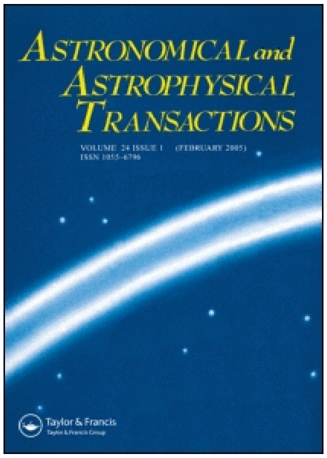

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GLOBAL INERTIAL WAVES IN THE SOLAR INTERIOR AND SMALL DEFORMATIONS OF THE SOLAR FIGURE

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The features of the background magnetic field evolution, such as the dipole–quadrupole cycle, the four-sectoral structure of the magnetic field polarity migration, and the sectoral structure of the polar facular and active region concentrations (Mikhailutsa and Makarova, 1994; 1997; Mikhailutsa, 1995) have been used for developing the qualitative characteristics of the supposed global inertial wave circulation in the deep interior of the Sun. An important observable consequence of that wave is a slight deformation of the solar figure. Based on recent results of measurements of the solar diameter at Tenerife and Locarno the expected small differences (of about 0.2″) in the values of the solar semidiameter have been found.

KEY WORDS Sun, magnetic field, solar figure, inertial waves

1 INTRODUCTION

At the present time there are two opposing views about large-scale patterns of the solar surface magnetic flux. One is that once flux erupts, large-scale patterns are simply the result of observable surface motions. The other view is that the large-scale patterns are controlled by processes deep within the Sun. The flux transport model, developed mainly by Wang, Sheeley and Nash (1991), has nicely explained in latitude-time coordinates (the one-dimensional Sun) nearly all the observations of large-scale magnetic flux patterns and their evolution. The model reproduces also some of the large-scale longitude structures seen in polar regions over the course of solar cycle 21 (Wang and Sheeley, 1994). There are some difficulties in the model in explaining the poleward surges of magnetic flux during some parts of the cycle, but on balance, the model works surprisingly well. There are physical objections to the first view, but we suppose that the main crucial question now is whether this model of the axisymmetrical or special local meridional magnetic-fluxmigration of the active region remainders is of any physical importance for the large-scale background magnetic pattern evolution? Another crucial question is whether the Carrington rotation and the differential rotation are a real rotation of the background magnetic patterns or whether the rotation rate might be used as a free parameter to study the evolution of this field in latitude–longitude coordinates? The positive answers to these questions lend physical support to the second view. We adopt the second view and take into account zonal-sectoral evolution of large-scale patterns in an attempt to create a satisfactory scenario for the solar cycle. The flux transport model cannot reproduce some observed phenomena, which are very important for the two-dimensional Sun. For example:

- At times of solar polar magnetic field reversal the large-scale magnetic flux patterns have a sectoral quadrupole-like structure dominately, together with a considerable equatorial dipole (Hoeksema and Scherrer, 1986; Hoeksema, 1991; Mikhailutsa, 1995). Why are there quadrupole and sectoral dipole inter-relations among unipolar patterns of the large-scale fields when magnetic flux arises on the solar surface (according to the first view) as bipolar ephemeral and active regions?
- The longitudinal distributions of the polar faculae and bright K Ca+ points have steady sectoral-hemispherical features at solar cycle minima, which are inconsistent with an axisymmetric surface motion scenario (Mikhailutsa and Makarova, 1994). Recently, the same steady sectoral-hemispherical features as were discovered for the polar faculae points were found for the sectoral extremes of sunspot region number in the rotational system $T = 27.0$ days synodic (Mikhailutsa and Makarova, 1997). This shows that the global solar activity is not a purely axisymmetric process. There are four sectors per solar rotation with inclined boundaries from the poles to the equator of global solar activity extremes.
- The connections between important eigenmodes of the large-scale-magnetic-field polarity distributions at epochs of minima and daily mean sunspot areas at epochs of maxima of solar cycles demonstrate that the sectoral mode ($m = 1$) is a key of solar cycle magnitudes and, in principle, the subsurface motion idea is better than the surface motion scenario (Mikhailutsa, 1993; 1994).

This evidence logically brings us to a solar cycle scenario with dual origin of surface magnetic field patterns. This concept has three fundamental points:

1. The solar interior has a general magnetic field pervading the whole Sun.
2. A giant hydromagnetic wave exists in the deep solar interior. This wave (probably a Rossby wave) has two periods along the solar equator, and its synodic period of rotation is nearly equal to 27.0 days.
3. There is only an omega-dynamo at the bottom of the convective zone. This dynamo generates the azimuthal-active-region-magnetic-field from the cyclic dipole-quadrupole field of a giant hydromagnetic wave.

Global hydromagnetic waves (or 'Rossby waves') as a source of sunspot activity were already considered about 30 years ago (Ward, 1965; Gilman, 1969a, b). There was, however, no broad acceptance of the importance of Rossby waves, because – as it is quite clear now – sunspots are not produced directly by that type of wave. There is a rather extensive literature about so-called r-modes (non-radial oscillations), which are a general type of wave that look like Rossby waves. The description of r-modes in stars has been presented in the book by Unno *et al.* (1989). In the common case, the r-mode can be described by the superposition of three kinds of waves: a high-frequency acoustic wave, a low-frequency prograde wave, and a low-frequency mode with mixed characters of inertia-gravity wave and prograde wave. The essential dynamic aspect of a low-frequency r-mode is the conservation of vorticity in an inelastic rotational medium. The Rossby wave appears if the effect of curvature of the medium stratification becomes important. Pure inertial waves appear in homentropic fluid layers whose restoring force stems from the Coriolis force. Wolff and Blizard (1986) collect and compute various properties of r-modes, concentrating on those that would occupy the solar convective envelope. They also discuss propagation characteristics, damping and frequencies. More recently Wolff (1996) suggested how beats between r-modes may govern solar activity and interact with convection. Rossby wave vortices have been invoked to explain some of the characteristics of the large-scale magnetic field on the solar surface (Tikhomolov, 1995).

The results concerning the dipole–quadrupole cycle of the background magnetic field of the Sun (Mikhailutsa, 1995) have provided a new point of view on the origin of the solar magnetic cycle: signs of a four-sectoral structure which should be seated deep in the solar interior are seen on the solar surface. We suppose that the possible source of the solar global magnetic fields with the sectoral properties is a global inertial wave (GIW) which circulates in the deep solar interior: with the aid of GIW the dual origin of the solar surface magnetic field can be explained quite logically. The large-scale background magnetic fields (with magnetic poles located in the coronal holes), have to be directly generated by the GIW. As the active regions it concerns, their magnetic field would be created from background magnetic fields by an omega-dynamo at the bottom of the convective zone.

It is the purpose of the present paper to discuss new observable aspects which support the existence of GIW in the Sun.

As part a recent study of sunspot active longitudes and polar facular numbers (Mikhailutsa and Makarova, 1997) a rigid rotation of the sectoral structure with a period of about 27.0 days (synodic) has been found. In Figure 1 the (slightly idealized) two possible schemes of the global polarity circulation of the background magnetic flux are shown in cylindrical coordinates of that rotation rate for two successive solar cycles. Note that neither direct tracer nor Doppler velocity measurements are available to test this idea, because it reveals the background magnetic flux migration. The possible large-scale meridional material flow velocity (if it exists) is of the order of 1 m s^{-1}, that is within the error limits of velocity observations so far. The scheme (a) corresponds to the dipole–quadrupole background magnetic field cycle, which can be presented in standard spherical harmonic terms

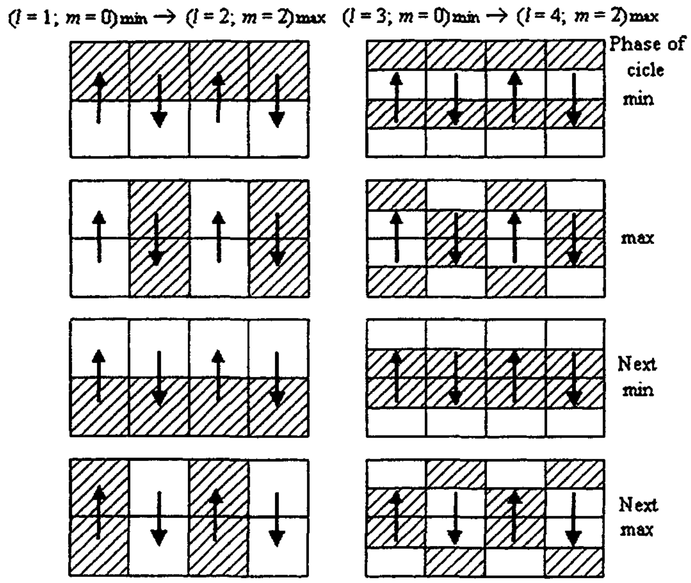

Figure 1 Idealized schemes of the large-scale polarity structure of the background magnetic field of the Sun at two successive solar cycle minima and maxima in cylindrical coordinates. The polarities are distinguished by dashes; arrows mark the direction of polarity migration in longitude sectors of the Sun. The longitudinal frame of reference corresponds to the polarity circulation. The scheme (a) corresponds to the dipole–quadrupole cycle ($l = 1; m = 0$) $\Rightarrow$ ($l = 2; m = 2$); the scheme (b) to the ($l = 3; m = 0$) $\Rightarrow$ ($l = 4; m = 2$) cycle.

as: ($l = 1, m = 0$) $\Rightarrow$ ($l = 2, m = 2$) cycle. The scheme (b) corresponds to the ($l = 3, m = 0$) $\Rightarrow$ ($l = 4, m = 2$) cycle. The last scheme is more attractive for the real Sun, because the ($l = 3$, $m = 0$) mode of the background magnetic field is maintained by the zonal structure of the filament distributions at solar cycle minima (Makarov and Mikhailutsa 1992). Both schemes can be incorporated in a model of a dipole-like $\Rightarrow$ quadrupole-like background magnetic field cycle, or in a common model of a zonal $\Rightarrow$ sectoral field polarity cycle.

It what follows we shall develop the properties of the GIW in accordance with Figure 1. The present paper consists of two parts: In the first part we outline some qualitative theoretical aspects of GIW in the Sun; in the second part we turn to the very important observational evidence in favour of the GIW velocity field manifesting itself on the solar surface.

2 WHY SHOULD GIWs EXIST IN THE SUN?

Many theories of global circulation have been advanced to explain the observed properties of differential rotation. All these theories have assumed that the large-scale material motions are symmetric with respect to the rotation axis. This is one of the reasons that there is so little reliable information about other possible global motions. The crucial question now is whether these departures from the time-and-longitude-averaged motion field are of any physical importance for the general circulation of the solar atmosphere. It is not possible to predict in advance the type of motion to be expected, because this critically depends on the eddy viscosity and other parameters which are poorly known for the Sun. Nevertheless the steady symmetric circulations are not able to build-up the observed dipole-quadrupole cycle of the background magnetic field and to maintain the fundamental sector structure of solar surface phenomena. Note that axisymmetric motions alone may be sufficient to accurately predict the location, shape and evolution of some coronal holes on a relatively short time interval, when sectors are usually blended by the more intense small-scale features. But bearing in mind the sequence of solar cycles, we can conclude that it is impossible to produce the dipole–quadrupole cycles by axisymmetric large-scale polarity circulation, because the sectoral polarity patterns can't be constructed from the zonal ones (and vice versa) by axisymmetric rearrangements. Having this fact in view, the possibility of an axisymmetric circulation must be called into question and the concept of some kind of r-mode wave should be accepted.

Gilman (1969a, b) has defined a Rossby wave as a nearly horizontal wave-like or eddy-like flow pattern in a rotating medium in which Coriolis forces nearly balance horizontal pressure forces (the so-called heliostrophic balance). If the Lorentz forces are also important in the balance, the wave is usually called a hydromagnetic inertial wave (Moffatt, 1978; Priest, 1982). The quasi-stable magnetic cyclicity should be maintained by steady global circulation, and the Lorentz forces should be much smaller than the Coriolis forces and the pressure forces. The Coriolis force is largest in the rapidly rotating layers of the solar atmosphere. According to some helioseismological data the inner part of the Sun ($r \leq 0.2R_s$) rotates about 1.5–2 times as fast as the solar surface (Duvall and Harvey, 1984; Kosovichev, 1988; Goode *et al.*, 1991; Jimenez *et al.*, 1994), and at those depths global inertial waves (GIW) can be generated. But there is virtually no information about the location of the possible rise in angular velocity, and the very existence of that rise is still highly controversial (Toutain and Kosovichev, 1994). The latest helioseismology results do not give a clear answer to that issue. For example, the results presented at the October (1996) IAU Symposium in Nice (mainly from SOHO and GONG) strongly suggest that the solar core is rotating no faster than 1.1 times the overlying layers. But this does not make the proposed mechanism of this paper less likely. The generation of r-modes mainly arises from the conservation of vorticity in an inelastic rotational medium where the variation of volume and density of a fluid element are physically important. As far the GIW is concerned, the density variation in buoyancy is not important and the particle motions in the wave have a circular orbit (Moffatt, 1978; Priest, 2982), or elliptical in general. That kind of particle motion is very convenient for the generation of a magnetic field. Since the frequencies of global r-modes (and of GIW, of course) are close to the rotation frequency of the Sun, it is necessary to explain how a 22-year magnetic cycle is generated in our scenario. Some speculation about it will be presented in the next section. It should also be pointed out that non-axisymmetric inertial waves are not the only possible response of the solar atmosphere to a latitudinal energy transport: one can also get a purely axisymmetric circulation in meridional planes, coupled with a zonal circulation. But all of these contrary arguments are not very productive in explaining the origin of the dipole–quadrupole cycle, where the sectoral oppositely directed migration of the background magnetic flux is taking place. This phenomenon is the basis of the concept of GIWs.

3 CONCEPT OF THE GIW

We shall develop here the qualitative aspect of the GIW concept for the Sun to find the observational evidence of its reality. The quantitative aspect of how GIW might be excited and damped and the diagram showing regions in the Sun in which these waves are trapped, propagating, evanescent, etc., will be the subject of a separate work.

The GIW is formed in the solar interior where Coriolis forces are balanced by the pressure forces. The dispersion relation for the GIW (Moffatt, 1978) is:

$$\omega = \pm\frac{2(\boldsymbol{\Omega}\cdot\boldsymbol{k})}{k}, \tag{1}$$

where $\boldsymbol{\Omega}$ is the vector of angular velocity, and $\boldsymbol{k}$ is the wave number vector. From Equation (1) we get the expressions for the group and phase velocities of the GIW:

$$\boldsymbol{V}_{\mathrm{g}} = \mathrm{grad}_{k}\omega = \mp\frac{2[\boldsymbol{k}\times[\boldsymbol{\Omega}\times\boldsymbol{k}]]}{k^3}, \tag{2}$$

$$\boldsymbol{V}_{\mathrm{ph}} = \frac{\omega\boldsymbol{k}}{k^2} = \pm\frac{2(\boldsymbol{\Omega}\cdot\boldsymbol{k})\boldsymbol{k}}{k^3}. \tag{3}$$

The particle motion of the inertial wave has a circular orbit, so any component of the velocity of the medium V_{m} can be represented in a spherical system of coordinates for the running wave as:

$$V_{\mathrm{m}} \sim \cos\left((\boldsymbol{k}\cdot\boldsymbol{r}) - \frac{2(\boldsymbol{\Omega}\cdot\boldsymbol{k})t}{k}\right) = \cos((\boldsymbol{k}\cdot\boldsymbol{r}) - 2t\Omega\cos\theta), \tag{4}$$

where t is the time, θ is the polar angle, and $\boldsymbol{r}$ is the radius vector. Due to the globality of the GIW for the Sun one can consider the superposition of many running waves in the solar interior that may give rise to the global standing wave. The velocity of the medium in such a wave can be described by the following expression:

$$V_{\mathrm{m}} \sim \cos(\boldsymbol{k}\cdot\boldsymbol{r})\cos(2t\Omega\cos\theta). \tag{5}$$

Which of these cases (global running or global standing wave) is realized in the Sun? The answer can be given by direct observations of the global velocity field or its results on the solar surface.

It should be emphasized that the short-period modulation of solar global cycles, which depend on solar latitude, is one of the ways to find observational evidence in favour of GIWs existing in the Sun.

We shall now derive expressions for the wave numbers of the GIW in the Sun and shall base our discussion on the four-sector structure of the background magnetic field polarities at the maxima of the solar cycle, and on the alternative directions of their magnetic flux migrations in neigbouring sectors, as shown schematically in Figure 1. This magnetic polarity structure can serve as an indicator that the wave number k_{p}, directed along the solar parallels, can be expressed as:

$$k_{\mathrm{p}} = \frac{2\pi}{\lambda_{\mathrm{p}}} \approx \frac{2}{R_{\mathrm{s}} \sin\theta}, \tag{6}$$

where λ_{p} is the wavelength of the GIW along the parallels, and R_{s} is the radius of the Sun. As for wavenumbers k_{t} directed along the solar meridians this can also be written (cf. Figure 1*a* and *b*) as:

$$\left\{\begin{array}{lcl} k_{\mathrm{t}} & = & \dfrac{2\pi}{\lambda_{\mathrm{t}}} \sim \dfrac{1}{R_{\mathrm{s}}}; \\ k_{\mathrm{t}} & = & \dfrac{3}{R_{\mathrm{s}}}. \end{array}\right. \tag{7}$$

As it is known that the background magnetic flux crosses the solar equator during the migration in sectors (Mikhailutsa, 1995), the GIW circulation must take place from one polar region of the Sun to the other. This means that the meridional component of the group velocity (V_{gt}) might have a non-zero value at the solar equator ($\theta = \pi/2$). During the course of our investigation it has been found that a non-zero value of V_{gt} can be attained only if the wavenumber k_{t} is proportional to $\cos\theta$. So we have correspondently:

$$\left\{\begin{array}{lcl} k_{\mathrm{t}} & \approx & j\dfrac{\cos\theta}{R_{\mathrm{s}}}; \\ k_{\mathrm{t}} & \approx & j\dfrac{3\cos\theta}{R_{\mathrm{s}}}, \end{array}\right. \tag{8}$$

where the j-function is either $+1$ or -1 in accordance with the sectors and solar hemispheres. Further we suggest that the wave number k_{r} directed along the radius, is much smaller than k_{t} and k_{p}. This means that the GIW circulates mainly along the θ and ϕ spherical coordinates. Taking this, together with Equations (2), (3), (6), and (8), into account we find:

$$\left\{\begin{array}{lcl} k_{\mathrm{r}} & \approx & j\dfrac{2\sin^4\theta - \sin^2\theta - 4}{2R_{\mathrm{s}}\sin^3\theta}; \\ k & = & \sqrt{k_{\mathrm{t}}^2 + k_{\mathrm{p}}^2 + k_{\mathrm{r}}^2} \approx j\dfrac{\sin^2\theta + 4}{2R_{\mathrm{s}}\sin^3\theta}; \\ k_{\mathrm{p}} & \approx & j\dfrac{2}{R_{\mathrm{s}}\sin\theta}, \end{array}\right. \tag{9}$$

for the dipole magnetic field polarity structure at solar cycle minima, and

$$\left\{\begin{array}{lcl} k_r & \approx & j\dfrac{18\sin^4\theta - 9\sin^2\theta - 4}{6R_s\sin^3\theta}; \\ k & \approx & j\dfrac{9\sin^2\theta + 4}{6R_s\sin^3\theta}; \\ k_p & \approx & j\dfrac{2}{R_s\sin\theta}, \end{array}\right. \tag{10}$$

for the octupole magnetic field polarity structure at solar cycle minima.

The model that describes the solar cycle scenario in detail would be much too complicated mathematically for us to be able to calculate exact predictions. One therefore has to make simplifying assumptions and approximations – and even then, the problem of extracting predictions remains a formidable one. The solution for the small amplitude disturbed fields of velocity $\boldsymbol{V}_m$, magnetic strength $\boldsymbol{h}_m$, and pressure P_m can be presented in the form of a wave (Lehnert, 1954):

$$(\boldsymbol{V}_m, \boldsymbol{h}_m, P_m) = \mathrm{Re}\,(\boldsymbol{V}_f, \boldsymbol{h}_f, P_f)\exp[i((\boldsymbol{k}\cdot\boldsymbol{r}) - (\omega + (\boldsymbol{V}_0\cdot\boldsymbol{k}))t)], \tag{11}$$

where $\boldsymbol{V}_f$, $\boldsymbol{h}_f$, P_f are the Fourier forms of the corresponding disturbed fields, and $\boldsymbol{V}_0$ is the homogeneous undisturbed velocity field component. The temporal term in Equation (11) consists of two parts. The first can be expressed, for example, as $\omega t = 2t\Omega\cos\theta$, which corresponds to the variations with periods from 13.5^d (for $\theta = 0°$) to 155^d (for $\theta = 85°$, for the Bartel rotational rate of the sector structure. The second, $(\boldsymbol{V}_0 \cdot \boldsymbol{k})t$, represents the much lower frequency of variations and should correspond to the 22-year solar background magnetic field cycle. In principle, the appropriate velocity field $\boldsymbol{V}_0$ can be constructed to explain the global solar cycle. The required magnitude of velocity is of the order of 1 m s^{-1}, but the reason of such kinematics remains outside of our frame of investigation. Equation (11) does not explain directly the origin of the solar global magnetic cycle. It shows us only a possible alternative way of searching the satisfactory scenario of that cycle. Up to now we have considered the radial wavenumbers of the GIW which were much less than the azimuthal ones: $k_r \ll k_t, k_p$. But in the real Sun, probably, there is a spectrum of k_r, in which the relation $k_r \sim k_t, k_p$ is also taking place. The last condition is very convenient for the global 22-year solar cycle generation because (as can be seen from Equations (1), (6) and (7)) the first temporal term in Equation (11) becomes equal to zero when $k_r = j\sin\theta/R_s$ and the cycle remains under the direction of the second term.

We see from Equations (6), (8), and (9) that the wavenumber values depend on the polar angle and azimuthal coordinates, but do not depend on the radial coordinate. This means that some particular modes of GIW are expected instead of a whole spectrum having various azimuthal, and latitudinal wavenumbers. Note again that the radial wavenumbers should differ strongly from that which are presented in Equations (9) and (10) to explain the 22-year solar magnetic cycle. The following analysis will be made on the basis of Equations (4), (5), and (8)–(10) without taking into consideration the radial wavenumbers.

4 AMPLITUDE OF SOLAR FIGURE DEFORMATIONS

The particle velocities of the GIW, depending on the direction of wave propagation, have radial components on the solar surface. As a result, a small deformation of the solar figure can arise in certain places. We have used this phenomenon to find observational evidence of the existence of GIW in the Sun.

Let us make some rough estimations: the linear approximation for solution in Equation (6) suggests that the magnitude of the disturbed velocity should be much less than the undisturbed velocity magnitude. It is easy to estimate the last value, which needs to explain the global solar cycle duration: $V_0 \approx R_s/T_c \approx 1$ m s^{-1} Further, due to the opposite directions of rotation of sectors and of the phase velocity latitudinal component (from Equations (2) and (3): $V_{gp} = -V_{php}$), the time of crossing over a sector by a phase signal can be found approximately as

$$\Delta t = \frac{1}{4}\left(\frac{1}{T_s} + \frac{1}{T_c}\right)^{-1} \approx 3.4 \text{ day}$$

for $T_s = 27.0^d$. Only during half of that time will the amplitude of deformation of the solar figure be increased, and so the peak of the amplitude reaches the value $\Delta r < 1/2 \cdot 1 \cdot 3.4 \cdot 86400 = 1.5 \times 10^5$ $(m) \approx 0.2''$. The magnitude of the difference between the mean and deformed values of the solar radius should be less than $0.2''$. It should be noted that the effect expected from an r-mode according to Volland (1992) and Wolff (1996) (at least for modes trapped in the convection zone) is several times less than the magnitude of the solar radius distortion estimated above. The estimated value of the solar figure deformation is less than the ordinary noise caused by the Earth's atmosphere (seeing). In such a case the direct methods of searching for those deformations are not applicable.

5 EXPERIMENTAL VERIFICATION OF THE GIW

The interest in possible variations of the solar diameter in space and time has been aroused by the results about a possible cycle-dependence of global solar properties (Delache *et.al.*, 1993). To solve this problem, the most accurately calibrated method for ground-based angular measurements of the solar diameter has been used, viz. drift-scan timing along circles of constant declination (for a detailed description see Wittmann, 1980; Wittmann, Bonet Navarro, and Wöhl, 1981).

In the years 1990–1995 several observational series comprising more than 5000 measured drift times have been obtained at Izaña/Tenerife and Locarno/Switzerland by means of two very similar telescopes (D = 45 cm Gregory–Coudé Telescope). For a detailed description of the observations and how to reduce them, cf. Wittmann, Alge and Bianda (1991) and Wittmann and Neckel (1996). One may be somewhat sceptical about deformations of the figure of the Sun, at least as far as the possibility of deducing them from the diameter measurements through the Earth's atmosphere is concerned. As a matter of fact almost all diameter variations seen at Locarno are of non-solar origin (mostly of atmospheric, partly also of instrumental and methodological origin), and almost all variations seen at Tenerife (which are considerably smaller than those at Locarno) may be attributed to local atmospheric effects (seeing) and – to a much lesser degree than at Locarno – to instrumental or methodological effects. The formal accuracy of solar semidiameter measurements at Tenerife is about $\pm 0.07''$ (or about ± 50 km) for the daily mean (from typically 16–32 individual measurements). We have applied the superimposed epoch method (SEM) in order to search for small systematic deformations in the diameter signal.

6 METHOD OF INVESTIGATION AND RESULTS

In our case we base the SEM on the inferred phases which have been calculated for all our diameter (or, rather, semidiameter) measurements. The calculation has been made for two alternative possible cases for the Sun in accordance with Equations (4) and (5), which correspond to running (R) or standing (S) GIWs, respectively:

$$P_{\mathrm{R}} \propto \cos[(\boldsymbol{k} \cdot \boldsymbol{r}) - 2t\Omega \cos\theta], \tag{12}$$

$$P_{\mathrm{S}} \propto \cos(\boldsymbol{k} \cdot \boldsymbol{r}) \cos(2t\Omega \cos\theta). \tag{13}$$

In these cases the inferred phase values can be directly or inversely proportional to the expected medium velocities. We are interesting in the short-period modulation term of the GIW, which can be seen in the solution (11). It is not known in advance what kind of background magnetic field polarity structure is present on the solar surface at solar cycle minima (see for example Figures 1*a* and *b*). If l is the standard harmonic number, then for the zonal dipole field polarity structure: $l = 1$, for the octupole structure: $l = 3$, etc. That number should define the value of the wave number component of the GIW along the solar meridians. Taking it into account, the scalar term $(\boldsymbol{k} \cdot \boldsymbol{r})$ in Equations (9) and (10) in the case of the drift scan measurements (which are performed along a circle of constant declination) can be expressed as (making use of Equations (6) and (8)):

$$(\boldsymbol{k} \cdot \boldsymbol{r}) = k_{\mathrm{p}} r_{\mathrm{p}} + k_{\mathrm{t}} r_{\mathrm{t}} = 4\pi \frac{t}{T_{\mathrm{s}}} + l\left(\frac{\pi}{2} - p\right) \sin p, \tag{14}$$

where p is the position angle of the rotational axis of the Sun (the sign of which is not important here). By virtue of Equation (14) the inferred phases can be calculated from:

$$P_{\mathrm{R}} = \cos\left[4\pi \frac{t + \Delta t_1}{T_{\mathrm{s}}} + l\left(\frac{\pi}{2} - p\right) \sin p - 4\pi \frac{t + \Delta t_2}{T_{\mathrm{s}}} \sin p\right], \tag{15}$$

$$P_{\mathrm{S}} = \cos\left[4\pi \frac{t + \Delta t_1}{T_{\mathrm{s}}} + l\left(\frac{\pi}{2} - p\right) \sin p\right] \cos\left[4\pi \frac{t + \Delta t_2}{T_{\mathrm{s}}} \sin p\right], \tag{16}$$

where Δt_1 and Δt_2 are parameters which are not known in advance and whose values provide the maximal direct or inverse proportionality between the inferred phases and the solar surface particle velocities.

Table 1. The optimal values of the parameters and the linear regression coefficients

l	Δt_1^{d}	Δt_2^{d}	$T_{\mathrm{s}}^{\mathrm{d}}$	$A('')$	B	ρ
1	1.94	3.46	27.00	0.008	−0.080	−0.80
3	1.45	3.47	26.98	0.004	−0.097	−0.87
5	0.20	3.47	26.94	0.010	−0.096	−0.83
7	−1.30	3.47	26.92	0.010	−0.096	−0.76

The limits for the possible values of Δt_1 and Δt_2 can be obtained in accordance with the value of the sector structure rotation $T_{\mathrm{s}} = 27.0^{\mathrm{d}}$, viz. $0^{\mathrm{d}} < \Delta t_1 < 3.5^{\mathrm{d}}$ and $0^{\mathrm{d}} < \Delta t_2 < 3.5^{\mathrm{d}}$. Taking into account that the real value of T_{s} can differ slightly from 27.0^{d} synodic, we have four unknown parameters for searching for the best agreement between the inferred phases and the diameter data: Δt_1, Δt_2, T_{s}, and l.

For the start epoch (which is arbitrary) we have used 7 July 1990 0h UT, and the inferred phases have been calculated for each day of solar diameter measurements. Then these phase values have been split into twenty clusters in accordance with phase-windows. The width of the phase windows was chosen as 0.2 in order to have more then 10 daily means fall in any phase-window. Neighbouring phase-windows were overlapped at half of their width.

We used the measured daily values of the solar semidiameters; their mean value $\langle R_{\mathrm{s}}\rangle$ was used to calculate the differences $\Delta R_i = R_i - \langle R_{\mathrm{s}}\rangle$ for each day of observation. Here i denotes the index of the phase-window. The ΔR_i were averaged in each phase-window, and the linear regression coefficient between the averaged $\langle \Delta R_i\rangle$ and the mean values of phases of windows $\langle P_i\rangle$ was computed according to:

$$\langle \Delta R_i\rangle = A + B\langle P_i\rangle. \tag{17}$$

From all possible sets of parameters Δt_1, Δt_2 and T_{s} we selected those which led to the largest value of B in Equation (17) and – at the same time – to the largest value of the correlation coefficient (ρ), for each l. We have checked four variants of l value: $l = 1$; $l = 3$; $l = 5$; and $l = 7$. It should be noted here that the experiment with inferred phases of the running wave (Equation (15)) has been a failure: we could not find a linear regression with a sufficiently large value of the correlation coefficient. But our experiment with inferred phases of the standing wave (Equation (16)) has been successful: in the course of our investigation an optimal set of parameters was found, which are presented in Table 1.

The best result (maximal ρ and B) was obtained for the case $l = 3$. The rotational rate of sectors increases with increasing zonal number l. Given the limited data of solar diameter measurements, we are not sure whether this fact is of any physical significance. It should be mentioned that any combination of the different l phases has not improved the degree of correlation presented in Table 1.

The sensitivity of the correlation coefficient and multiplier B to the choice of parameters is as follows. If Δt_1 and Δt_2 have optimal values, but the value of T_{s} changes in the range $T_{\mathrm{opt}} \pm 0.01^{\mathrm{d}}$ then the 0.04 percentages of the difference of the value of T_{s} leads to 10 percentages of the difference of the value of ρ and to 20

Table 2. Average deviation of the actual semidiameter from the total mean semidiameter as a function of the phase-window

Phase-windows	$\Delta R_i('')$	*Number of days*	*Dispersion* $\sigma('')$
m9: −1.0−−0.8	+0.052	17	0.22
m8: −0.9−−0.7	+0.110	18	0.25
m7: −0.8−−0.6	+0.070	18	0.27
m6: −0.7−−0.5	+0.015	17	0.28
m5: −0.6−−0.4	+0.073	17	0.34
m4: −0.5−−0.3	+0.095	23	0.27
m3: −0.4−−0.2	+0.035	26	0.25
m2: −0.3−−0.1	+0.031	37	0.28
m1: −0.2−−0.0	+0.028	50	0.26
m0: −0.1−+0.1	−0.019	55	0.25
p1: +0.0−+0.2	−0.044	46	0.30
p2: +0.1−+0.3	−0.027	32	0.32
p3: +0.2−+0.4	−0.046	26	0.24
p4: +0.3−+0.5	−0.023	21	0.34
p5: +0.4−+0.6	−0.023	19	0.32
p6: +0.5−+0.7	−0.056	21	0.26
p7: +0.6−+0.8	−0.024	23	0.27
p8: +0.7−+0.9	−0.060	19	0.26
p9: +0.8−+1.0	−0.109	11	0.42

percentages of the difference of the value of B. It is a very high level of sensitivity to the optimal value of T_s.

Further, if T_s has an optimal value, but Δt_1 and Δt_2 change in the range of $\pm 0.1^d$, then the value of ρ decreases to 0.1 and the value of B decreases to 0.01. It can be concluded that the presented solution of the parameters Δt_1 and Δt_2 is slightly better than other possible combinations of these parameters in the range of $\Delta t_{\rm opt} \pm 0.1^d$.

So, we have found that the correlation coefficient and the multiplier B were very sensitive to the value of the parameter T_s and moderately sensitive to the values of Δt_1 and Δt_2.

Table 2 shows the results for the optimal set of parameter values for the best variant of $l = 3$.

The values of dispersion in the last column of Table 1 exceed the averaged $\langle \Delta R_i \rangle$ in each phase-window. This is an indication that the signal is fairly weak, and that it is necessary to use additional data to ultimately verify our results. In SEM the result confidence can be estimated by comparison of two magnitudes: $(N \Delta R_i)$ and (3σ). From Table 2 one can find that most of $(N \Delta R_i)$ exceed the value of (3σ), so the result is statisticaly reliable. Figure 2 shows the linear regression pattern of the data presented in Table 2. The confidence band of 0.95, drawn by dashed lines, was chosen to demonstrate the most distant points from the regression line. These points, however, do not disturb the general tendency.

Further indication of the confidence of our results could be obtained by a Monte Carlo study of noise sensitivity. This could tell us how frequently the high degree of

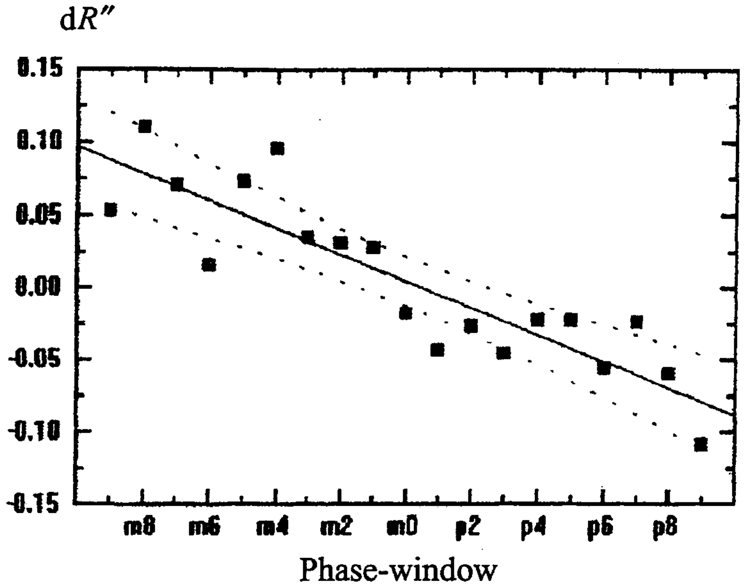

Figure 2 Linear regression pattern for the case of zonal wavenumber of $l = 3$. The dashed lines correspond to a confidence band of 0.95.

correlation obtained in Table 2 might arise simply by chance. This has been done in two different ways:

1. Given the normal distribution of all daily-mean ΔR, a histogram of which is shown in Figure 3, we have simulated each ΔR. In that case, the daily-mean values appear randomly, without influence of the number of individual measurements of each day. In result, 200 sets of simulated data were constructed. The correlation coefficient of each set did not exceed 0.2. So, it can be concluded that the high degree of correlations obtained in Table 2 can arise by chance with a probability of less than 0.005.

2. Given the observational noise level (daily dispersion), we have carried out a computer simulation of daily individual measurements by using a normal distribution. Note that each simulated daily mean has been calculated from 20 simulated individual measurements (as was in real observations). In this way 50 sets of the simulated daily mean data have been obtained. For each set of simulated data the correlation coefficient was computed under the unchangeable optimal values of the parameters. For the case of $l = 3$ we have found that the correlation coefficient values varied in the range from 0.82 to 0.89 with a peak of occurences near 0.86. So, the high degree of correlation is practically insensitive to noise. This is so because each daily mean was calculated from many individual measurements and the superimposed epoch method was used.

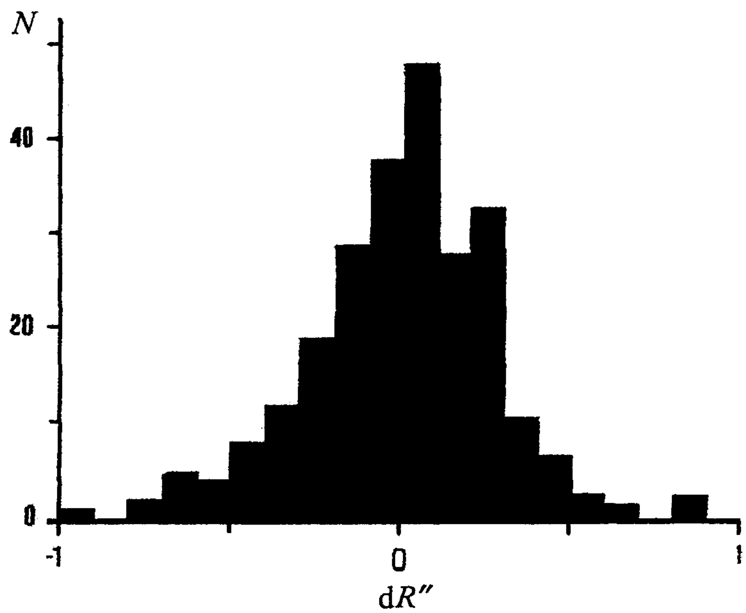

Figure 3 Histogram of all daily means ΔR.

7 DISCUSSION AND CONCLUSIONS

Let us first discuss the properties of the phase of the standing wave (P_S) as observed at the solar limb from the Earth. At the solar equator ($\theta = 90°$) the phase changes periodically with time (or solar longitude), and the corresponding deformation of the solar equatorial belt is elliptical. For any other latitude both multipliers of P_S in Equation (16) change with different periods in time, which in common cases are not proportional. So the corresponding deformation of the solar figure at those latitude belts is not stationary, and this is the reason for searching Δt_1 and Δt_2 in Equation (16) to find the moment of time when the value of P_S and the deformation of the solar figure change approximately in phase for any positional angle p.

Nobody doubts that the origin of the solar magnetic cycle is one of main questions of solar physics. Here we presented conceptual work based on the properties of evolution of the background magnetic field and, as a result, on the supposition of the existence of GIWs in the solar interior. It goes without saying that this problem needs further investigation, especially as far as the observational support for the concept of GIWs is concerned. We are far from believing in the impossibility of a random result, but at the present time it can be concluded that the physical signal probably exists because:

1. There is a strong correlaton between the mean values of the phase and the differences of solar semidiameters for certain sets of physical parameters (which we have found from a physical point of view);

2. the formulae for the calculation of phases have been derived from the physical model;

3. the signal appeared strongly only for the sector rotational periods which practically coincided with the famous Bartel rotational period.

It should be emphasized that our results can be verified by means of new measurements of the solar diameter whose accuracy exceeds $0.1''$.

The GIWs zonal wavenumber, which corresponded to the octupole zonal structure of the background magnetic field at solar cycle minima, gives the best correlation with the solar diameter variations. The measurements of the solar diameter by the drift scan technique are confined to the solar equatorial belt (viz. $\pm 26°$ solar latitude). That latitudinal belt is too narrow for the exact determination of the GIW zonal wavenumber and, thus, it should be made on the basis of other solar diameter measurements covering all positional angles of the limb.

Some estimation of the period of solar interior rotation in an inertial frame can be done on the basis of our results. The first-order approximation to the frequency of the r-mode demonstrates that the flow pattern drifts westward (opposite to the sense of surface rotation) at the speed $-2\Omega/l(l+1)$, if we observe it in a frame rotating with the Sun (Saio, 1982). But the drift speed of the pattern seen from an inertial frame is $\Omega - 2\Omega/l(l+1)$. The period of the four-sector structure rotation $\approx 27.0^{\mathrm{d}}$ associates with the synodic period of the wave node rotation (boundaries of the sectors). Keeping this in mind, the rotational rate of the solar interior where the wave is born can be estimate correspondingly for ($l = 2$; $m = 2$), and ($l = 4$; $m = 2$) sector structure as: $\Omega = 3/2(\Omega_{\mathrm{syn}} + \Omega_{\mathrm{Earth\ orbit}}) \approx 1.93 \times 10^{-6}\ \mathrm{s}^{-1}$; $T \approx 16.8^{\mathrm{d}}$, and $\Omega = 10/9(\Omega_{\mathrm{syn}} + \Omega_{\mathrm{Earth\ orbit}}) \approx 3.21 \times 10^{-6}\ \mathrm{s}^{-1}$; $T \approx 22.6^{\mathrm{d}}$. The last estimation falls into line with the latest results from helioseismology, which strongly suggest that the solar core is rotating no faster than 1.1 times the overlying layers.

Turning finally to interpretation, which is not the main subject of the present paper, let us mention that there is reason to assume the particle motion in GIW as a source of the background solar magnetic field. Thus we can expect a spatial correlation between these phenomena, at least to a certain extent. In a general way the large-scale interrelationships between global polarity structures in successive solar cycles (Mikhailutsa, 1993; 1994) look like a logical result of the existence of GIWs in the Sun.

We found many similarities between the GIW concept and the solar cycle theory as proposed by Dicke (1979a) and several earlier theories (see e.g. Cowling, 1953), which were essentially based on the assumption that the solar cycle is a manifestation of a hydromagnetic oscillation in which the entire Sun participates. For example, Dicke (1979b) concluded that the sunspot cycle shows no statistical indication of a random walk in phase and that the sunspot cycle seems to be coupled flexibly to some internal chronometer (perhaps a hydromagnetic oscillator). If the core of the Sun is perturbed, as in Dicke's theory, GIW will arise immediately, and a zonal-sectoral (dipole–quadrupole) global- background-magnetic-field cycle is generated. It should be mentioned here that a recent variant of Dicke's concept has been made by Peter Sturrock (1996).

The question of far-reaching importance is: Can the spherical geometry of a rotating gaseous object with a radial stratification of the medium be a sufficient condition for generating GIWs? If so, there are probably other stars with larger amplitudes of figure deformations as in case of the Sun.

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