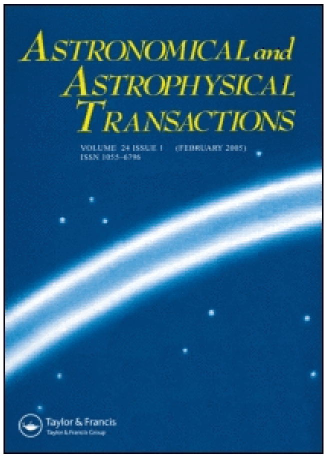

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CONCEPTUAL PROBLEMS OF FRACTAL COSMOLOGY

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This report continues the recent Peebles–Turner debate 'Is cosmology solved?' and considers the first results for Sandage's programme for 'Practical cosmology'. A review of conceptual problems of modern cosmological models is given, among them: the nature of space expansion; recession velocities of distant galaxies greater than the velocity of light; the cosmological Friedmann force; continuous creation of gravitating mass in Friedmann's equation; cosmological pressure is not able to produce work; cosmological gravitational frequency shift; the Friedmann–Holtsmark paradox; the problem of the cosmological constant; Einstein's and Mandelbrot's cosmological principles; the fractality of the observed galaxy distribution; Sandage's 21st problem, the Hubble–de Vaucouleurs paradox; and the quantum nature of the gravity force.

KEY WORDS Cosmology, Friedmann model, fractals, – general

1 IS COSMOLOGY SOLVED?

A debate under the title 'Is cosmology solved?' was held recently at the Smithsonian National Museum of National History. James Peebles and Michael Turner presented two different views of the problem. According to Turner (1999a) cosmology was solved in 1998 by the theory of inflation and cold dark matter; while according to Peebles (1999) 'many commonly discussed elements of cosmology are still on dangerous ground'. Recent discoveries of the dominating contribution of the cosmological constant into the dynamics of the expansion and fractality of the large-scale galaxy distribution have demonstrated how modern powerful observations can dramatically change the common view on cosmological physics. As Lawrence Krauss said: 'One thing is already certain. The standard cosmology of the 1980s, postulating a flat universe dominated by matter, is dead' (Krauss, 1999).

Five years ago '23 astronomical problems for the next three decades' were formulated by Allan Sandage at the conference on Key Problems in Astronomy and Astrophysics (Sandage, 1995). Problems 15–23 relate to 'Practical cosmology' and recent observations shed light on some of them. It is now clear that these problems have roots in the foundations of cosmological models and this is why it is the right time for an analysis of the basis of contemporary cosmology.

This report is devoted to a continuation of this debate and relates especially to conceptual aspects of cosmological models, which are sharpened by recent observations and have been only little discussed previously.

2 BUILDING BLOCKS OF COSMOLOGICAL MODELS

Any cosmological model contains several fundamental hypotheses which determine the interpretation of observable phenomena. A classification of possible relativistic cosmologies in accordance with basic initial assumptions has been discussed by Baryshev *et al.* (1994). Modern cosmological theory includes in particular, as fundamental building blocks, the theory of gravitational interaction, global matter distribution, the origin of the cosmic microwave background radiation, the mechanism of cosmological redshift, evolution and the arrow of time.

The most important elements of any cosmological model are the *cosmological principle* and the *relativistic gravity theory*, tying the main conceptual problems of cosmology closely with recent studies of large-scale matter distribution and the investigation of the physics of gravitational interaction.

Modern astrophysical observations give an empirical foundation of cosmological models. The main task of observational cosmology is to compare predictions of cosmological theories with real data and to select viable models. During the last decade observations are developing exponentially and this opens new horizons for cosmological theory.

Below we give an analysis of the contemporary state of modern cosmology with special emphasis on conceptual aspects of cosmological models.

3 THE STANDARD MODEL

The Friedmann–Lemaitre–Robertson–Walker (FLRW) cosmological model is currently accepted as the Standard Model (SM) for all interpretations of observed astrophysical data. For this conference it is interesting to note that Alexander Friedmann found his famous solution in 1922–1924 working (partly) here in St Petersburg University, and at the same time George Gamow was a student of our university (together with other brilliant students such as Lev Landau, Dmitrij Ivanenko, Vladimir Fok, and Viktor Ambartzsumyan).

3.1 Einstein's Cosmological Principle

The first basic element of the SM is Einstein's cosmological principle. The cosmological principle, in fact, is the hypothesis that the universe is spatially homogeneous and isotropic on 'large scales' (see e.g. Weinberg, 1972; Peebles, 1993; Peacock, 1999). Homogeneity of the matter distribution plays a central role in the expanding universe model, because homogeneity implies that the recession velocity is proportional to distance. This means that the linear velocity–distance relation $V = Hr$, identified with the observed Hubble law, is valid at scales where matter distribution can be considered on average uniform. Hence the words 'large scales' have an exact meaning in FLRW cosmology as the scales where the linear velocity–redshift relation starts to exist.

The homogeneity and the isotropy of the distribution of matter in space mean that starting from the scale r_{hom} for all scales $r > r_{\text{hom}}$ we have

$$\varrho(\boldsymbol{r}, t) = \varrho(t), \tag{1}$$

$$p(\boldsymbol{r}, t) = p(t). \tag{2}$$

It has been extensively discussed whether the homogeneity of the Universe is to be expected from general physical arguments. However within the SM one cannot account for the homogeneity and this means that homogeneity must be accepted as a phenomenon to be explained by some future deeper theory.

3.2 General Relativity

The second fundamental element of the SM is general relativity (GR), which is a geometrical gravity theory (as an alternative to the quantum field approach, see e.g. Feynman, 1971; Baryshev, 1996). GR was successfully tested in the weak gravity condition of the solar system and binary neutron stars. It is assumed that GR can be applied to the Universe as a whole.

According to GR, gravity is described by a metric tensor g^{ik} of a Riemannian space. The 'field' equations in GR (Einstein–Hilbert equations) have the form:

$$\Re^{ik} - \frac{1}{2} g^{ik} \Re = \frac{8\pi G}{c^4} T^{ik}_{(m)} + g^{ik} \Lambda, \tag{3}$$

where $\Re^{ik}$ is the Ricci tensor, $T^{ik}_{(m)}$ is the energy–momentum tensor (hereafter EMT) for all kinds of matter, and Λ is the famous cosmological constant, which does not depend on time and space coordinates. Note that gravity in GR is not matter, so T^{ik}_{m} does not contain the EMT of the gravity field. Solutions of Eq. (3) for an unbounded homogeneous matter distribution (Eqs. (1, 2)) are the basis of the FLRW cosmological model.

3.3 Space Expansion Paradigm

An important consequence of homogeneity and isotropy is that the line element may be presented in the Robertson–Walker form:

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - S(t)^2\,\mathrm{d}\chi^2 - S(t)^2 I_k(\chi)^2 (\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \tag{4}$$

where χ, θ, ϕ are 'spherical' comoving space coordinates, t is the synchronous time coordinate, $I_k(\chi) = \sin(\chi)$, χ, $\sinh(\chi)$ corresponding to curvature constant values $k = +1, 0, -1$ respectively and $S(t)$ is the scale factor.

The *expanding space paradigm* is that the proper metric distance r of a body with fixed comoving coordinate χ from the observer is:

$$r = S(t)\chi \tag{5}$$

and increases with time t as the scale factor $S(t)$. Note that the physical dimension of the metric distance $[r]$ = cm, hence if $[S]$ = cm then χ is the dimensionless comoving coordinate distance. In fact χ is the spherical angle and $S(t)$ is the radius of the sphere (or pseudosphere) in the embedding four-dimensional Euclidean space. Hence r is the 'internal' proper distance on the three-dimensional hypersurface of the embedding space. In other words r and χ are Euler and Lagrangian comoving distances, respectively.

Use is often made also of 'cylindrical' comoving space coordinates μ, θ, ϕ, for which the interval

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - S(t)^2 \frac{\mathrm{d}\mu^2}{1 - k\mu} - S(t)^2 \mu^2 (\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2). \tag{6}$$

In this case the metric distance l

$$l = S(t)\mu \tag{7}$$

is the 'external' distance from the z-axis in embedding Euclidean four-dimensional space. It is thus important to use different designations for the different distance interval defined by Eq. (4) and Eq. (6) (see e.g. Peacock, 1999, p. 70).

The relation between these two metrical distances is

$$r = S(t) I_k^{-1}(l/S), \tag{8}$$

were I_k^{-1} is the inverse function for I_k.

3.4 Cosmological Redshift

The expansion of space induces wave stretching of the travelling photons via Lemaitre's equation, i.e.:

$$(1 + z) = \frac{\lambda_0}{\lambda_1} = \frac{S_0}{S_1}, \tag{9}$$

where z is the cosmological redshift, λ_1 and λ_0 are the wavelengths at emission and reception, respectively, and S_1 and S_0 the corresponding values of the scale factor. Equation (9) is usually obtained from the radial null-geodesies ($\mathrm{d}s = 0$, $\mathrm{d}\theta = 0$, $\mathrm{d}\phi = 0$) of the RW line element.

According to the expanding space paradigm, the cosmological redshift is not the familiar Doppler effect but is a new physical phenomenon (see the discussion in Harrison, 1993; 1995). This is clear by comparison between the relativistic Doppler and cosmological FLRW velocity–redshift relations.

3.5 Friedmann's Equation

The behaviour of the scale factor with time $S(t)$ is governed by Einstein's equations (3) which can be written in the form:

$$\mathfrak{R}_i^k - \frac{1}{2}\delta_i^k \mathfrak{R} = \frac{8\pi G}{c^4} T_i^k, \tag{10}$$

where the total EMT is given by

$$T_i^k = T_{(m)\,i}^k + T_{(r)\,i}^k + T_{(v)\,i}^k. \tag{11}$$

Here the indexes m, r, v denote matter, radiation and vacuum, respectively. In comoving coordinates the total EMT has the form:

$$T_i^k = \mathrm{diag}\,(\varrho c^2, -p, -p, -p), \tag{12}$$

where $\varrho = \varrho_m + \varrho_r + \varrho_v$ is the total density and $p = p_m + p_r + p_v$ is the total pressure. For radiation $p_r = (1/3)\varrho_r c^2$ and for the vacuum $p_v = -\varrho_v c^2$.

In the case of homogeneity, Einstein's equations are directly reduced to Friedmann's equation, which may be presented in the form:

$$\frac{\mathrm{d}^2 S}{\mathrm{d}t^2} = -\frac{4\pi G}{3} S \left(\varrho + \frac{3p}{c^2}.\right) \tag{13}$$

From the Bianchi identity it follows that the continuity equation is

$$\dot{\varrho} = -3\left(\varrho + \frac{p}{c^2}\right)\frac{\dot{S}}{S} \tag{14}$$

which must be added to Eq. (13). Because Lagrangian comoving coordinates do not depend on time, one may rewrite Eq. (13) using Eq. (5) as

$$\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{GM_g(r)}{r^2}, \tag{15}$$

where the gravitating mass $M_g(r)$ is given by

$$M_g = M_m + M_r + M_v \tag{16}$$

and contributions from matter, radiation and vacuum are

$$M_m(r) = \frac{4\pi}{3}\left(\varrho_m + \frac{3p_m}{c^2}\right) r^3, \tag{17}$$

$$M_r(r) = \frac{4\pi}{3} 2\varrho_r r^3, \tag{18}$$

$$M_v(r) = -\frac{4\pi}{3} 2\varrho_v r^3. \tag{19}$$

Solving the Friedmann equation (15) one finds the dependence on time for the metric distance $r(t)$ or the scale factor $S(t)$.

3.6 *Cosmological Parameters*

The FLRW model has two generally used parameters. The Hubble parameter $H = \dot{S}/S$ and the deceleration parameter $q = -\ddot{S}S/\dot{S}^2$ which for the present time t_0 are $H(t_0) = H_0$ and $q(t_0) = q_0$ respectively.

Use is frequently also made of the density parameter $\Omega = \varrho/\varrho_{\rm cr}$ where the critical density is

$$\varrho_{\rm cr} = \frac{3H^2}{8\pi G}. \tag{20}$$

Equation (15) may also be written in the form

$$q = \frac{1}{2}\Omega\left(1 + \frac{3p}{\varrho c^2}\right), \tag{21}$$

where Ω, p, ϱ are the total quantities (see Eq. 12).

The old standard model has the following parameters

$$\Omega_0 = \Omega_{(m)\,0} = 1, \quad \Omega_v = 0, \quad q_0 = 0.5. \tag{22}$$

The new version of SM which is currently accepted is

$$\Omega_0 \approx 1, \quad \Omega_m \approx 0.2, \quad \Omega_v \approx 0.8, \quad q_0 \approx -0.7, \quad H_0 = 65 \pm 10\,\text{km s}^{-1}\,\text{Mpc}^{-1}. \tag{23}$$

This means that the expansion of the present universe is accelerated and that the dominant force in the Universe is the cosmological antigravity of the vacuum (see the discussion by Krauss, 1999). Sandage's problems 18 and 20 are related to the value of the parameters q_0 and Ω_0, and recent observations of distant supernovae now specify their values (see discussion in Section 4.8).

3.7 *Mattig's Distance-Redshift Relation*

In the case of a matter-dominated FLRW model there is a very important explicit relation between the cosmological redshift and the metrical distance at the present epoch $t = t_0$. The relation was first derived by Mattig (1958) and has the form (see e.g. Peacock, 1999)

$$l_m(z, q_0) = S_0 I_k(\chi) = \frac{c}{H_0}\frac{zq_0 + (q_0 - 1)((2q_0 z + 1)^{1/2} - 1)}{q_0^2(1+z)}, \tag{24}$$

where $l_m(z, q_0)$ is the cylindrical metric distance, χ is the spherical comoving coordinate distance, q_0 is the deceleration parameter, z is the cosmological redshift, $I_k = \sin\chi$, χ, $\sinh\chi$ for $k = +1, 0, -1$ respectively, and the scale factor is

$$S_0 = \frac{c}{H_0}\sqrt{\frac{k}{2q_0 - 1}}. \tag{25}$$

To calculate the internal metrical distance r_m for a known Hubble constant, deceleration parameter and redshift, one must then use also relation (8) between l and r.

3.8 Observable Quantities

The basic relation for the calculation of different observable quantities within FLRW model is the connection between metric r_m, angular r_a and luminosity r_{lum} distances in the expanding universe:

$$r_m = r_a(1+z) = \frac{r_{lum}}{(1+z)}. \quad (26)$$

Using the Eq. (26) one may calculate theoretical predictions such as angular size–redshift, magnitude–redshift, count–magnitude and count–redshift relations.

Classical cosmological tests, such as $\Theta(z)$, $m(z)$, $N(m)$, are based on these relations and actually give practical tools for the estimation of the observed values of the main cosmological parameters (see e.g. Baryshev *et al.*, 1994; Peacock, 1999).

3.9 Successes of the Standard Model

According to modern cosmological textbooks (see e.g. Peebles, 1993; Peacock, 1999) the Standard Model is the homogeneous FLRW model of the Universe, which begins from a singularity and has expanded in a near homogeneous way from a denser hotter state when the cosmic background radiation was thermalized.

There is definite success in the application of the SM to the observed Universe. Indeed, there are no gravitational, photometric and thermodynamic paradoxes in the SM, because the age of the Universe is finite and rather small, equal to the age of a solar-like normal star.

In the SM, space has been filled with blackbody radiation, the cosmic microwave background radiation (CMBR). The number of CMBR photons per unit volume at redshift z is

$$n(z) = n_0(1+z)^3, \quad (27)$$

when n_0 is the present value of the number density. As the Universe expands CMBR preserves a blackbody spectrum with temperature

$$T(z) = T_0(1+z), \quad (28)$$

where T_0 is the present temperature of the CMBR. Equations (27) and (28) are used back to $z \sim 10^{10}$. The observed thermal spectrum of the CMBR is considered as the greatest success of the SM.

In the SM, the Universe was hot and dense enough to drive thermonuclear reactions that changed the chemical composition of matter. The values of the abundances left over from this hot epoch depend on the cosmological parameters. Knowing the present temperature and assuming a value for the present matter density, the thermal history of the Universe is fixed. If the matter is uniformly distributed and lepton numbers are comparable to the baryon number, this is sufficient to fix the final abundances of the light elements. The observed light element abundances of ^{4}He, ^{2}He, ^{3}He and ^{7}Li are in good agreement with SM predictions.

4 CONCEPTUAL PROBLEMS OF THE STANDARD MODEL

In parallel with the successes of the SM there are several deep conceptual puzzles which have no convincing explanation yet and which need more careful analysis at the present time when the foundations of the SM are under consideration.

4.1 The Nature of the Expansion of Space

According to SM the space of our Universe is described by the RW metric (see Eq. (4) and Eq. (6)). In mathematical language our three-dimensional space at the fixed cosmic time is just a hypersphere in four-dimensional embedding Euclidean space. Hence space expansion simply means that the radius of the hypersphere grows with time and the three-dimensional volume of space continuously increasing, i.e. for an internal three-dimensional observer, space is continuously created. The puzzling physical problem is that the space in physics is not empty but relates to the physical vacuum, so the physics of space creation needs to be explained.

Another problem is how to measure space expansion. Indeed if our Galaxy does not expand then it is a hopeless problem to verify this new physical phenomenon by laboratory experiments and one has to only believe in the theoretical interpretation of the cosmological redshift.

4.2 Recession Velocities of Distant Galaxies Greater than the Velocity of Light

The *exact relativistic* expression for recession velocity, or the 'space expansion' velocity, or the rate of increase of the metric distance r, for a body with fixed χ directly follows from Eq. (5):

$$V_{\mathrm{exp}} = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t}\chi = \frac{\mathrm{d}S}{\mathrm{d}t}\frac{r}{S} = H(t)r = c\frac{r}{r_{\mathrm{H}}}, \tag{29}$$

where $H(t) = \dot{S}/S$ is the Hubble constant (which actually depends on time) and $r_{\mathrm{H}} = c/H(t)$ is the Hubble distance at time t.

The exact relativistic velocity–distance relation is Eq. (29) and it is linear for all distances r. It means that for $r > r_{\mathrm{H}}$ we get $V_{\mathrm{exp}} > c$ and the question arises of why general relativity violates special relativity. The usual answer is that the space expansion velocity is not the ordinary velocity of a body in space, hence it has no ordinary limit by the velocity of light. This question is tightly connected with the fact mentioned above, that the space expansion redshift and the Doppler redshift are quite different physical phenomena (see the discussion in Harrison, 1993).

4.3 Cosmological Friedmann Force

Friedmann's equation (15) in fact presents the *exact relativistic* cosmological Friedmann force acting on a test galaxy with mass m placed at a distance r from any fixed point at the origin of the coordinate system:

$$F_{\mathrm{Fr}}(r) = m\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -\frac{GmM_g(r)}{r^2}. \tag{30}$$

It looks like the usual Newtonian equation of motion of a test particle. Such a similarity was first found by Milne (1934) and McCrea and Milne (1934) and created a problem in cosmology because Eq. (30) has no relativistic restrictions such as the limit by the velocity of light and general retarded response effects. The root of the puzzle lies in the derivation of Friedmann's equation, which utilizes the comoving coordinates r and synchronous universal cosmic time t.

For example, the critical density (Eq. (20)) of the FLRW universe does not depend on the velocity of light and is simply the Newtonian pulsation formula. The superluminous expansion velocity (Eq. (29)) is also a consequence of this *non-relativistic* character of Friedmann's equation.

4.4 Continuous Creation of Gravitating Mass

The most puzzling property of the FLRW model is the dependence of gravitating mass in Eq. (15) on the cosmic time t. Indeed, in the case of ordinary matter the density $\varrho \sim r^{-3}$ and the gravitating mass Eq. (17) does not depend on time. However in the case of radiation the density is $\varrho \sim r^{-4}$ and the gravitating mass of radiation will be

$$M_r(r) = \frac{4\pi}{3} 2\varrho_r r^3 \sim r^{-1}(t). \tag{31}$$

This means that the mass of radiation is continuously disappeared in the expanding universe. As noted by Peebles (1993, p. 139): 'The resolution of this apparent paradox is that ... there is not a general global energy conservation law in general relativity theory'.

The next strange example is the vacuum, where the density ϱ_v is a constant in time, so the gravitating mass of the vacuum will be

$$M_v(r) = -\frac{4\pi}{3} 2\varrho_v^3 \sim r^3(t). \tag{32}$$

This means that vacuum antigravity continuously increases in time due to the continuous creation of gravitating (actually 'antigravitating') vacuum mass.

4.5 Cosmological Pressure Is Not Able to Produce Work

It was noted by Harrison (1981; 1995) that in a homogeneous unbounded expanding universe there is no pressure gradient and so the first law of laboratory thermodynamics

$$\frac{\mathrm{d}E}{\mathrm{d}t} + p\frac{\mathrm{d}V}{\mathrm{d}t} = 0 \tag{33}$$

is not applicable. Indeed in the case of the FLRW model we may imagine the whole universe partitioned into macroscopic cells, each of comoving volume V, and all having contents in identical states. The $-p\,dV$ energy lost from any one cell cannot reappear in neighbouring cells because all cells experience identical losses. So the usual idea of an expanding cell performing work on its surroundings cannot apply in this case. As Edward Harrison emphasized: 'The conclusion, whether we like it or not, is obvious: energy in the universe is not conserved' (Harrison, 1981, p. 76).

4.6 Cosmological Gravitational Frequency Shift

In 1947 in the classic paper 'Spherical symmetrical models in general relativity' by Sir Hermann Bondi, it was shown that, at least for small redshifts, the total cosmological redshift of a distant body is due to two causes: the velocity shift (Doppler effect) due to the relative motion of source and observer, and the global gravitational shift (Einstein effect) due to the difference between the potential energy per unit mass at the source and at the observer.

This means that the spectral shift depends not only on the conditions at the source and at the observer but also on the distribution of matter in the intervening space around the source. In the case of small distances Bondi derived a simple formula for the redshift which is simply the sum of Doppler and gravitational effects, and which explicitly showed that 'the sign of the velocity shift depends on the sign of v, but the Einstein shift is easily seen to be towards the red' (Bondi, 1947, p. 421).

Hence according to Bondi the cosmological gravitational frequency shift is redshift. It was shown by Baryshev *et al.* (1994) that from Mattig's relation (24) it follows directly for the case of $z \ll 1$, $v/c \equiv x = r/r_{\rm H}$ that

$$z_{\cos} \approx x + \frac{1+q_0}{2}x^2 = \left(\frac{v}{c} + \frac{1}{2}\frac{v^2}{c^2}\right) + \frac{q_0}{2}x^2 \tag{34}$$

is the sum of the Doppler and gravitational redshifts:

$$z_{\cos} \approx z_{\rm Dop} + z_{\rm grav}, \tag{35}$$

where the cosmological gravitational redshift is

$$z_{\rm grav} = \frac{\Delta\varphi(r)}{c^2} = \frac{1}{2}\frac{GM(r)}{c^2 r} = \frac{1}{4}\Omega_0 x^2. \tag{36}$$

Here r is the distance between the observer and the source, and the source is in the centre of the sphere.

An ambiguity arises when one considers the observer at the centre and a galaxy at the edge of the sphere. In this case one may conclude that the cosmological gravitational shift is a blueshift (see Zeidovich and Novikov, 1984, p. 97 and Peacock, 1999, problem 3.4).

It is interesting that for a fractal matter distribution in which $M(r) \sim r^D$ with fractal dimension $D = 2$ the cosmological gravitational redshift gives the linear distance–redshift relation and becomes an observable physical phenomenon.

4.7 Friedmann-Holtsmark Paradox

According to Friedmann's equation there is a cosmological force Eq. (30) acting on a galaxy situated at distance r from another fixed galaxy. The value of the cosmological force is equal to the value of the Newtonian force for a finite spherical ball with radius r around the fixed galaxy. So this cosmological force increases up to infinity when a galaxy is infinitely far. Moreover the Friedmann force determines the dependence on time of the scale factor $S(t)$, so it plays a fundamental role in the SM.

This is in apparent contradiction with the well-known Holtsmark result for the probability density of the force acting between particles in infinite Euclidean space in the case of $1/r^2$ behaviour of the elementary force (see Holtsmark, 1919; Chandrasekhar, 1941). Due to the isotropy of the particle distribution the average force is equal to zero and there is a finite value of the fluctuating force, which is determined by the nearest neighbour particles. Hence in infinite Euclidean space with homogeneous Poisson distribution and Newtonian gravity force there is no global expansion or contraction, but there are density and velocity fluctuations caused by gravity force fluctuations.

Recently it was found by de Vega and Sanchez (1999) that the ground state of the self-gravitating Newtonian gas is a fractal mass distribution with fractal dimension $D \approx 2$. Probably the final state of an initially Poissonian self-gravitating gas will be this deVS ground state. Future N-body simulations can check this possibility.

4.8 The Problem of the Cosmological Constant

The claim 'New observations have smashed the old view of our universe' opened the January 1999 issue of *Scientific American*, devoted to a special report on the revolution in cosmology because of new observations of very distant supernovae. Two independent groups of astronomers (Riess *et al.* 1998; Perlmutter, *et al.*, 1999) have constructed the magnitude–redshift relation for about fifty SNIa in distant galaxies within the redshift interval 0.1–1.0. The result was completely unexpected because it showed a significant deflection from the prediction of the standard model for $\Omega_m = 1$. To fit the observational data one needs a positive cosmological constant giving $\Omega_v \approx 0.8$. So what Einstein called 'the biggest blunder of my life' now became the biggest news in cosmology.

This is a quite unexpected solution of Sandage's 18th problem because in the framework of the FLRW model it means that the observed universe is accelerating under a mysterious repulsive force which dominates the dynamics of the universe. Within the old version of SM the cosmological constant 'naturally' had zero value and so did not participate in the present-time dynamics of the universe.

Using Eqs. (12) and (20) we get for the observed vacuum density

$$\varrho = 6.8 \times 10^{-30} \Omega_v h_{60}^2 \ (\mathrm{g\ cm^{-3}}), \tag{37}$$

where $h_{60} = H_0/60$ (km s^{-1} Mpc^{-1}) is the normalized Hubble constant.

This result is very hard to explain theoretically. Indeed a naive theoretical estimation of the energy density of the vacuum includes the sum of zero-point energies of all physical quantum fields, which must be calculated up to certain high-energy cutoff $k_{\max}$

$$\varrho \approx \frac{\hbar}{c} \frac{k_{\max}^4}{16\pi^2}. \tag{38}$$

If one takes as the cutoff the Planck energy $E_{\rm Pl} = m_{\rm Pl}c^2$ with $m_{\rm Pl} = \sqrt{\hbar c/G}$ then $k_{\max} \approx E_{\rm Pl}/\hbar c$ and the theoretical value of the vacuum density will be

$$\varrho_v \approx \varrho_{\rm Pl} = \frac{c^5}{G^2\hbar} = 5.46 \times 10^{93} \text{ g cm}^{-3}. \tag{39}$$

Hence the theoretical expectation for the cosmological constant exceeds the observed value by 123 orders of magnitude.

Weinberg (1989) considered various possible solutions of this problem based on different approaches: all these approaches show that the cosmological constant problem has a great impact on other areas of physics and astronomy. Weinberg notes: 'More discouraging than any theorem is the fact that many theorists have tried to invent adjustment mechanisms to cancel the cosmological constant, but without any success so far.'

Another problem connected with a non-zero cosmological constant or cosmological scalar field ('*quintessence*') was the puzzle of the continuous creation of the corresponding gravitating mass in Friedmann's equation (15) mentioned above. Indeed, the density of the vacuum does not change with time, hence its mass within a comoving radius $r(t)$ grows with time as $r^3(t)$. In the case of quintessence the dependence on time is defined by the product $\varrho_v(t)r^3(t)$ as follows from Eq. (32).

Finally, the observed approximate equality of matter and vacuum densities at the present epoch leads to a puzzling 'fine tuning' or coincidence: the density of ordinary matter rapidly decreases as the universe expands but the density of the vacuum is fixed, so why, despite these opposite behaviours, do the two densities have nearly the same value today?

5 COSMOLOGICAL PRINCIPLE

One of the fundamental elements of modern cosmology is the cosmological principle (CP) and it is very important to understand its different formulations and applications. Sometimes misleading claims appear in the literature about the CP, especially when a fractal matter distribution is discussed.

5.1 Einstein's Cosmological Principle

In the section devoted to the SM we already mentioned a formulation of Einstein's CP, which states that the universe is homogeneous (constant density) and isotropic (the same in all directions). In modern cosmological textbooks there is also another weaker formulation of the CP: the universe has no centre and is isotropic at any place, or that humans are not privileged observers.

There is a widely spread opinion that from isotropy and the absence of a preferred centre one may deduce the homogeneity of the universe (see e.g. Peacock, 1999, p. 65). Strictly speaking this inference is true only for a continuous matter distribution and is not true for discrete sets (e.g. fractals).

5.2 Fractality of the Observed Galaxy Distribution

For a long time, astronomers used only photographic plates of the sky as the basic means for galaxy structure studies with no direct observations of the three-dimensional large-scale matter distribution.

Recently, several three-dimensional maps of galaxy distribution have become available, based on massive redshift measurements. Surveys such as CfA, SSRS, Perseus–Pisces, IRAS, LEDA, APM–Stromlo, Las Campanas, and ESP for galaxies, and Abell and ACO for galaxy clusters have detected remarkable structures such as filaments, sheets and voids. The galaxy maps now probe scales up to $200h_{60}$ Mpc and they show that large-scale structures are common features of the local universe.

Pietronero and collaborators (see Pietronero (1987) and review by Sylos Labini *et al.* (1998) for a comprehensive discussion of the subject), by using the methods of modern statistical physics, have shown that, in the various surveys, the galaxy distribution exhibits fractal behaviour with dimension $D \approx 2$ at least up to 200 Mpc and the size of the upper cutoff, if it exists, must be more than 200 Mpc (see the web page devoted to the debate on fractality of galaxy distribution http://pil.phys.uniroma1.it).

It is important to note that according to recent $N(r)$ count–distance analysis of the complete sample of KLUN spiral galaxies by Teerikorpi *et al.* (1998), it was shown that the number of galaxies increases as $r^{(2.2\pm0.2)}$ up to distance $r \approx 200$ Mpc. This result solves the old controversy between the observed local inhomogeneous galaxy distribution and the $N(m)$ count-magnitude relation with the $0.6m$ law. Now the direct $N(r)$ count–distance relation is in accordance with a fractal galaxy distribution up to 200 Mpc.

5.3 Mandelbrot's Cosmological Principle

The homogeneity of visible matter up to a hundred Mpc is now disproved by direct observations of the spatial galaxy distribution. But is the cosmological principle true? From Einstein's CP of homogeneity and isotropy it follows that the Universe is the same in every place and in every direction. However it is possible to formulate a more general CP which possess these properties in an inhomogeneous discrete matter distribution.

This is the Mandelbrot cosmological principle, which states that in a statistical sense an inhomogeneous fractal matter distribution in the Universe is isotropic around any structure point and has no centre (Mandelbrot, 1977; 1982). In the fractal Universe the density of matter depends on the scale of statistical averaging and may even be zero for infinite distances. So the fractal Universe is not 'An unprincipled Universe' as claimed by Coles (1998), but is simply a Universe obeying a more general cosmological principle.

Isotropy of a fractal distribution means that the usual arguments for homogeneity based on observed isotropy are not generally valid. The only convincing test of fractality is the direct study of the space galaxy distribution by measuring redshifts for huge numbers of galaxies. Projects such as 2dF and Sloan will soon show the true nature of the visible matter distribution up to scales approaching the Hubble radius.

5.4 Einstein–Mandelbrot Cosmological Principle

There is some astrophysical evidence for possible homogeneity at very large scales close to the Hubble radius. For example from the isotropy of CMBR it follows that at scales of about several thousands Mpc electromagnetic radiation fills the Universe homogeneously, because photons cannot cluster as usual matter.

If the fractal distribution of ordinary matter extends up to scales where the density of radiation dominates, then one has a Universe which is essentially fractal inside the Hubble radius and which is essentially homogeneous outside the Hubble radius. For this case one may say that the Einstein–Mandelbrot cosmological principle of no centre and statistical isotropy is valid at all scales.

6 SANDAGE'S 21st PROBLEM: THE HUBBLE–DE VAUCOULEURS PARADOX

The discovery of a fractal galaxy distribution within scales of about 200 Mpc has created a new puzzle in cosmology. Indeed, the SM assumption of homogeneity 'leads to the prediction of Hubble's law – that the apparent recession velocity of a galaxy is proportional to its distance – for that is the only expansion law allowed by homogeneity' (Peebles, 1993, p. 5). Consequently, without direct information about the real spatial distribution of matter in the Universe, it was usually claimed that from the linear Hubble law it follows that the Universe is homogeneous just from the scales where the linearity of the Hubble law was found.

In an important earlier paper, Sandage *et al.* (1972) were the first to note the surprising coexistance of the linear Hubble law and local inhomogeneites. Actually they used the observed linearity of the Hubble law at small distances as a strong argument against de Vaucouleurs' hierarchical universe. Later in 1995 in the list of 'Astronomical Problems for the Next Three Decades' Sandage devoted the 21st problem to this subject in the form of the question: 'Are there significant velocity deviations from the pure cosmological expansion?'

6.1 Statement of the HdeV Paradox

According to modern observations based on Cepheid distances to local galaxies, Tully–Fisher distances from the KLUN program, and Supemovae Ia distances (see Teerikorpi, 1997; Ekholm *et al.*, 1999) the linear Hubble law is well established starting from scales of about 1 Mpc.

But, as we have already mentioned, studies of the three-dimensional galaxy universe have shown that de Vaucouleurs' prescient view on matter distribution (de Vaucouleurs, 1970) is valid at least in the range of scales $\sim$ 1–200 Mpc (Sylos Labini *et al.*, 1998).

The Hubble and de Vaucouleurs laws describe very different aspects of the Universe, but both have in common universality and observer independence. This makes them fundamental cosmological laws and it is important to investigate the consequences of their coexistence at the same length scales (see Baryshev, *et al.* 1998).

A puzzling conclusion is that the strictly linear redshift–distance relation is observed deep inside the fractal structure, i.e. for distances less than the homogeneity scale r_{hom}:

$$(r < r_{\mathrm{hom}}) \ \& \ (cz = H_0 r). \tag{40}$$

This empirical fact presents a profound challenge to the standard model in which homogeneity is the basic explanation of the Hubble law, and 'the connection between homogeneity and Hubble's law was the first success of the expanding world model' (Peebles *et al.*, 1991).

In fact, within the SM one would not expect any neat relation of proportionality between velocity and distance for nearby galaxies, which are members of large-scale structures. However, contrary to expectation, modern data show a good linear Hubble law even for nearby galaxies. This leads to a new observationaly established puzzling fact that the linear Hubble law is not a consequence of a homogeneity of visible matter, just because visible matter is distributed inhomogeneously.

6.2 Possible Solutions of the HdeV Paradox

Up to now several possible solutions of the HdeV paradox have been suggested. The first one (Baryshev *et al.*, 1998; Durrer and Sylos Labini, 1998) is based on the assumption of the existence of uniformly distributed dark matter starting just from the halos of galaxies; in this case the standard FLRW solution exists. However, then the fractal distribution of luminous matter (galaxies) can appear only from a special choice of initial conditions and hence has no fundamental meaning.

The second solution is to accept a very low value for the global average density (Baryshev *et al.*, 1998; Humphreys *et al.* 1998; Gromov *et al.*, 1999). However in this case when the value of the upper cutoff scale of the fractal structure is large, the low density contradicts the available estimates of the density of the baryonic luminous and dark matter.

Other solutions of the HdeV paradox are based on cosmological models more general than FLRW. For instance Lemaitre–Tolman–Bondi (LTB) models are exact non-linear solutions of Einstein's equations under the assumptions of spherical symmetry, pressureless matter and no spherical layers intersecting. In the frame of LTB cosmological models non-simultaneous bang time (Gromov *et al.* 1999) and the Λ-term (Baryshev, *et al.*, 1999) allow the linear Hubble law to be compatible with a fractal structure having an upper cutoff.

A very different possibility of solving the HdeV paradox comes from the recent discovery by de Vega and Sanchez (1999), that self-gravitating (via Newtonian gravity) N-body systems have a quasi-equilibrium state which is fractal in its structure with a fractal dimension of about 2. So, self-gravity naturally leads to fractality and the actual problem is how to explain the appearance of the Hubble law inside this structure. As shown by Baryshev (1981) (see also Baryshev *et al.*, 1994; 1998) the cosmological gravitational redshift effect gives the linear redshift–distance relation just for a fractal structure with $D = 2$, which is actually observed at least up to scales about 200 Mpc. For such a model the main problem is a high value of dark matter coupled with fractal visible matter needed for an explanation of the observed value of the Hubble constant.

7 QUANTUM NATURE OF THE GRAVITY FORCE

The roots of many of the conceptual problems of modern cosmology discussed above actually lie in gravity theory. In fact, all fundamental forces in physics (strong, weak, electromagnetic) are quantum in nature, (i.e. there are quanta of corresponding fields which carry the energy–momentum of physical interactions), while GR presents the geometrical interpretation of the gravity force (i.e. the curvature of space itself but not as matter in space) which, as is well known, excludes the concept of localizable gravity energy. This is why the main problem of GR is the absence of the energy of the gravity field or pseudo-tensor character of gravity EMT (see Landau and Lifshitz, 1971; and for a recent attempt to construct gravity EMT see Babak and Grishchuk, 1999). Together with GR the energy problem comes to cosmology and is the cause of some conceptual problems of SM.

The quantum field approach to the gravity force was considered by Feynman (1971) in his *Lectures on Gravitation*. Within the field approach, gravity is a kind of matter, i.e. a tensor field in Minkowski space, and this means that its quanta–gravitons carry the energy–momentum of the gravitational interaction. As Feynman emphasized 'the geometrical interpretation is not really necessary or essential to physics' (Lecture 8, p. 110) and a field gravity theory (FGT) may be constructed with the usual field-theoretical techniques. This means that Minkowski space allows one to define the EMT of the gravity field and conservation laws without 'pseudo'-problems, and also utilize the usual quantum mechanics and quantum field theory. The main advantage of the field gravity theory is that it gives a positive and localizable energy density of gravitational field which allows one to get gravitons as the energy quanta of the field.

In the case of the weak field approximation both theories give the same predictions for classical relativistic gravity effects. But in the scope of FGT there are also new relativistic effects even in weak fields and profoundly different predictions in the case of strong gravity fields. For instance, it can be shown, that within FGT the positive energy density of the gravity field excludes the possibility of black holes and in cosmology there is an expansion of matter in space but there is no expansion of space. So the observed cosmological redshift may be related to the Doppler effect and the cosmological gravitational redshift. A general discussion of the geometrical and field approaches to gravity may be found in Baryshev (1996).

The modern state of gravity theory and experiments was analyzed by Damour (1999), who emphasized that existing tests of GR does not exclude a more general quantum gravity theory which may have very different predictions for strong field effects. In particular, the possible existence and observational tests for a scalar gravitational field have been discussed by Damour (1999) and Baryshev (1995; 1996; 1997). The problem of non-zero mass for the graviton was analyzed by Visser (1998), who showed that in this case the strong field effects and cosmological solutions will differ dramatically from GR.

It is important to note that study of the scalar part of the gravitational field and the mass of the graviton is not an 'academic' problem but has practical importance, because the corresponding theories wil be experimentally tested in the near future by using gravitational wave observatories (such as LIGO and VIRGO) which will start to operate in two years.

8 CONCLUSIONS

Two major building blocks of modern cosmological models are the cosmological principle and the theory of gravitation. Correspondingly the main conceptual problems of cosmology are related to studies of large-scale matter distribution and the physics of the gravitational interaction. There are fundamental problems in cosmology which are still unsolved and have not yet even been analyzed, so the opinion that 'cosmology is solved' is a dream far from reality. Moreover deep conceptual puzzles which we have discussed above actually leave no room for 'cosmologists' arrogance' (see Turner, 1999b) with the existing standard model. The main conclusions of this report are the following:

- The time for fractal cosmology is coming, so the old cosmological principle of homogeneity must be replaced by the new more general cosmological principle of fractality. The new cosmological principle is fully compatible with the reasonable requirements of the equivalence of all observers and the condition of local isotropy around any structure point. The case of an Einstein–Mandelbrot Universe where the essential fractal matter distribution at small and intermediate scales becomes homogeneous at very large scales is a particular model of fractal cosmology.

- The paradox of a linear Hubble law within the fractal visible matter distribution implies a high value of homogeneous dark matter, or a very low value of the asymptotic FLRW background, or application of more general cosmological models such as the Lemaitre–Tolman–Bondi model.
- Modern relativistic quantum field theory shows that future gravity theory will be more general than general relativity. Within the framework of the quantum field approach to gravity there are unexplored possibilities such as the scalar part of the gravity field and the non-zero rest mass of the graviton.
- Crucial future observational tests are needed to make a distinction between rival cosmological models. Among them: the fractal dimension and maximum scale of fractality of the spatial galaxy distribution (2dF, SLOAN); the detection of gravitational waves (LIGO, VIRGO); physical properties of high-redshift galaxies, radio galaxies and quasars (HDFS).

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