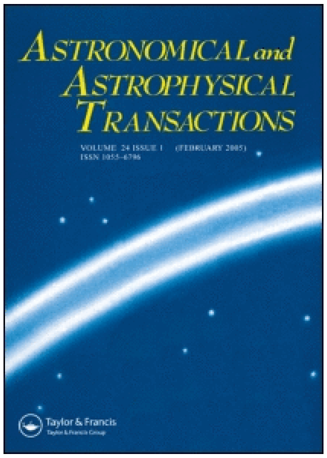

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 January 2000
To cite this Article: Labini, F. Sylos (2000) 'Scale invariance of galaxy clustering', Astronomical & Astrophysical Transactions, 19:3, 397 - 411
To link to this article: DOI: 10.1080/10556790008238586
URL: http://dx.doi.org/10.1080/10556790008238586

Astronomical and Astrophysical Transactions, 2000, Vol. 19, pp. 397–411
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SCALE INVARIANCE OF GALAXY CLUSTERING

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(Received January 28, 2000)

The fact that galactic distribution exhibits fractal properties has been well established for 20 years. Nowadays, the controversy concerns the range of the fractal regime, the value of the fractal dimension and the eventual presence of a cross-over to homogeneity. In the debate about galaxy correlation there are different questions which can be addressed separately: Which statistical methods are able to properly detect scale invariance and describe, in general, the properties of irregular and regular distributions? What are the implications for cosmology of the fractal behaviour of galactic structures, up to a certain scale λ_0? What is the homogeneity scale λ_0, i.e. the scale beyond which the galaxy distribution has an eventual cross-over to homogeneity? These are three different, but related, problems, which must be considered in different steps, from the point of view of data analysis as well as from the theoretical perspective.

KEY WORDS Distribution of galaxies, fractal structures, homogeneity scale

1 INTRODUCTION

The assumption of homogeneity in the distribution of matter lies at the heart of the Big Bang cosmology. The nature of the evidence, if any, for this assumption has, however, been the subject of very considerable controversy (Davis, 1997; Pietronero *et al.*, 1997). A central point made by Pietronero (1987) has been that the standard methods of analysis of galaxy red-shift catalogues, which provide the most direct probe of the (luminous) matter distribution, actually assume homogeneity implicitly. In this report we review the main points of this controversy: we discuss the different methods of analysis of redshift samples, and the corresponding results. The basic point we try to clarify is: what do we learn from the redshift surveys? We show that complementary to the adoption of a new method of analysis there are important theoretical implications for the usual scenario of galaxy formation.

2 THE PROBLEM OF LARGE-SCALE STRUCTURE DISTRIBUTION

Nowadays there is general agreement about the fact that galactic structures are fractal up to a distance scale of $\sim$ 30–40h^{-1} Mpc (Sylos Labini *et al.*, 1998; Joyce *et al.*, 1999) and the increasing interest about the fractal versus homogeneous distribution of galaxy in the last year has focused mainly on the determination of the homogeneity scale λ_0.* Instead, we would like to discuss three important and different aspects of this problem which, we believe, have not been considered appropriately in the debate. The main point we would like to stress is that galaxy structures are fractal no matter what the cross-over scale, and this fact has never been properly appreciated.

Methodological Point

The major problem from the point of view of data analysis is to use statistical methods which are able to properly characterize scale invariant distributions, and hence which are also suitable to characterize an eventual cross-over to homogeneity. Our main contribution (Pietronero, 1987; Coleman and Pietronero, 1992; Sylos Labini *et al.*, 1998) in this respect has been to clarify that the usual statistical methods (correlation function, power spectrum, etc.) are based on the assumption of homogeneity and hence are not appropriate to test it. Instead, we have introduced and developed various statistical tools which are able to test whether a distribution is homogeneous or fractal, and to correctly characterize the scale-invariant properties. Such a discussion is clearly relevant also for the interpretation of the properties of artificial simulations. The agreement about the methods to be used for the analysis of future surveys such as the Sloan Digital Sky Survey (SDSS) and the Two Degrees Fields (2dF) is clearly a fundamental issue.

Implication of the Fractal Structure up to Scale λ_0

The fact that galactic structures are fractal, no matter what the homogeneity scale λ_0, has deep implications for the interpretation of several phenomena such as the luminosity bias, the galaxy-cluster mismatch, the determination of the average density, the separation of linear and non-linear scales, etc., and on the theoretical concepts used to study such properties. For example the properties of dark matter are inferred from those of visible matter, and hence they are closely related. If one now observes different statistical properties for galaxies and clusters, this necessarily implies a change of perspective on the properties of dark matter.

*See the web page http://pil.phys.uniroma1.it/debate.html where all these materials have been collected (Teerikorpi *et al.*, 1998; Coles, 1998; Scaramella *et al.*, 1998; Wu *et al.*, 1999; Cappi *et al.*, 1998; Martinez, 1999; Hatton, 1999; Chown, 1999; Landy, 1999; Joyce *et al.*, 1999).

Determination of the Homogeneity Scale λ_0

This is clearly a very important point which is at the basis of the understanding of galaxy structures and more generally of the cosmological problem. We distinguish here two different approaches: direct tests and indirect tests. By direct tests, we mean the determination of the conditional average density in three-dimensional surveys, while with indirect tests we refer to other possible analyses, such as the interpretation of angular surveys, the number counts as a function of magnitude or of distance or, in general, the study of non-average quantities, i.e. when the fractal dimension is estimated without making an average over different observers (or volumes). While in the first case one is able to have a clear and unambiguous answer from the data, in the second one is only able to make some weaker claims about the compatibility of the data with a fractal or a homogeneous distribution. However, also in this second case, it is possible to understand some important properties of the data, and to clarify the role and the limits of some underlying assumptions which are often used without critical perspective.

3 THE HOMOGENEITY SCALE

The proper methods to characterize irregular as well as regular distributions have been discussed by Coleman and Pietronero (1992) and Sylos Labini *et al.* (1998) in a detailed and exhaustive way. The basic point is that, so long as a system shows power law correlations, the usual $\xi(r)$ analysis (Peebles, 1980) gives an incorrect result, since it is based on the *a priori* assumption of homogeneity. In order to check whether homogeneity is present in a given sample one has to use the conditional density $\Gamma(r)$ defined as (Pietronero, 1987)

$$\Gamma(r) = \frac{\langle n(r_*) n(r_* + r)\rangle}{\langle n \rangle} = \frac{BD}{4\pi} r^{D-3}, \tag{1}$$

where the last equality holds in the case of a fractal distribution with dimension D and prefactor B. In the case of a homogenous distribution ($D = 3$) the conditional density equals the average density in the sample. Hence the conditional density is a suitable statistical tool to identify fractal properties (i.e. power law correlations with codimension $\gamma = 3 - D$) as well as homogeneous ones (constant density with sample size). If there exists a transition scale λ_0 towards homogenization, we should find $\Gamma(r)$ constant for scales $r \gg \lambda_0$.

Basically λ_0 is related to the maximum size of voids: the average density will be constant, at least on scales larger than the maximum void in a given sample. Several authors have approached this problem by looking at void distributions. For example El-Ad and Piran (1997) have shown that the SSRS2 and IRAS 1.2 Jy redshift surveys are dominated by voids: they cover $\sim 50\%$ of the volume. Moreover the two samples show very similar properties even if the IRAS voids are $\sim 33\%$ larger than the SSRS2 ones because they are not bounded by narrow angular limits as the

SSRS2 voids. The voids have a scale of at least $\sim$ 40–50h^{-1} Mpc and the largest void in the SSRS2 sample has a diameter of $\sim 60h^{-1}$ Mpc, i.e. comparable to the Bootes void. The problem is to understand whether such a scale has been fixed by the samples' volume, or whether there is a tendency not to find larger voids: in this case one would have (weaker) evidence for the homogeneity scale. In any case, we note that the homogeneity scale cannot be smaller than the scale of the largest void found in these samples and that one has to be very careful when comparing the size of the voids to the effective depth of catalogues. For example in the Las Campanas Redshift Survey, even if it is possible to extract sub-samples limited at $\sim 500h^{-1}$ Mpc, the volume of space investigated is not so large, as the survey is made by thin slices. In such a situation a definitive answer to the dimension of the voids, and hence to the existence of the homogeneity scale, is rather difficult and uncertain.

Another complementary way to study the eventual cross-over to homogeneity of galaxy distribution is represented by the morphological signatures identified by tools such as the Minkowski functionals. Kerscher *et al.* (1998), by analysing the IRAS samples, have found that there are large fluctuations in the clustering properties as seen in a large difference between the northern and southern parts of the catalogue on scales of $\sim 100h^{-1}$ Mpc. These fluctuations remain discernible even on the scale of $200h^{-1}$ Mpc and this is again a sign of the inhomogeneous character of galaxy structures at these scales. There are several other approaches to this problem, but we believe that the analysis via the conditional average density is the more stable and powerful to understand the correlation and statistical properties of a given sample of galaxies.

4 THE STANDARD STATISTICAL METHODS

The problems of the standard analysis can easily be seen from the fact that for the case of a fractal distribution the standard 'correlation function' $\xi(r)$ in a spherical sample of radius R_s, is given by

$$\xi(r) = \frac{3-\gamma}{3}\left(\frac{r}{R_s}\right)^{-\gamma} - 1. \tag{2}$$

Hence for a fractal structure the 'correlation length' r_0 (defined by $\xi(r_0) = 1$) is not a scale characterizing any intrinsic property of the distribution, but just a scale related to the size of the sample. If, on the other hand, the distribution is fractal up to some scale λ_0 and homogeneous beyond this scale, it is simple to show that (if $\lambda_0 < R_s$, i.e. the cross-over to homogeneity is well inside the sample size)

$$r_0 = \lambda_0 2^{-1/\gamma}. \tag{3}$$

The correlation length does in this case have a real physical meaning (when measured in samples larger than λ_0), being related in a simple way to the scale characterizing homogeneity. In the case $D = 2$ we have $r_0 = \lambda_0/2$[†]. Finally it should be noticed that $\xi(r)$ is a power law only for

$$\left(\frac{3-\gamma}{3}\right)\left(\frac{r}{R_s}\right)^{-\gamma} \gg 1 \tag{4}$$

hence for $r \ll r_0$: for larger distances there is a clear deviation from the power law behaviour due to the definition of $\xi(r)$. This deviation, however, is just due to the size of the observational sample and does not correspond to any real change of the correlation properties. It is clear that if one estimates the $\xi(r)$ exponent at distances $r \lesssim r_0$, one systematically obtains a higher value of the correlation exponent due to the break of $\xi(r)$ in the log–log plot. In this respect it is useful to compute the log derivative of Eq. (2) with respect to $\log(r)$:

$$\gamma' = \frac{\mathrm{d}(\log(\xi(r))}{\mathrm{d}\log(r)} = -\frac{2\gamma r_0^{\gamma} r^{-\gamma}}{2r_0^{\gamma} r^{-\gamma} - 1}, \tag{5}$$

where r_0 is defined by $\xi(r_0) = 1$. The tangent to $\xi(r)$ at $r = r_0$ has a slope $\gamma' = -2\gamma$. It is clear that even if the distribution has fractal properties, it is very difficult to recover the correct slope from the study of the $\xi(r)$ function. The $\xi(r)$ is intrinsically problematic to this end.

In Figure 1 we show the results of the analysis of all the available galaxy samples through the conditional density (Sylos Labini *et al.*, 1998; Joyce *et al.*, 1999a, b), while in Figure 2 we show the behaviour of the standard $\xi(r)$ in the same catalogues. One may note that the different data are in rather good agreement when analysed by $\Gamma(r)$ and give complex information when seen from the perspective of $\xi(r)$. As we discuss below, this complex situation has given rise to some confused concepts such as the luminosity bias or galaxy-cluster mismatch.

5 OTHERS 'CHARACTERISTIC' LENGTH SCALES

The usual analysis finds that rms fluctuations of the observed galaxy density field are very large on small scales, of the order of unity within spheres of $8h^{-1}$ Mpc dropping as a power law as a function of scale, becoming a few percent at several tens h^{-1} Mpc. In this perspective it therefore makes sense to refer the density field of galaxies to its mean. Let $\rho(r)$ be the observed galaxy density field; the density fluctuation field is defined as

$$\delta(r) = \frac{\rho(r) - \langle\rho\rangle}{\langle\rho\rangle}. \tag{6}$$

This quantity can be measured in redshift samples. The problem in this case is the same one which enters in the definition of r_0: one is comparing the amplitude

[†]This calculation assumes a simple matching of a fractal onto a pure homogeneous distribution. For any particular model with fluctuations away from perfect homogeneity, the numerical factor will differ slightly depending on how precisely we define the scale λ_0.

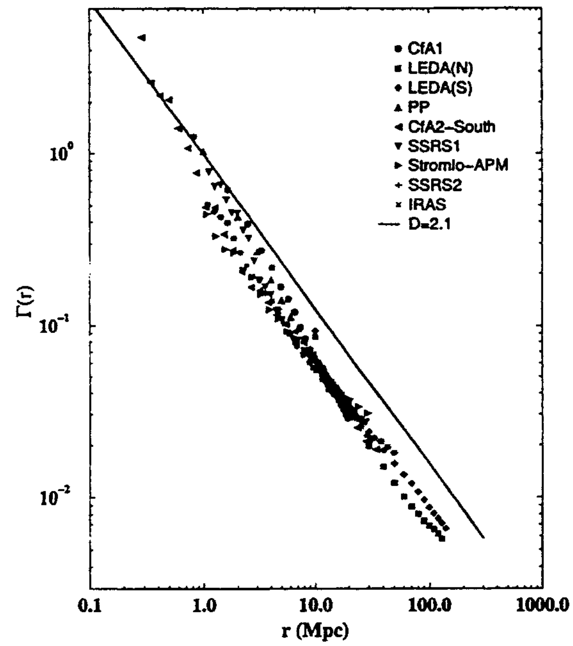

Figure 1 Conditional average density computed for various different galaxy surveys (from Sylos Labini *et al.*, 1998). The power law behaviour corresponds to a fractal structure with dimension $D \approx 2$.

of fluctuations to the mean density. As an example, one can consider a portion of a fractal structure of size R_s and study the behaviour of $\delta N/N$. The average density is just given by Eq. (1) while the overdensity δN, as a function of the size r ($r \leq R_s$) of a given structure is:

$$\delta N = \frac{N(r)}{V(r)} - \langle n \rangle = \frac{3}{4\pi} B(r^{-(3-D)} - R_s^{-(3-D)}). \tag{7}$$

We have therefore

$$\frac{\delta N}{N} = \left(\frac{r}{R_s}\right)^{-(3-D)} - 1. \tag{8}$$

Clearly for structures that approach the size of the sample, the value of $\delta N/N$ becomes very small and eventually becomes zero at $r = R_s$.

Another typical length scale which is usually defined in the study of redshift samples is the scale at which the power spectrum (hereafter PS) of the *density fluctuations* has a turnover: $\mathrm{d}P(k(\lambda_f))/\,\mathrm{d}k = 0$. Essentially all the currently elaborated models of galaxy formation (e.g (Peebles, 1993) *assume large-scale homogeneity* and predict that the galaxy PS, which is *the PS of the density contrast*, decreases both toward small scales and toward large scales, with a turnaround somewhere in the

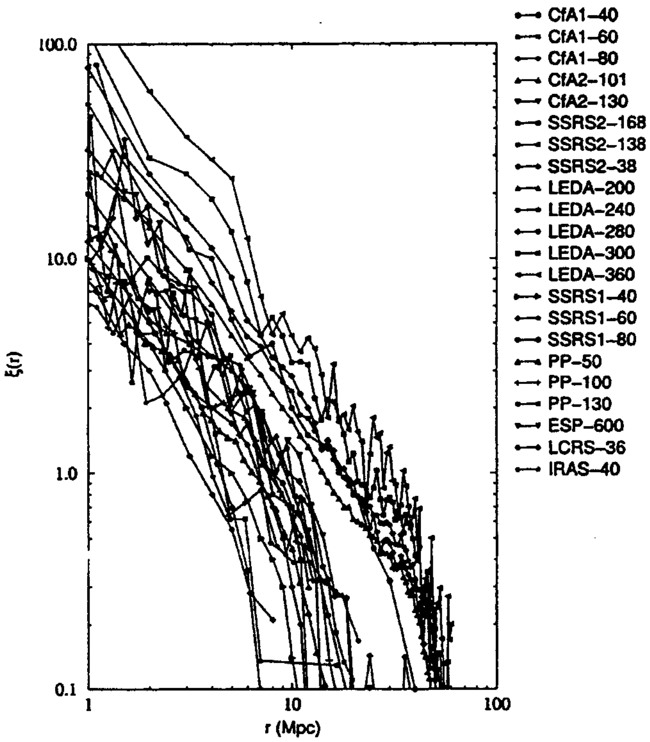

Figure 2 The standard correlation function $\xi(r)$, computed for the same galaxy samples of Figure 1 (from Sylos Labini *et al.*, 1998)

middle, at a scale λ_f that can be taken as separating 'small' from 'large' scales. Because of the homogeneity assumption, the PS amplitude should be independent of the survey scale, any residual variation being attributed to luminosity bias (or to the fact that the survey scale has not yet reached the homogeneity scale). However, the crucial clue to this picture, the firm determination of the scale λ_f is still missing, although some surveys do indeed produce a turnaround scale around $100h^{-1}$ Mpc. Recently, the CfA2 survey analysed by Park *et al.* (1994) (PVGH), showed a $n = -2$ slope up to $\sim 30h^{-1}$ Mpc, a milder $n \approx -1$ slope up to $200h^{-1}$ Mpc, and some tentative indication of flattening on even larger scales. PVGH also find that deeper subsamples have higher power amplitude, i.e. that the amplitude scales with the sample depth.

It is simple to show (Sylos Labini *et al.*, 1998) that both features, bending and scaling, are a manifestation of the finiteness of the survey volume, and that they cannot be interpreted as convergence to homogeneity, nor to PS flattening. The systematic effect of the survey's finite size is in fact to suppress power at large scales, mimicking a real flattening. In fact we have shown that even a fractal distribution of matter, which never reaches homogeneity, shows a sharp flattening and then a turnaround. In particular, it is possible to show that in a spherical sample of radius R_s, which contains a portion of a fractal structure with dimension $D = 2$, the PS turnover scale is given by

$$\lambda_f \approx 1.45 R_s \tag{9}$$

and hence it is another quantity related to the sample size rather than being an intrinsic characteristic scale of galaxy distribution.

6 BIAS AND DARK MATTER

We have discussed the concept of correlation and bias, as it is usually defined in the literature, in a series of papers (Gabrielli *et al.*, 1999; Gabrielli and Sylos Labini, 1999). We review here the main points of this discussion. The concept of bias, i.e. the relative abundance and distribution of objects of different mass, was originally introduced by Kaiser (1984) to explain the different amplitudes of the correlation function $\xi(r)$ found for galaxies and galaxy clusters. Afterwards it has also been invoked to explain the increasing amplitudes of $\xi(r)$ for galaxies with brighter luminosity. Finally, it is used to describe the 'clustering' of dark matter relative to that of visible matter. In general it is believed that objects of different mass have different clustering properties, i.e. 'correlation lengths', the latter increasing with the object's mass: the highest peaks of the density field are more 'strongly clustered' than the density field itself. We have shown (Gabrielli and Sylos Labini, 1999; Gabrielli *et al.*, 1999) that in the general case of distributions with a well-defined average density, the value at fixed r of $\xi(r)$ is only related to the amplitude of the local fluctuation with respect to the average density (Gaite *et al.*, 1999; Gabrielli and Sylos Labini, 1999; Gabrielli *et al.*, 1999) and it does not give any information of the spatial extension of structures in the system. Let us see this point in more detail.

The simplest assumption to describe the distribution of mass in the universe is that distribution of galaxies is a good tracer of the distribution of dark matter. A specific model has been suggested by Kaiser (1984) in which galaxies and galaxy clusters represent different high-density peaks of the mass density field. Then the term biasing has been used to refer to a number of different but related effects (Strauss and Willick, 1995). The so-called peak biasing model originally proposed by Kaiser (1984) makes a definitive prediction of the relation between the correlation function of galaxies of different masses, galaxy clusters (which we generally call *objects*) and dark matter (dm), at least at large scale:

$$\xi_{\mathrm{obj}}(r) = b_{\mathrm{obj}}^2 \xi_{\mathrm{dm}}(r), \tag{10}$$

b_{obj} being the corresponding bias parameter, and $\xi_{\mathrm{dm}}(r)$ is the correlation function of 'dark matter', i.e. of the underlying density field. Rather than being one bias parameter for the correlations of galaxies, there is an undetermined number of such parameters. The bias parameter b_{obj} for each class of objects is now one of the fundamental parameters included both in the theoretical model, and in the interpretation of galaxy correlation. For instance, for what concerns the clustering of galaxies of different luminosity (mass) (Park *et al.*, 1994; Benoist *et al.*, 1996) the biasing is usually referred to as *luminosity bias*, while for the case of galaxy clusters it has been introduced in the *clustering-richness relation* (Bahcall and Soneira, 1983). Moreover the 'bias parameter' plays a crucial role in the interpretation of the peculiar velocities of galaxies and clusters as well as of the anisotropies of the CMBR (Strauss and Willick, 1995).

The incorrect definition of 'correlation length' used in cosmology (Peebles, 1980) is not just a question of semantics (Gaite *et al.*, 1999), but it has generated confusion even when the average density of the system is a well-defined property, especially for what concerns the concept of bias (Gabrielli *et al.*, 1999; Gabrielli and Sylos Labini, 1999). For instance, we have shown (Gabrielli and Sylos Labini, 1999) that Eq. (10) increases the amplitude of $\xi(r)$ and hence the amplitude of the fluctuations with respect to the average density, but the typical dimension of the structures of fluctuations remains the same. In order to illustrate more clearly this point, let us recall briefly the concept of correlation (see Gabrielli and Sylos Labini (1999) for a more detailed discussion). If the presence of an object at the point r_1 influences the probability of finding another object at r_2, these two points are correlated. Hence there is a correlation at the scale distance r if

$$G(r) = \langle n(\mathbf{0})n(\mathbf{r})\rangle \neq \langle n\rangle^2, \tag{11}$$

where we average over all occupied points of the system chosen as origin and on the total solid angle supposing statistical isotropy. On the other hand, there is no correlation if

$$G(r) = \langle n\rangle^2. \tag{12}$$

The proper definition of λ_0, the *homogeneity scale*, is the length scale beyond which the average density becomes well-defined, i.e. there is a cross-over towards homogeneity with a flattening of $G(r)$. The length-scale λ_0 represents the typical dimension of the voids in the system. On the other hand, the *correlation length* r_c. separates correlated regimes of the fluctuations with respect to the average density from uncorrelated ones, and it can be defined only if a cross-over towards homogeneity is shown by the system, i.e. λ_0 exists (Gaite *et al.*, 1999). In other words r_c defines the organization in geometrical structures of the fluctuations with respect to the average density. Clearly $r_c > \lambda_0$: only if the average density can be defined may one study the correlation length of the fluctuations from it. In the case in which λ_0 is finite and then $\langle n\rangle > 0$, in order to study the correlations properties of the fluctuations around the average and then the behaviour of r_c, can we introduce the correlation function $\xi(r)$.

We note that if $\lambda_0 \ll R_s$, λ_0 has nothing to do with questions like 'what is the typical size of structures in the system?' or 'up to which length-scale is the system clusterised?' (Gaite *et al.*, 1999). The answer to this question is strictly related to r_c and not to λ_0. The length scale r_c characterizes the distance over which two different points are correlated (clusterized). In fact, this property is related not to how large the fluctuations are with respect to the average (λ_0), but to the length extension of their correlations (r_c).

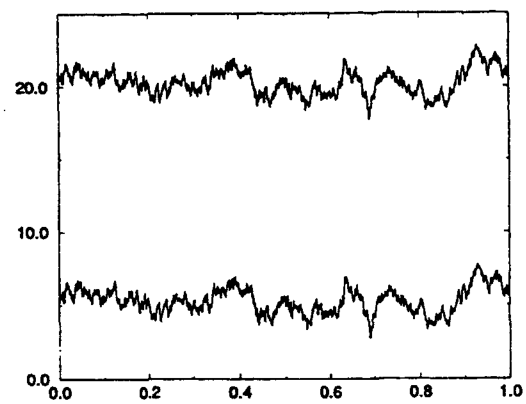

Figure 3 Guassian fluctuations with correlation up to a scale $r_c \approx 0.1$ in the density field superimposed on a uniform background. The background density (and hence the average density) is smaller for the lower density field than for the upper one, but the correlation length is the same for the two distributions. The amplitude of $\xi(r)$ at the same distance scale is clearly larger for the lower distribution than for upper one: this is because the amplitude of the fluctuations with respect to the average density is larger. The correlation length r_c is finite and it is related to the largest spatial extension of the fluctuation structures. Beyond r_c the distribution of the fluctuations from the average density is completely random.

To be more specific, let us consider a fixed set of density fluctuations. They can be superimposed to a different value of a uniform density background. The larger this background the lower λ_0, but obviously the length scale of the correlations (r_c) among these fluctuations is not changed, i.e. they are clusterized independently of the background (see Figure 3). The conclusion (Gabrielli and Sylos Labini, 1999) is that a linear amplification of $\xi(r)$

$$\xi'(r) = A\xi(r) \tag{13}$$

doesn't change r_c (which can be finite or infinite) but only λ_0, i.e. if $A > 1$ we need larger subsamples to have a good estimation of $\langle n \rangle$, but it doesn't change the characteristic length (correlation length) of the structures. For a more detailed discussion of the concept of bias we refer to Gabrielli *et al.* (1999) and Gabrielli and Sylos Labini (1999).

7 WHAT DO WE LEARN FROM GALAXY CATALOGUES?

As we have already mentioned the usual concept of bias arises from the interpretation of the results of the $\xi(r)$ analysis. The concept of bias fixes the relative distribution of galaxies of different types, clusters and dark matter. In general (Strauss and Willick, 1995) one assumes that there exists a direct proportionality between the density fluctuations of galaxies δ_g and dark matter δ_{DM}

$$\delta_g = b\delta_{DM} \tag{14}$$

and the same concept applies to galaxies of different masses and galaxy clusters. Under this assumption, the biasing parameter b is independent of location. One case uses the two-point correlation function $\xi_{gg}(r)$ and the mass autocorrelation function $\xi_{\rho\rho}(r)$ to define the bias factor

$$b = \left(\frac{\xi_{gg}(r)}{\xi_{\rho\rho}(r)}\right)^{1/2}. \tag{15}$$

Let us see in more detail the origin of the concept of bias as given in Eq. (15). It is a well-known observational fact that galaxies of different morphological types have different clustering properties. For example, the most luminous elliptical galaxies usually reside in the clusters cores, at local density maxima, and are not present in low-density fields, so that these objects seem to be the product of dense environments. There are various other morphological facts of this type (Sylos Labini *et al.*, 1998) which support the fact that brighter (more massive) galaxies are more *clustered* than for example spirals (less massive). The different clustering properties have been interpreted, through the $\xi(r)$ analysis, as a different *amplitude of correlation* for different galaxy types. In particular while for the general galaxy field the correlation length is $r_0 \approx 5h^{-1}$ Mpc, for the brighter galaxies ($L > L^*$) it has been found (Park *et al.*, 1994; Benoist *et al.*, 1996) that $r_0 \approx 16h^{-1}$ Mpc. This trend seems to be confirmed also by the cluster (more massive than galaxies) distribution for which $r_0 \approx 25h^{-1}$ Mpc (Bahcall and Soneira, 1983).

On the contrary from our interpretation there follows a number of important implications in this respect. As a cross-over to homogeneity has not been found, *all* the length scales found by the $\xi(r)$ analysis are artifacts of an inconsistent data analysis. The 'correlation lengths' $r_0 = 5, 16, 25, \ldots h^{-1}$ Mpc are not real physical characteristic scales, but just fractions of sample sizes. Brighter objects allow one to investigate a larger volume of space. Hence, for example the sample size of cluster catalogues is usually larger than that of galaxy samples. This simple observation explains why one obtains different correlations lengths, and in general why the correlation length seems to increase as a function of the luminosity of objects. To this qualitative observation, one may add a detailed study of the available galaxy and cluster samples. This has been done in a detailed way by our group and we refer to (Sylos Labini *et al.*, 1998) for an exhaustive explanation of this fact.

Therefore, contrary to the usual interpretation, we have shown that the segregation of giant galaxies in clusters arises as a consequence of self-similarity of matter distribution, and that in this case the only relevant parameter is the *exponent* of the correlation function, while the amplitude is a spurious quantity that has no direct physical meaning and depends explicitly on the sample size. Let us explain this important point in more detail.

In a well-known review on the galaxy luminosity function (LF) Binggeli *et al.* (1988) state that '*as the distribution of galaxies is known to be inhomogeneous on* *all scales up to a least* $100h^{-1}$ *Mpc, a rich cluster of galaxies is like a Matterhorn in a grand Alpine landscape of mountain ridges and valleys of length up to 100 Km'*. This point of view can be seen in the light of the concept of multifractality of the mass distribution. The main observational aspects of galaxy luminosity and space distributions are strongly related and can be naturally linked and explained as a multifractal (MF) distribution. The concept of MF is appropriate to discuss physical systems with local properties of self-similarity, in which the scaling properties are defined by a continuous distribution of exponents. Roughly speaking one can visualize this property as having different scaling properties for different regions of the system (see (Coleman and Pietronero, 1992; Sylos Labini and Pietronero, 1996) for a more detailed discussion). The fundamental point which is dismissed in the usual picture is that *the whole matter distribution, i. e. weighing each point by its mass, is self-similar.* This situation requires the generalization of the simple fractal scaling to a MF distribution in which a continuous set of exponents is necessary to describe the spatial scaling of peaks of different weight (mass or luminosity). *In this respect the mass and space distributions become naturally entangled with each other.*

The MF implies a strong correlation between spatial and mass distribution so that the number of objects with mass M at the point $\boldsymbol{r}$ per unit volume $\nu(M, \boldsymbol{r})$ is a function of space and mass and *it is not separable in a space density multiplied by a mass (or luminosity) function* (Binggeli *et al.*, 1988). This means that we *cannot* express the number of galaxies $\nu(M, x, y, z)$ lying in volume $\mathrm{d}V$ at (x, y, z) with mass between M and $M + \mathrm{d}M$ as the *product* of a space density and a luminosity function. This would assume that galaxy positions are not correlated with their luminosities, while the observations show just the opposite. Moreover we cannot define a well-defined average density, independent of sample depth as for the simple fractal case. It can be shown (Sylos Labini and Pietronero, 1996) that the mass function of a MF distribution, in a well-defined volume, has indeed a Press–Schechter behaviour.

The continuous set of exponents which describes a MF distribution can characterize completely the galaxy distribution when one considers the mass (or luminosity) of galaxies in the analysis. In this way much observational evidence is linked together and arises naturally from the self-similar properties of the distribution. Considering a MF distribution, the usual power-law space correlation properties correspond just to a single exponent of the MF spectrum: such an exponent simply describes the space distribution of the support of the MF measure. Furthermure the shape of the luminosity function (LF), i.e. the probability of finding a galaxy of a certain luminosity per unit volume, is related to the MF spectrum of exponents. We have shown that, under MF conditions, the LF is well approximated by a power law function with an exponential tail. Such a function corresponds to the Schechter LF observed in real galaxy catalogues. In this case the shape of the LF is almost independent of the sample size, but the amplitude of the LF depends on the sample size as a power law function.

These results have important consequences from a theoretical point of view. In fact, when one deals with self-similar structures the relevant physical phenomenon which leads to scale-invariant structures is determined by the *exponent* and *not the amplitude* of the physical quantities which characterize such distributions.

The geometric self-similarity has deep implications for the non-analyticity of these structures. In fact, analyticity or regularity would imply that at some small scale the profile becomes smooth and one can define a unique tangent. Clearly this is impossible in a self-similar structure because at any small scale a new structure appears and the distribution is never smooth. Self-similar structures are therefore intrinsically irregular at all scales and correspondingly one has to change the theoretical framework into one which is capable of dealing with non-analytical fluctuations. This means going from differential equations to something like the renormalization group to study the exponents. For example the so-called 'biased theory of galaxy formation' (Kaiser, 1984) is implemented considering the evolution of density fluctuations within an analytic Gaussian framework, while the non-analyticity of fractal fluctuations implies a breakdown of the central limit theorem which is the cornerstone of Gaussian processes (Sylos Labini *et al.*, 1998).

In this scheme *the space correlations and the luminosity function are then two aspects of the same phenomenon, the MF distribution of visible matter.* The more complete and direct way to study such a distribution, and hence at the same time the space and the luminosity properties, is represented by the computation of the MF spectrum of exponents. This is the natural objective of theoretical investigations in order to explain the formation and the distribution of galactic structures. In fact, from a theoretical point of view one would like to identify the dynamical processes which can lead to such a MF distribution.

In this perspective, it would be extremely interesting to study the distribution of dark matter and to determine its correlation exponent. It could be that dark matter is distributed like a homogeneous fluid, having hence $D = 3$ even at small scale. In this way one may save the usual FRW metric (which needs a homogeneous density to be developed), while a substantial revision to the models of galaxy formation is required. On the contrary if dark matter is found to have the same distribution as luminous matter, than a basic revision of the theory must be considered. In fact, if dark matter is essentially associated to luminous matter, then the use of the FRW metric is not longer justified. This does not necessarily imply that there is no expansion or no Big Bang. It implies, however, that these phenomena should be described by more complex models (Sylos Labini *et al.*, 1998).

It is worth noticing that from an observational point of view there are various arguments for the proposition that galaxies are fair tracers of the mass. For example no survey, in 21-cm, infrared, ultraviolet or low optical surface brightness has revealed a void population. There is a straightforward interpretation: the voids are nearly empty because they contain little mass.

8 CONCLUSION

In the discussion about the theoretical implication of our results, we should not forget the invisible, 'dark' matter, which is thought to account for at least 90 per cent of the mass in the Universe. Apart from the galaxy rotation curves, which is different evidence, the exotic forms of dark matter are introduced to explain the observed puzzling properties of visible matter. Actually in the most recent propositions there are two weird forms of DM which add to about 98% of the total matter. So the standard interpretation is entirely based on unknown entities whose properties are defined just to explain the observed data. In our approach we show the correct statistical properties of visible matter which are different from the usual ones. These results in the above perspective have important implications for the eventual DM which, however, has now to be reconsidered in the new perspective. The properties of dark matter in the standard picture are inferred from the observed properties of visible matter and radiation. Now one studies changes in these properties and in this respect they will have consequences on dark matter too (Baryshev *et al.*, 1998; Durrer and Sylos Labini, 1998; Gabrielli and Sylos Labini, 1999).

For some questions the fractal structure leads to a radically new perspective and this is hard to accept. But it is based on the best data and analyses available. It is neither a conjecture nor a model, it is a fact. The theoretical problem is that there is no dynamical theory to explain how such a fractal Universe could have arisen from the pretty smooth initial state we know existed in the Big Bang. However this is a different question. The fact that something can be hard to explain theoretically has nothing to do with whether it is true or not. Facing a hard problem is far more interesting than hiding it under the rug by an inconsistent procedure. For example some interesting attempts to understand why gravitational clustering generates scale-invariant structures have been recently proposed by de Vega *et al.* (1996a, b, 1998). Indeed this will be the key point to understand in the future, but first we should agree on how these new 3d data should be analysed. In addition, the eventual cross-over to homogeneity has also to be found with our approach. If for example homogeneity would really be found say at $\sim 100h^{-1}$ Mpc, then clearly all our criticism of the previous methods and results still holds fully. In summary the standard method cannot be used either to disprove homogeneity, nor to prove it. One has simply to change methods.

Acknowledgements

I warmly thank Y. Baryshev, R. Durrer, A. Gabrielli, M. Joyce, M. Montuori, and L. Pietronero with whom various parts of this work have been done in fruitful collaborations. I also thank the organizers for this very interesting conference. This work has been partially supported by the EC TMR Network 'Fractal structures and self-organization' ERBFMRXCT980183 and by the Swiss NSF.

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