f}_1(u-v)$ is known and it is a 'good' function of u and v). In practice, equation (21) requires numerical calculations to find $A(\omega_0)$. However, there is a specific case, namely a power-law spectrum $I(\omega) \propto \omega^n$, which has astrophysical importance and allows one to find an analytical solution. Assuming $A(\omega_0)$ to have a power-law form and using the new variable $\xi = \omega_0/\omega$, we find:

$$I(\omega) \equiv K_1\omega^n = \omega^{m+2} \int_0^\infty K_2\xi^m f_1(1/\xi)\,\mathrm{d}\xi, \tag{23}$$

$$A(\omega_0) \equiv K_2\omega_0^m,$$

$$K_1 = K_2 \int_0^\infty \xi^m f_1(1/\xi)\,\mathrm{d}\xi, \tag{24}$$

$$m = n - 2. \tag{25}$$

As can be seen from (25), the function $A(\omega_0)$ is steeper than $I(\omega)$.

To analyse the evolution of the initially power-law decomposition $A(\omega_0)$ we use the conservation law (12), the relation (16) and its inverse:

$$\omega_0(0) = \omega_0(l)\left(1 - 5\int_0^l \mu\, \mathrm{d}l\omega_0^5(l)\right)^{-1/5}. \tag{26}$$

Note from Eq. (16) that the function $A_l(\omega_0)$ is truncated at frequency $\omega_c = (5\int_0^l \mu\,\mathrm{d}l)^{-1/5}$, which is valid for arbitrary $A(\omega_0)$. For $\omega_0 \le \omega_c$ we have

$$\begin{aligned} A_l(\omega_0) &= \left(\frac{\omega_0(0)}{\omega_0(l)}\right)^2 \left(1 - 5\int_0^l \mu\,\mathrm{d}l\omega_0^5(l)\right)^{-6/5} A(\omega_0^{(0)}(\omega_0(l))) \\ &= K_2\omega_0^m \left(1 - 5\int_0^l \mu\,\mathrm{d}l\omega_0^5(l)\right)^{-(m+8)/5}, \end{aligned} \tag{27}$$

if $A(\omega_0)$ is given by (24). Depending on the spectral index $n \equiv m+2$, the behaviour of $A(\omega_0)$ near the critical frequency ω_c differs qualitatively. 'Spectral' decomposition of the reprocessed spectrum, $A_l(\omega_0)$, has an integrable singularity if $n > -6$ and is sharply cut at $\omega_0 = \omega_c$ for $n = -6$. In the case of hard spectra with $n < -6$, $A_l(\omega_0)$ smoothly decreases to zero near $\omega_0 = \omega_c$.

In the general case, the solution of the radiation transfer equation (8) is given by:

$$\begin{aligned} F(l,\omega) &= \int_0^{\omega_c} A_l(\omega_0) f_1\left(\frac{\omega}{\omega_0}\right) \mathrm{d}\omega_0, \\ \omega_c &= \left(5\int_0^l \mu\,\mathrm{d}l\right)^{-1/5}. \end{aligned} \tag{28}$$

It is evident that for soft spectra with spectral index $n \ge -6$ the only feature implied by photon splitting is a cut-off at $\omega = \omega_c$. For hard spectra the singularity in (27) has a width

$$\left(1 - \left(\frac{2-n}{8}\right)^{1/5}\right)\omega_c < 0.2\omega_c;$$

it is significantly smaller than the width of the self-similar solution $f_1(\omega/\omega_c)$ itself. Thus, it is reasonable to think of $A_l(\omega_0)$ as a function consisting of a truncated power-law and the delta-function at $\omega = \omega_c$. Consequently, the resulting spectrum turns out to be a superposition of the initial spectrum without its hard-energy tail and the self-similar solution $A_1 f_1(\omega/\omega_c)$. The total intensity is concentrated near the singularity of $A_l(\omega_0)$ and depends on the spectral index. Two results are possible. An observer will see either a flattening near ω_c or a bump-like feature for harder spectra with $n > -6$. Another important feature of the solution (27) is that for any $l > 0$, $A_l i(\omega_0)$ has one and the same shape with only the energy scale decreasing as ω_c. Such self-similar behaviour is a direct consequence of the fact that a pure power-law spectrum has no intrinsic scale.

Note that all results obtained above are applicable not only for pure power-law spectra but also for spectra that allow a power-law approximation with spectral index less than -1 and even for exponential hard-energy tails which can be considered as a limiting case of a power-law with infinite value of the spectral index. However, any spectrum with a maximum at a frequency greater than ω_c has to be considered in a different way. Fortunately, the weak dependence of ω_c on optical depth simplifies the situation a lot. It appears that this or that specific spectral shape does not contribute much to the final spectrum provided most of energy in the initial spectrum is concentrated at frequencies $\omega > \omega_c$ (for further discussion see Derishev *et al.* (1997)).

5 CONCLUSIONS AND CONSTRAINTS

Photon splitting appears to be an important mechanism of spectra formation in highly magnetized environments. Neutron stars represent that class of astrophysical objects where required fields can exist. In addition, fields of the order of B_{cr} could possibly be created due to plasma turbulence in a system of colliding neutron stars which is considered as a possible progenitor of cosmological gamma-ray bursts.

The result of photon splitting differs a lot for transparent and opaque media. In the first case an approximation of single splitting can be employed. One-mode photon decay then produces the distinct feature of the outgoing spectrum – very high degree of polarization, up to 100%. Photons with the electric vector parallel to the magnetic field will be observed in all energy ranges. Having a polarization-sensitive instrument, operating at sub-MeV energies, one will be able to determine the frequency at which the degree of polarization drops below 50%, thus giving a rough estimate of the magnetic field strength. The strong angular dependence of the photon splitting cross-section makes possible the existence of γ-sources with regularly pulsating spectra – young neutron stars with rotation axis and magnetic dipole axis not yet aligned. Another interesting feature is generated if the initial spectrum has a 0.5 MeV annihilation line and undergoes single photon decay. The shape of the resulting broad maximum resembles that originating from single Compton scattering.

The case when a medium is opaque to scattering on electrons and positrons is completely different. Due to multiple scattering, polarization modes chaotically transform to each other and polarization-averaged radiation transfer equation can be used. It was shown in Section 3 that this equation possesses a set of self-similar solutions that simply scale with optical depth for splitting. Scaling with the field strength was pointed out in the work of Baring (1995), where a stationary equation with uniformly distributed sources was used, though the solution of the Cauchy problem seems to be more adequate.

The qualitative features of the spectrum in an energy range of the order of or greater than the boundary value $\hbar\omega_c$ are clear from the analysis which we performed. First of all, the splitting leads to a lack of hard photons compared to the equilibrium thermal spectrum (they are transferred to a softer region of the spectrum). The flattening or even an elevation of the spectrum in the immediate vicinity of the boundary energy are unambigous signatures of the hardness of the initial spectrum which was subject to the photon splitting effect. In the absence of the low-energy background, an arbitrary given distribution of photons condenses from the region of energies greater than $\hbar\omega_c$ to the self-similar spectrum. Identification of the above features in the spectra of neutron stars can be an indirect proof of the existence of photon splitting.

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