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LOWER CHROMOSPHERE OSCILLATIONS NEAR 4 mHz

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This paper is devoted to the study of spatial characteristics and height stratification of the 4-min oscillations. The line-of-sight velocity observations were made using a local differential method to ensure spatial filtering of the oscillations in a given range of wave numbers K . Particular emphasis is placed on the distinctive properties of the differential method, a knowledge of which can be a help in avoiding errors when interpreting observational results obtained by this method. To encompass the height range from the upper atmosphere to the lower chromosphere including the temperature minimum zone, a group of spectral lines was used: Fe I λ 5434 Å, Ba II λ 4554 Å, Na DI λ 5896 Å, and H β λ 4861 Å. For each line, the observations were made in several parts of the wing. In the region of high values of $K = 2\text{--}4 \text{ Mm}^{-1}$, a maximum $K \sim 0.7$ shows up. It is suggested that within the fine zone of temperature minimum there occurs a transformation of the oscillation frequency which ensures the upward propagation of the oscillations and the transport of energy to the chromosphere.

KEY WORDS Sun, solar oscillations, lower chromosphere, 4-min oscillations

1 INTRODUCTION

In the author's opinion, solar oscillations with a period of 4 minutes are as yet imperfectly understood and are of interest for helioseismology. This derives not only from their being intermediate between the well-known 5-min (photosphere) and 3-min (chromosphere) oscillations but also from certain distinguishing features of their observation. It should be noted that when interpreting solar oscillation spectra, the possibility always exists of assigning these oscillations either to the high-frequency 'tail' of the 5-min maximum or to the low-frequency 'head' of the 3-min maximum.

However, some factors make this difficult. In the first place, there is theoretical work (for example, Ando and Osaki, 1977) stating that the frequency of 4.3 mHz belongs to the resonance p -mode of the chromosphere. Secondly, there is experimental work showing such power spectra of short-period oscillations where the 4 mHz peak occupies a prominent place with respect to 3 mHz and 6 mHz. For example,

there are 'Sun-as-a-star' observations where the 4-min period is dominant (Leifsen and Maltby, 1990), while clear observational evidence of a similar character is still missing for the 3-min oscillations. From what has been said, it appears that it is more reasonable to treat the 4-min oscillations as a subject of helioseismology in its own right. Later in the paper we will attempt to demonstrate the validity of this point of view.

Now, what do we know about the 4-min oscillations? Soon after the discovery of the 5-min oscillations, it was recognized that the average period of the oscillations decreases with increasing height in the atmosphere (Noyes and Leighton, 1963). Already in early observations of solar oscillations in the case of reasonably long time series this period was observed both in the line-of-sight velocity signal (Fraser, 1968) and in the signal of the longitudinal component of the magnetic field measured in the magnesium line $\lambda 5173 \text{ \AA}$ at the disk centre (Tannenbaum *et al.*, 1971), as well as in the oscillation spectrum of the mean solar magnetic field (Ioshpa *et al.*, 1973). In Stanford observations of the line-of-sight velocity measured as the difference between the central ($0.5R$) and annular ($0.55\text{--}0.8R$) zones of the visible solar image in the $\lambda 5250 \text{ \AA}$ line (Scherrer *et al.*, 1983), the 4-min maximum is four times smaller than the 5-min maximum. At that stage in the research, the 4-min oscillations formed part of a common group referred to as the spectrum of five-minute solar oscillations. There is also some other evidence for the manifestations of this period. In observations in the line Mg I $\lambda 4571 \text{ \AA}$ and Ti II $\lambda 4572 \text{ \AA}$ at frequencies near 4 mHz, a stable phase shift of 270° and a maximum of coherence between the line-of-sight velocity signal and the intensity signal of the line core were observed by Kulaczewski (1992). This period was also detected when measuring the slope of the spectral line Fe I $\lambda 5247 \text{ \AA}$ profile (Kobanov, 1993). Furthermore, it is exceeded in value by the 5-min maximum in the power spectrum. The variation spectrum of the solar diameter includes the 4-min maximum alongside the other maxima (Brown *et al.*, 1978; Rosch and Yerle, 1983). It is interesting to note that in a paper by Jain and Roberts (1996) devoted to the explanation for the frequency shift of the p -modes with activity cycle, the frequency of 4 mHz is one of two peculiar points on the frequency axis, at which the shift changes sign. In theoretically calculated oscillation spectra for main sequence stars, the 4-min period is one of the main periods (Christensen-Dalsgaard and Frandsen, 1983). The variation spectrum of the energy flux, determined from measured line-of-sight velocity oscillations (Staiger, 1987), has its centre of gravity near 4 mHz. The 4-min maximum was found in oscillations of the central intensity Na I D1 and D2 lines (Kariyappa, 1996). In the past decade, interesting results on the 4-min oscillations were obtained in the infrared spectral region. In observations in the line OH $\lambda 11.065 \text{ mkm}$ it was found that in the power spectrum of the line-of-sight velocity signal the 4.3 mHz maximum is present alongside 3 mHz and ranks below the latter by a factor of 2.5, while in the spectrum of the intensity signal measured from the line core, they exchange their values, with 4.3 mHz becoming dominant (Deming *et al.*, 1986). The authors of this paper argue that oscillations at 4.3 mHz frequency have a horizontal wavelength $> 19 \text{ Mm}$ and are a fundamental chromospheric mode. Later (Leifsen and Maltby, 1990), from observations of intensity fluctuations of the continuous spectrum in the

6.5 nm band centred on λ 2.23 μm , it was found that oscillations with 4.3 mHz frequency are prevalent in the signal spectrum of the full solar disk and are absent from the signal obtained with the aperture of ~ 1 arcmin. On this basis, it was concluded that the 4-min oscillations have a global character. However, the authors of the cited reference are inclined to believe that the 4.3 mHz oscillations are the high-frequency 'tail' of the spectrum of the five-minute oscillation p -modes (a nice 'tail' – larger than the main maximum!).

Such is the variegated picture of research into the 4-min oscillations at this juncture. The above brief analysis raises the following questions:

1. Is it possible to obtain (using the differential method) line-of-velocity power spectra which would be dominated by the 4-min period for the range ℓ 300–400?
2. What, after all, are the spatial characteristics of the observed oscillations?
3. What height layer in the solar atmosphere is responsible for the observed period?

It is the objective of this paper to look for additional experimental evidence regarding these parameters and to extract the 4-min oscillations from a mixture of a large number of different periods and spatial sizes.

2 METHOD

The observations were made at the horizontal solar telescope of the Sayan observatory using a special differential method (Kobanov, 1983; 1985). To gain a better understanding of the distinctive properties of this method when used to investigate the wave and oscillatory processes and to correctly interpret results obtained by this method, we now briefly compare it with other methods. It is generally believed that when the one-element photodetector is illuminated by the light from the full solar disk, global oscillations of low degree are measured, and when the light comes from a small element of the solar surface, local oscillations of high degree ℓ are measured. Actually, however, in the latter case the signal formation involves oscillations of all degrees ℓ starting with $\ell = 0$, except for the highest values of ℓ , for which the horizontal wavelength is comparable to the instrument's entrance aperture. By increasing the entrance aperture, there is a corresponding reduction in contribution of short-wavelength components. This enables filtering of spatial frequencies with a moving short-wavelength boundary. The question arises of whether it is possible to carry out filtering with a moving long-wavelength boundary. Differential methods designed for investigating local wave processes are able to cope well with this operation (Kobanov, 1983; 1985). Let us consider the simplest case where the measured signal is the difference of values of the parameter under investigation in two equal-sized elements of the solar image $S_1 = S_2$ separated by a distance L . In the

simplest form, the filtering properties of the method are expressed by the formula,

$$A' = A \left| \sin \frac{\pi L}{\lambda} \right|, \quad (1)$$

where A and A' are the oscillation amplitudes at the filter input and output, respectively, L is the distance between the centres of the image elements, and λ is the oscillation wavelength measured in the same units as L . At small S this formula holds for all $\lambda \gg S$ and implies that $A' = A$ for the following series of wavelengths $\lambda_0 = 2L$; $\lambda_1 = 2L/3$; ..., $\lambda_i = 2L/(2i + 1)$; ...

For $\lambda > \lambda_0$, A decreases smoothly and asymptotically approaches zero. By varying L , it is possible to select λ_0 , and this will involve a variation of $\lambda_1, \dots, \lambda_i$, to suppress high-frequency harmonics by increasing S , and a more exact formula will then hold:

$$A' = \frac{\lambda}{\pi S} A \left| \sin \frac{\pi S}{\lambda} \sin \frac{\pi L}{\lambda} \right|. \quad (2)$$

To minimize the contribution from high-frequency harmonics, $S = 0.6L$ is used (Kobanov, 1985). It is hence clear that differential measurements have sharply defined filtering properties which must be taken into account when interpreting observations. By way of illustration we now consider the findings reported by Deming *et al.* (1986). The conclusion about a global character of the 4-min oscillations is based on two observational facts. They imply that when $L = 19$ Mm ($26''$), no 4-min oscillations are present in the differential signal spectrum, while the signal measured from one part of the solar image does involve this period. But as we have assured ourselves above, the differential signal becomes zero when $L = n\lambda$, where n is an integer. Thus, if $L = 26''$, then the zero signal may well have been given by $\lambda_1 = 26''$; $\lambda_2 = 13''$; $\lambda_3 = 8.7''$, etc. In an earlier study made at the same instrument (Glenar *et al.*, 1986) it was shown that when $L = 30''$ the 4.3 mHz maximum is present in the differential signal spectrum. In such a situation, the disappearance of the 4.3 mHz maximum when $L = 26''$ may be interpreted in favour of $\lambda = 26''$, rather than the reverse as done by the authors of the cited reference. Another feature of our differential measurements is that they depend not only on the value of L but also on the direction of L . This attribute is particularly useful when investigating the oscillatory processes in extended objects (along the filament axis, on the chromospheric network line, etc.) or when attacking problems similar to those which are solved using the 'time-distance' method (Duvall and Kosovichev, 1996). In current research, series longer than 70 hours are used when identifying the modes with degrees $\ell = 300$ –500 (Braun, 1995). This is dictated by the fact that in the absence of filtering the initial signal contains thousands of modes of different degrees ℓ which in the spectra analysed give a huge number of overlapping peaks. The mean lifetime of individual modes in this region of values of ℓ , according to current data, does not exceed 2–3 hours (Chen *et al.*, 1996). A paradoxical situation is obvious: the mode persists for a few hours, while identifying it requires observations longer than 70 hours. In differential measurements used by this author, the signal contains only components in a given direction and wavelength range, while a large

Table 1. Brief information on observational dates

| <i>Spectral line</i> λ , Å | <i>distance from core</i> $\Delta\lambda$, mÅ | <i>L</i> , <i>arcsec</i> | <i>N</i> , <i>number</i> <i>observations</i> | <i>T</i> , <i>hours</i> |
|---------------------------------------|---|-----------------------------|---|----------------------------|
| H β 4861 | 40, 80, 140 | 4, 11, 12, 20 | 15 | 13.4 |
| Fe I 5434 | 30, 60, 100 | 4, 11, 12, 20 | 16 | 14.1 |
| Na DI 5896 | 40, 100 | 11, 12 | 5 | 3.8 |
| Ba II 4554 | 30, 60 | 4, 11 | 3 | 2.6 |

number of modes outside this interval will be suppressed. Such filtering makes it possible to identify modes of high degree ℓ without resorting to ultrahigh resolution spectra that require long observational series. Time series with a duration from 1-3 hours become suitable for this purpose.

3 OBSERVATIONS AND DATA TREATMENT

This paper uses line-of-sight velocity oscillation measurements acquired by the differential method in a number of spectral lines during the time interval 1991–1997. The observations were made in the following spectral lines: Fe I λ 5434 Å, λ 5123 Å; H β λ 4861 Å; Na DI λ 5896 Å; Ba II λ 4554 Å. When working with broad lines covering a relatively large height range (Fe I λ 5434 Å, H β λ 4861 Å), the observations were carried out alternatively in three different parts of the wing.

The size of the entrance aperture was varied depending on the chosen line and the value of L . Areas of an undisturbed atmosphere near the disk centre, either along the meridian or along the latitude line, as a rule, were selected. Using a set of polarization prisms it was possible to select the following main distances between observational areas: 4", 11" and 20". Tracking of the Sun was done accurate to about 1". The image motion caused by solar rotation was compensated by a slow scanning with the error not exceeding 0.3" per hour. In many cases the quality of observational data was inadequate to ascertain the presence of the 4-min period without recourse to methods of spectral analysis. Time series shorter than 30 min were not considered in this analysis. The maximum duration of a single observation was about 2 hours. The characteristics of some of the observational runs are listed in Table 1. As is evident from Table 1, a significant part of the observations was done in two strong Fraunhofer lines: Fe I λ 5434 Å and H β λ 4861 Å. The former is a non-magnetosensitive line and is extensively used in the study of oscillations in the photosphere and lower chromosphere (Lites, 1992). The latter refers to chromospheric lines and spans the height range up to 2 thousand kilometres. Additionally, in the lines Fe I λ 5434 Å and λ 5123 Å some observational series were obtained in which oscillations in the slope of the profile of these lines were measured using a modulational method (Kobanov, 1993). The observational sequences used in the analysis comprised time series consisting of 200–600 quantities each. On removal of the trend and subtraction of the mean value, a standard (multiple of 2)

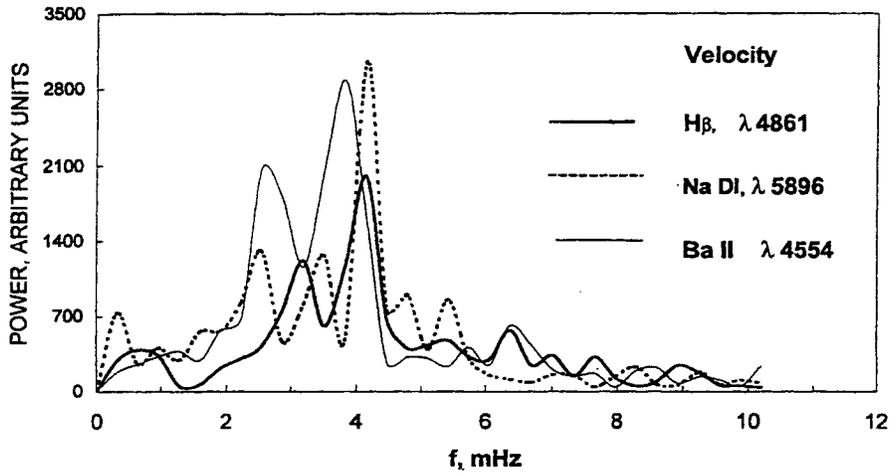


Figure 1 Examples of velocity power spectra, observed in some spectral lines.

length of the series was obtained. In our case they were the values 256 and 512 points, for which purpose the series was terminated or was completed with zeroes. After that, the procedure of fast Fourier transform was applied.

4 RESULTS, ANALYSIS AND DISCUSSION

Characteristically, oscillations with 4 mHz frequency are observed to a certain extent in all lines used. The 4-min period was assumed to be present in the observational series if the region 3.75–4.5 mHz of the power spectrum showed a predominant peak or a peak constituting no less than 30 maximum peak in a corresponding power spectrum. Figure 1 shows examples of power spectra of the line-of-sight velocity signal in an undisturbed atmosphere for different spectral lines. Common to these spectra is the value of the spatial filtering parameter $L = 11''$, which corresponds to the acoustic p -mode with $\ell = 274$. In these spectra, the peak near 4 min is dominant and is by about a factor of 1.5 larger than the 5-min oscillations in power. The maxima at 5–6 mHz corresponding to the 3-min oscillations are significantly excelled by them. Noteworthy is the outward resemblance of two spectra in Figure 1. They represent the power spectra calculated from the line-of-sight velocity observations in the wing of the H_{β} λ 4861 Å, $\Delta\lambda = 140$ mÅ, and for the Ba II λ 4554 Å, $\Delta\lambda = 60$ mÅ, respectively.

It is interesting to note that the measured variations in the slope of the wing of the spectral line clearly show the presence of 4-min oscillations (Figure 2) even for a purely photospheric line, Fe I λ 5123 Å, as well as in the wing of the line Fe I λ 5434 Å ($\Delta\lambda = 80$ mÅ). The latter line under the same conditions ($\Delta\lambda = 80$ mÅ) but for the line-of-sight velocity signal reveals very weak 4-min oscillations which

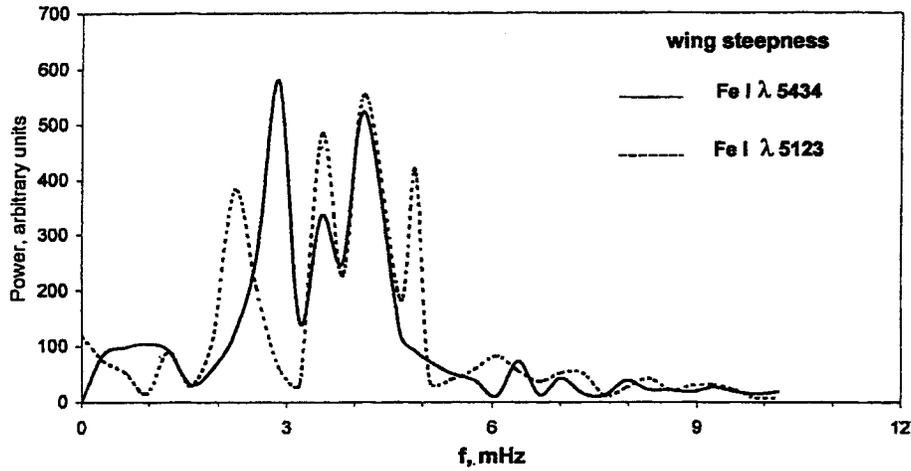


Figure 2 The power spectra of spectral line steepness variations.

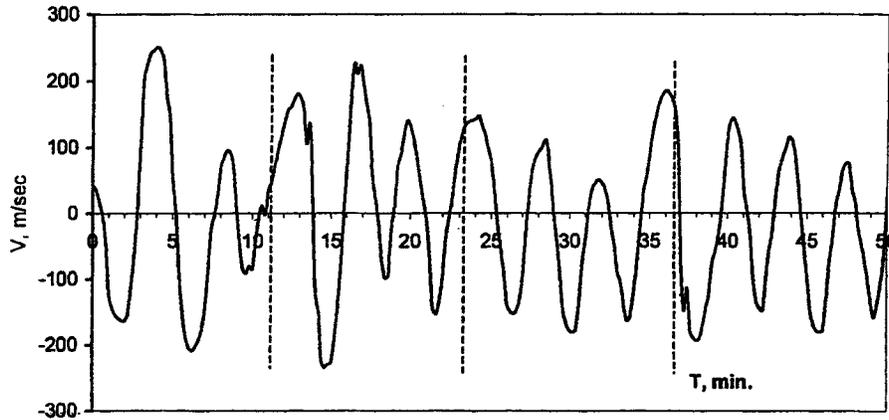


Figure 3 Velocity oscillations, observed in the Na DI spectral line; entrance slit $2 \times 4''$, $L = 11''$.

in some instances disappear completely (Figures 3 and 4). It is not an easy matter to determine the height scale for each kind of observation. Based on published data (Staiger, 1987; Sykowski *et al.*, 1988; Lites, 1992; Steffens *et al.*, 1995) we used the following height scale: H_{β} core 2000 km, Na DI core - 700-900 km, Fe I λ 5434 Å core - 700 km, and Ba II λ 4554 Å core - 600-700 km. Figure 5 plots the distribution of spectral maxima for the line-of-sight signal in different spectral lines as a function of the frequency and height. Each time series is represented in this diagram by a single main maximum. In the majority of cases the error of determination of the position of a maximum on the frequency axis $150 \mu\text{Hz}$. The scale of the Y axis is represented as arbitrary height levels h_1-h_7 listed in Table 2. From the diagram

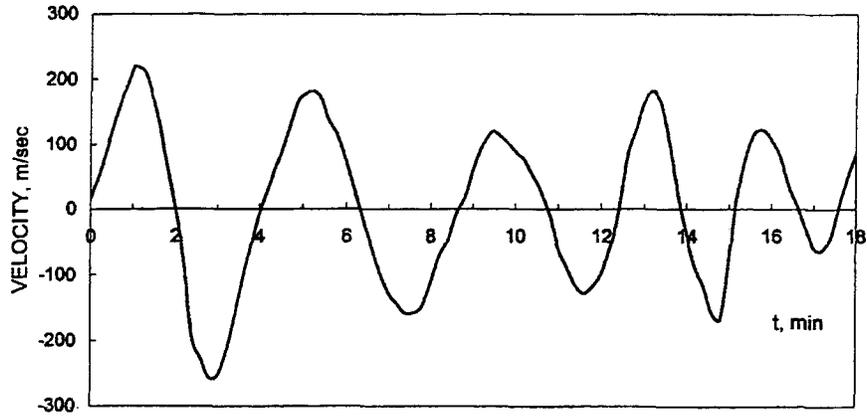


Figure 4 Velocity oscillations, observed in the H_{β} λ 4861 Å spectral line; entrance slit $2 \times 4''$, $L = 12''$.

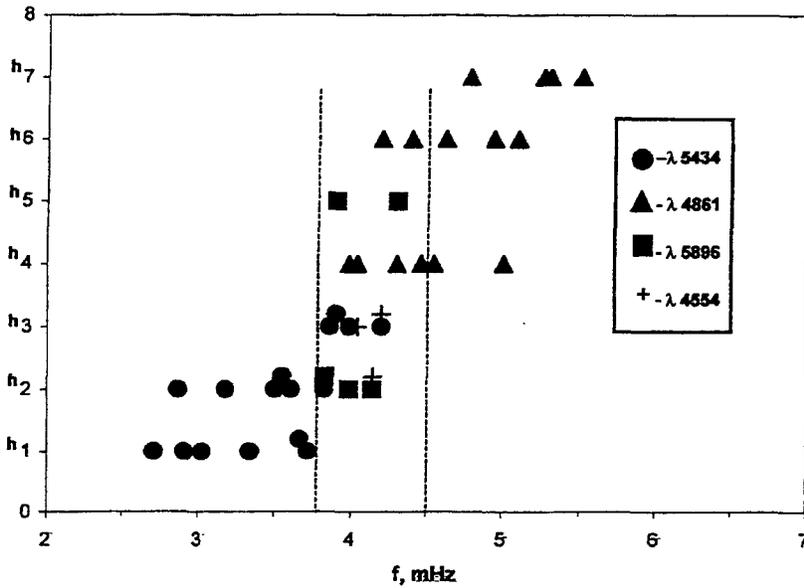


Figure 5 Diagram, illustrating the distribution of main spectral maxima depending on the velocity oscillation frequency and the height in the solar atmosphere. The height scale on the axis Y is represented as arbitrary levels h_1 – h_7 listed in Table 2. Dashed lines show the 4-min range.

Table 2. Arbitrary height levels

| h_1 | h_2 | h_3 | h_4 | h_5 | h_6 | h_7 |
|-------------------------------------|---|---|--|-------------------------------------|---|---|
| Fe I 5434 $\Delta\lambda=100$ mÅ | Fe I 5434 $\Delta\lambda=60$ mÅ Ba II 4554 $\Delta\lambda=60$ mÅ Na DI 5896 $\Delta\lambda=100$ mÅ | Fe I 5434 $\Delta\lambda=30$ mÅ Ba II 4554 $\Delta\lambda=40$ mÅ | H β 4861 $\Delta\lambda=140$ mÅ | Na DI 5896 $\Delta\lambda=40$ mÅ | H β 4861 $\Delta\lambda=80$ mÅ | H β 4861 $\Delta\lambda=40$ mÅ |

in Figure 5 it is evident that the main maxima in the frequency range 3.75–4.5 mHz correspond to heights bounded by the levels h_2 , h_6 . This interval spans the height range from the uppermost photospheric layers to the lower chromosphere including the greater part of the temperature minimum zone. One can say with a fair degree of confidence that the 4-min oscillations are observed mainly in the temperature minimum zone where they dominate over oscillations of other periods. Consider further the manner in which the main maxima of the 4-min range are distributed depending on the parameter L that determines spatial filtering. For the sake of convenience, we use the ratio $P = N_L^{4m}/N_L$, where N_L is the total number of observations for a given value of L ; and N_L^{4m} is the number of observations of a given value of L which contain the 4-min main maximum. On the diagram in Figure 6 one can see that for two close values of $L = 11''$ and $L = 12''$, P is 0.69 and 0.545, respectively, while a significantly smaller P corresponds to the other two values of L . Although four points do not provide satisfactory resolution in spatial wavelength, it can be said with sufficient assurance that for the 4-min oscillations the size smaller than $26''$ is not the least of the factors and probably even plays a crucial role. This result is contradictory to the conclusions drawn by Deming *et al.* (1986). As has been pointed out above, this could arise from the fact that the differential observations were misinterpreted by the authors.

It is also important to ascertain whether the 4-min oscillations are a simple mixture of the 5-min and 3-min oscillations which act in a concerted manner.

High-quality primary observational data permits in some cases a direct analysis of measurements to be made. Figure 3, for example, presents an entire observational series. The line-of-sight velocity measurements were made in the Na DI line, $\Delta\lambda = 40$ mÅ, $L = 12''$, with the entrance aperture $2 \times 4''$. We arbitrarily divided the whole series into four intervals, in which dynamic changes of the signal are traceable. A larger amplitude is noticed to correspond to a longer period. In any case this is true if in each interval two first periods are compared with the subsequent two or three periods. The same conclusion can also be arrived at by analysing the fragments in Figure 4.

This may be interpreted as meaning that higher-frequency oscillations of the lower layers are the source for the upper layers (Fleck and Schmitz, 1991). When the photospheric 5-min oscillations force their way upwards, they act to drive the 4-min oscillations in the temperature minimum zone and these, in turn, are the

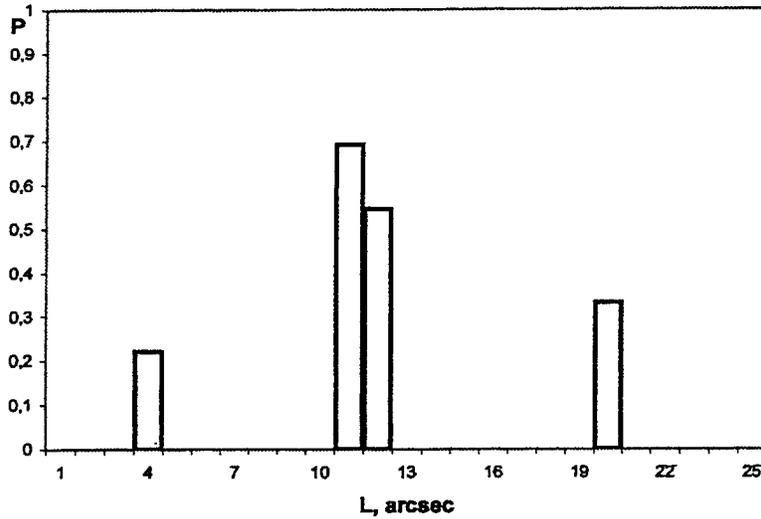


Figure 6 Distribution of the main spectral maxima as a function of the spatial parameter L .

triggers of 3-min oscillations in the chromosphere. The time characteristic of the measured signal would suggest that these oscillations in the range of the spatial scales under consideration ($\ell \sim 200\text{--}500$) are damped ones. It is notable that Braun (1995) reported power spectra for $\ell = 226$ and $\ell = 473$ which contain 4-min maxima. These spectra were obtained by using long-duration (67.7 hours) line-of-sight velocity observations in an undisturbed solar region at altitudes corresponding to the temperature minimum zone (spectral line Ca II K $\lambda 393$ nm). In spite of the very high spectral resolution the four-minute maxima are relatively broad, which seems to be associated with a short lifetime of these modes. It may be suggested that the temperature minimum zone is a peculiar kind of oscillation frequency transformer which ensures the propagation of the oscillations to higher chromospheric layers. Since this zone is relatively thin, and the oscillations there still do not dissipate, it seems appropriate to use it in estimating the energy transported upward through oscillations. It may well turn out in this case that for such estimations it will suffice to observe the 4-min oscillations only. This would alleviate the problem significantly.

Let us briefly summarize the results obtained in this work.

With moderate spatial resolution and a duration of about 1 hour, we were able to obtain spectra where the 4-min period is significant.

Oscillations with 4-min period are dominant in the temperature minimum zone.

Line-of-sight velocity oscillations at frequencies near 4 mHz are recorded with confidence in the range of spatial scales 4–20'' (3–15 Mm) and have a maximum at 11–12''. This result is inconsistent with conclusions drawn by other authors (Deming *et al.*, 1986; Leifsen and Maltby, 1990) about the spatial scales of the 4-min oscillations.

In the range of heights and spatial sizes treated in this study, the oscillations appear as trains including 2–4 full oscillations. The oscillation amplitude and period decrease from the beginning to the end of the train.

A distinguishing characteristic of the 4-min oscillations is that they manifest themselves sufficiently clearly in variations in the slope of the spectral line profile and at lower altitudes compared to the line-of-sight velocity signal.

Differential line-of-sight velocity measurements made in the real-time mode permit time series of a moderate and short duration to be used to record the p -modes in the range of high values of ℓ .

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