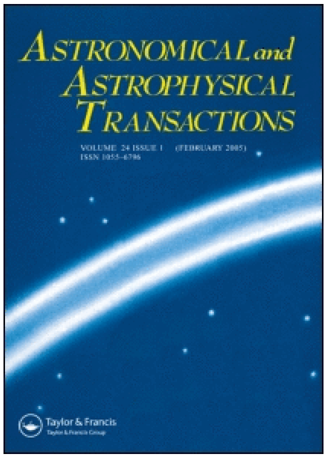

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Yu. V. Barkin [a]
[a] Sternberg Astronomical Institute, Moscow, Russia

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DYNAMIC REGULARITIES OF PLATE MOTION

Yu. V. BARKIN

Sternberg Astronomical Institute, Moscow, Russia

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On the basis of an analysis of dynamic characteristics of plates (vectors of angular momenta and momenta, their components, etc.), important dynamic regularities (laws) in positions and motions of the plates were discovered.

KEY WORDS Plate motion, regularities, Earth dynamics

1 MAIN DYNAMIC CHARACTERISTICS OF PLATES

In this paper, the main dynamic characteristics of plate motion are determined and analysed. From these we get vectors of momenta ($\boldsymbol{q}_\sigma$), angular momenta ($\boldsymbol{G}_\sigma$) of the spherical plate motion, their components, etc. These characteristics are determined for plate motion in the kinematic theory HS2-NUVEL1 (Gripp and Gordon, 1990) with respect to the mantle geocentric reference system $Oxyz$, the axes of which in the given geological epoch coincide with corresponding axes of the Greenwich reference system.

The dynamic characteristics of the plate were calculated for a concrete model (Barkin, 1995a; 1996a). In this model the plates are considered as rigid unchangeable spherical thin covers. The contours of these covers correspond to their present boundaries (in the given geological epoch). The mean power of the ocean areas of the plates was taken to be 47 km, and the mean power of the continental areas was taken to be 114 km. The average density of the plates was taken to have an effective value of 3.3 g cm^{-3}.

In the calculations of the global characteristics of the plates (in particular, $\boldsymbol{q}_\sigma$, $\boldsymbol{G}_\sigma$), the details of the motion of the plate particles in the narrow boundary zones of the plates are not considered in this paper.

This simple model of the plates has been used for different geodynamic studies. These applications have confirmed its effectivness and suitability for a study of the secular motion of the Earth's pole (Barkin, 1995a, b; 1996b), for a study of geometrical and kinematic regularities in plate motions (Barkin, in press, a, b) and

Table 1. Kinematic characteristics of spherical plate motion in the HS2-NUVEL1 theory.

N	*Plate*	$\Omega_{\sigma x}$	$\Omega_{\sigma y}$	$\Omega_{\sigma z}$	Ω_{σ}	$\varphi_{\Omega\sigma}$	$\lambda_{\Omega\sigma}$
1	EA	0.034	0.055	−0.064	0.091	−44°5	58°9
2	AF	0.144	0.013	−0.017	0.146	−6°8	5°3
3	PA	0.001	0.487	−0.846	0.976	−60°1	89°9
4	NoA	0.107	−0.023	−0.259	0.281	−67°1	−102°2
5	SoA	0.030	0.107	−0.301	0.320	−69°7	74°4
6	AN	0.042	0.094	−0.026	0.106	−14°3	66°0
7	AU	0.564	0.506	0.129	0.769	9°7	41°9
8	IN	0.491	0.199	0.158	0.553	16°6	22°1
9	NA	−0.001	−0.320	0.329	0.458	45°8	−90°2
10	AR	0.492	0.165	0.155	0.542	16°7	18°6
11	PH	0.683	−0.245	−0.847	1.116	−49°4	−19°7
12	CO	−0.534	−1.098	0.406	1.287	18°4	−121°4
13	CA	0.081	−0.003	−0.156	0.176	−62°6	−2°0
14	JF	0.392	0.679	−0.544	0.954	−34°8	60°0

of ordered plate positions and motions (Barkin, 1996b), for prediction of the drift of the Earth's mass centre (Barkin, 1996c; 1997), for a study of the global structure and motion of the lithosphere (Barkin, 1996a), and other geodynamic effects.

In accordance with the HS2-NUVEL1 kinematic theory, plates P_σ accomplish a spherical motion which define, for the present geological epoch, angular velocities $\boldsymbol{\Omega}_\sigma$. The modulus of these vectors, $\Omega_\sigma = |\boldsymbol{\Omega}_\sigma|$, the geographical coordinates of their

Table 2. Masses and spherical coordinates of the plate centres of mass.

N	*Plate*	m_σ	$r_{C\sigma}$	$\varphi_{C\sigma}$	$\lambda_{C\sigma}$
1	EA	4.360	0.816	49°3	87°6
2	AF	3.441	0.866	8°3	14°3
3	PA	2.959	0.765	1°8	201°6
4	NoA	2.380	0.865	58°8	267°7
5	SoA	2.176	0.902	−18°6	306°7
6	AN	2.097	0.883	−28°1	125°7
7	AU	2.065	0.888	−85°1	62°8
8	IN	0.447	0.964	18°8	77°2
9	NA	0.415	0.964	−21°0	267°8
10	AR	0.376	0.980	21°8	46°2
11	PH	0.137	0.985	19°1	134°4
12	CO	0.122	0.993	14°0	289°7
13	CA	0.101	0.993	7°8	267°7
14	JF	0.019	0.999	45°0	232°7

Table 3. Axial and centrifugal moments of inertia of the plates.

N	*Plate*	A_σ	B_σ	C_σ	F_σ	E_σ	D_σ
1	EA	1.072	0.805	0.683	−0.071	0.095	0.348
2	AF	0.304	0.869	0.902	0.159	0.020	−0.036
3	PA	0.402	0.678	0.704	0.134	−0.024	0.019
4	NoA	0.653	0.483	0.299	−0.004	−0.043	−0.220
5	SoA	0.461	0.318	0.534	−0.214	−0.093	0.102
6	AN	0.387	0.341	0.456	−0.150	0.123	−0.153
7	AU	0.583	0.535	0.128	0.018	−0.015	−0.041
8	IN	0.163	0.025	0.162	0.034	0.008	0.035
9	NA	0.133	0.024	0.117	0.005	0.001	0.039
10	AR	0.065	0.058	0.087	0.042	0.026	0.027
11	PH	0.027	0.026	0.042	−0.021	−0.010	0.010
12	CO	0.028	0.004	0.029	−0.007	0.002	−0.007
13	CA	0.025	0.001	0.025	0.002	−0.001	−0.004
14	JF	0.002	0.002	0.001	0.001	−0.001	−0.001

poles, i.e., latitude $\varphi_{\Omega\sigma}$, and longitude $\lambda_{\Omega\sigma}$, and the projections $\Omega_{\sigma x}$, $\Omega_{\sigma y}$, $\Omega_{\sigma z}$ with respect to the reference system $Oxyz$ are presented in Table 1. The values of Ω_σ, $\Omega_{\sigma x}$, $\Omega_{\sigma y}$ and $\Omega_{\sigma z}$ are given in units of $^\circ$ my^{-1} and $\varphi_{\Omega\sigma}$, $\lambda_{\Omega\sigma}$ are in degrees.

Let m_s be the mass of the plate P_σ; and let $x_{C\sigma}$, $y_{C\sigma}$, $z_{C\sigma}$ and $r_{C\sigma}$, $\varphi_{C\sigma}$, $\lambda_{C\sigma}$ be the rectangular and spherical coordinates of its centre of mass. Here $r_{C\sigma}$ is the modulus of the radius-vector $\boldsymbol{r}_{C\sigma}$ of the mass centre of the plate C_σ and $r_{C\sigma} = |\boldsymbol{r}_{C\sigma}| = \sqrt{x_{C\sigma}^2 + y_{C\sigma}^2 + z_{C\sigma}^2}$; $\varphi_{C\sigma}$ and $\lambda_{C\sigma}$ are the latitude and the longitude of the point C_σ in the main reference system $Oxyz$. Values of these coordinates have been calculated for the proposed plate model by the trapezoid method (Barkin, 1996c). These values are presented in Table 2.

The calculated values of the axial $(A_\sigma, B_\sigma, C_\sigma)$ and centrifugal $(F_\sigma, E_\sigma, D_\sigma)$ moments of inertia of the plate P_σ $(\sigma = 1, 2, \ldots, 14)$ with respect to the reference system $Oxyz$ are presented in Table 3.

In this paper, we consider the main and medium lithosphere plates for which the following abbreviated notations are used in the tables and in the text of the paper: EA – Eurasian, AF – African, PA – Pacific, NoA – North Amencan, SoA – South Amencan, AN – Antarctic, AU – Australian, IN – Indian, NA – Nasca, AR – Arabian, PH – Philippines, CO – Cocos, CA – Carribean, JF – Juan de Fuca.

The parameters in Tables 2 and 3 are the principal ones for the plates. They are determined by known volume integrals, the approximate values of which were calculated by the method of trapezoids.

In accordance with method of Barkin (1995a, 1996c), every plate was represented as a system of trapezoids and for every trapezoid the integration was performed in a finite form, and the corresponding parameters from Tables 2 and 3 were formed as a result of the summation of these integrals.

The momentum vector $\boldsymbol{q}_s$ of the plate P_s is equal to the product of its mass and the velocity of its centre of mass:

$$\boldsymbol{q}_\sigma = m_\sigma \boldsymbol{v}_{C\sigma} = m_\sigma(\boldsymbol{\Omega}_\sigma \times \boldsymbol{r}_{C\sigma}) \tag{1}$$

and the components of this vector with respect to the mantle reference system are:

$$\begin{aligned} q_{\sigma x} &= m_\sigma(z_{C\sigma}\Omega_{\sigma y} - y_{C\sigma}\Omega_{\sigma z}), \\ q_{\sigma y} &= m_\sigma(x_{C\sigma}\Omega_{\sigma z} - z_{C\sigma}\Omega_{\sigma x}), \\ q_{\sigma z} &= m_\sigma(y_{C\sigma}\Omega_{\sigma x} - x_{C\sigma}\Omega_{\sigma y}). \end{aligned} \tag{2}$$

For vector (1), with respect to the origin of the reference system, its modulus (q_σ) and geographic coordinates of the pole ($\varphi_{q\sigma}$ is the latitude, $\lambda_{q\sigma}$ is the longitude) are given by the formulae:

$$\begin{aligned} q_\sigma &= \sqrt{q_{\sigma x}^2 + q_{\sigma y}^2 + q_{\sigma z}^2}, \\ \sin\varphi_{q\sigma} &= \frac{q_{\sigma z}}{q_\sigma}, \\ \tan\lambda_{q\sigma} &= \frac{q_{\sigma y}}{q_{\sigma x}}. \end{aligned} \tag{3}$$

The vector of the angular momentum $\boldsymbol{G}_\sigma$ of the plate P_σ is

$$\boldsymbol{G}_\sigma = J_\sigma \boldsymbol{\Omega}_\sigma, \tag{4}$$

where

$$J_\sigma = \begin{pmatrix} A_\sigma & -F_\sigma & -E_\sigma \\ -F_\sigma & B_\sigma & -D_\sigma \\ -E_\sigma & -D_\sigma & C_\sigma \end{pmatrix} \tag{5}$$

is the tensor of inertia of plate P_σ with respect to the reference system $Oxyz$, and $\boldsymbol{\Omega}_\sigma$ is the vector of the angular velocity of the plate P_σ in the HS2-NUVEL1 kinematic theory.

The components of vector (4), (5) in the reference system $Oxyz$ are:

$$\begin{aligned} G_\sigma x &= A_\sigma\Omega_{\sigma x} - F_\sigma\Omega_{\sigma y} - E_\sigma\Omega_{\sigma z}, \\ G_\sigma y &= -F_\sigma\Omega_{\sigma x} + B_\sigma\Omega_{\sigma y} - D_\sigma\Omega_{\sigma z}, \\ G_\sigma z &= -E_\sigma\Omega_{\sigma x} - D_\sigma\Omega_{\sigma y} + C_\sigma\Omega_{\sigma z}. \end{aligned} \tag{6}$$

The modulus of the vector and the geographic coordinates of its pole on the spherical surface of the Earth (latitude $\varphi_{G\sigma}$, longitude $\lambda_{G\sigma}$) are:

$$\begin{aligned} G_\sigma &= \sqrt{G_{\sigma x}^2 + G_{\sigma y}^2 + G_{\sigma z}^2}, \\ \sin\varphi_{G\sigma} &= \frac{G_{\sigma z}}{G_\sigma}, \\ \tan\lambda_{G\sigma} &= \frac{G_{\sigma y}}{G_{\sigma x}}. \end{aligned} \tag{7}$$

Table 4. Modulus of the vectors $\boldsymbol{q}_\sigma$ and $\boldsymbol{G}_\sigma$, and the geographic coordinates of their poles.

N	*Plate*	q_σ	$\varphi_{q\sigma}$	$\lambda_{q\sigma}$	G_σ	$\varphi_{G\sigma}$	$\lambda_{G\sigma}$
1	EA	0.320	13°0	342°0	0.106	−38°5	56°4
2	AF	0.131	30°9	279°3	0.047	−22°3	344°0
3	PA	2.160	28°3	110°6	0.702	−59°5	103°9
4	NoA	0.381	−17°6	209°2	0.119	−41°0	310°8
5	SoA	0.626	−15°2	211°4	0.183	−67°0	83°1
6	AN	1.423	59°8	148°9	0.398	9°6	44°8
7	AU	0.184	−0°3	156°1	0.053	1°2	65°3
8	IN	0.190	70°1	237°1	0.076	11°0	346°4
9	NA	0.169	−1°4	358°3	0.055	68°0	272°4
10	AR	0.089	67°3	242°9	0.026	−8°1	323°9
11	PH	0.090	26°7	34°3	0.032	−57°8	72°6
12	CO	0.021	−24°5	206°2	0.005	−61°0	349°1
13	CA	0.055	61°9	12°6	0.012	28°0	173°4
14	JF	0.004	21°4	345°8	0.001	15°3	25°4

Calculated parameters (3) and (7) for this model of the plates and their motion in accordance with the HS2-NUVEL1 theory are presented in Table 4.

In parallel with the main mantle reference system $Oxyz$ we introduce the new inclined reference system $Ox_{\mathrm{L}}y_{\mathrm{L}}z_{\mathrm{L}}$ (lithosphere reference system, LRS). This reference system was established as a result of an analysis of the regularities and details in the plate positions and motions (Barkin, 1996b, c). Its origin coincides with the origin of the main reference system. The orientation of the axes $Ox_{\mathrm{L}}y_{\mathrm{L}}z_{\mathrm{L}}$ with respect to the axes $Oxyz$ is defined by the Eulerian angles:

$$\Psi_{\mathrm{L}} = -3.4°, \quad \Theta_{\mathrm{L}} = 26.7°, \quad \Phi_{\mathrm{L}} = 0.0°.$$

Projections of the vectors $\boldsymbol{q}_\sigma$ and $\boldsymbol{G}_\sigma$ on the coordinate axes of the new reference system are defined by the following transformation formulae:

$$\begin{aligned} a^{\mathrm{L}}_{\sigma x} &= L_{11}a_{\sigma x} + L_{21}a_{\sigma y} + L_{31}a_{\sigma z}, \\ a^{\mathrm{L}}_{\sigma y} &= L_{12}a_{\sigma x} + L_{22}a_{\sigma y} + L_{32}a_{\sigma z}, \\ a^{\mathrm{L}}_{\sigma z} &= L_{13}a_{\sigma x} + L_{23}a_{\sigma y} + L_{33}a_{\sigma z}, \\ a &= (q, G), \end{aligned} \tag{8}$$

where the direction cosines are:

$$\begin{aligned} L_{11} &= \cos\Psi_{\mathrm{L}}\cos\Phi_{\mathrm{L}} - \sin\Psi_{\mathrm{L}}\sin\Phi_{\mathrm{L}}\cos\Theta_{\mathrm{L}}, \\ L_{21} &= \sin\Psi_{\mathrm{L}}\cos\Phi_{\mathrm{L}} + \cos\Psi_{\mathrm{L}}\sin\Phi_{\mathrm{L}}\cos\Theta_{\mathrm{L}}, \\ L_{31} &= \sin\Phi_{\mathrm{L}}\sin\Theta_{\mathrm{L}}, \\ L_{12} &= -\cos\Psi_{\mathrm{L}}\sin\Phi_{\mathrm{L}} - \sin\Psi_{\mathrm{L}}\cos\Phi_{\mathrm{L}}\cos\Theta_{\mathrm{L}}, \end{aligned}$$

Table 5. Modulus of momenta q_σ (1 unit = 10^{-3} mR ° my^{-1}) and of the angular momenta G_σ (1 unit = 10^{-2} ° my^{-1}) of the plate and coordinates of the poles of the vectors $\boldsymbol{q}_\sigma$, $\boldsymbol{G}_\sigma$; $\varphi^L_{q\sigma}$, $\lambda^L_{q\sigma}$ and $\varphi^L_{G\sigma}$, $\lambda^L_{G\sigma}$ (1 unit = 1 degree). (The plate motion is described by the HS2-NUVEL1 theory); ratios q_σ/G_σ and angles α_σ.

N	*Plate*	q_σ	$\varphi^L_{q\sigma}$	$\lambda^L_{q\sigma}$	G_σ	$\varphi^L_{G\sigma}$	$\lambda^L_{G\sigma}$	q_σ/G_σ	α_σ
1	EA	0.3163	20°05	350°07	0.1072	−59°92	30°20	3.0	86°4
2	AF	0.1314	60°27	286°27	0.0473	−11°63	335°24	2.8	81°7
3	PA	2.1681	−0°10	108°04	0.7023	−82°48	181°94	3.1	87°8
4	NoA	0.3808	1°67	213°64	0.1186	−16°45	300°94	3.2	86°9
5	SoA	0.6256	1°45	214°53	0.1832	−82°41	290°90	3.4	89°7
6	AN	0.1841	−11°95	159°11	0.0534	−25°82	62°33	3.4	90°8
7	AU	1.2476	39°86	122°72	0.3678	−14°87	45°32	3.4	90°2
8	IN	0.3212	53°80	122°33	0.1016	16°85	3°13	3.2	92°4
9	NA	0.1687	−0°38	357°84	0.0505	82°67	80°31	3.3	89°4
10	AR	0.0753	76°07	137°01	0.0207	14°29	320°32	3.6	89°6
11	PH	0.0666	6°02	43°12	0.0201	−83°33	26°94	3.3	89°6
12	CO	0.0562	45°45	49°08	0.0168	21°56	162°47	3.3	89°8
13	CA	0.0112	−9°09	214°24	0.0034	−46°78	313°58	3.3	89°7

$$
\begin{aligned}
L_{22} &= -\sin\Psi_{\mathrm{L}}\sin\Phi_{\mathrm{L}} + \cos\Psi_{\mathrm{L}}\cos\Phi_{\mathrm{L}}\cos\Theta_{\mathrm{L}},\\
L_{32} &= \cos\Phi_{\mathrm{L}}\sin\Theta_{\mathrm{L}},\\
L_{13} &= \sin\Psi_{\mathrm{L}}\sin\Theta_{\mathrm{L}},\\
L_{23} &= -\cos\Psi_{\mathrm{L}}\sin\Theta_{\mathrm{L}},\\
L_{33} &= \cos\Theta_{\mathrm{L}}.
\end{aligned}
\tag{9}
$$

The coordinates of the poles of the vectors $\boldsymbol{q}_\sigma$, $\boldsymbol{G}_\sigma$, related to point O, in the lithosphere reference system, are $\varphi^{\mathrm{L}}_{q\sigma}$, $\lambda^{\mathrm{L}}_{q\sigma}$ and $\varphi^{\mathrm{L}}_{G\sigma}$, $\lambda^{\mathrm{L}}_{G\sigma}$. These parameters are given by the formulae:

$$
\sin\varphi^{\mathrm{L}}_{a\sigma} = \frac{a^{\mathrm{L}}_{\sigma z}}{a_\sigma}, \quad \tan\lambda^{\mathrm{L}}_{a\sigma} = \frac{a^{\mathrm{L}}_{\sigma y}}{a^{\mathrm{L}}_{\sigma x}}, \quad (a = q, G). \tag{10}
$$

The values of the characteristics G_σ, $\lambda^{\mathrm{L}}_{G\sigma}$ $\varphi^{\mathrm{L}}_{G\sigma}$ and q_σ, $\lambda^{\mathrm{L}}_{q\sigma}$ $\varphi^{\mathrm{L}}_{q\sigma}$ calculated by means of (8)–(10) are shown in Table 5. In this table the ratios (q_σ/G_σ) of the modulus of the vectors $\boldsymbol{q}_\sigma$, $\boldsymbol{G}_\sigma$ and the angles α_σ between these vectors,

$$
\alpha_\sigma = \arccos\frac{q_{\sigma x}G_{\sigma x} + q_{\sigma y}G_{\sigma y} + q_{\sigma z}G_{\sigma z}}{G_\sigma q_\sigma} \tag{11}
$$

are also presented.

In Fig. 1, the positions of the poles of the vectors $\boldsymbol{q}_\sigma$ and $\boldsymbol{G}_\sigma$ are mapped by circles and squares. Here the continent positions are presented in standard projection but with respect to LRS. The role of the Earth's equator is here played by

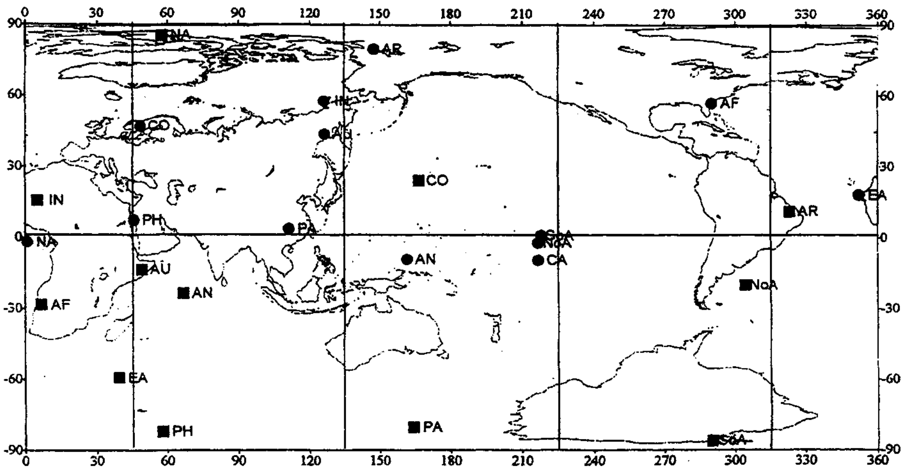

Figure 1 Positions of the poles of the momenta (•) and of the angular momenta (■) of the plates in the lithosphere reference system (HS2-NUVEL1 theory).

the lithosphere equator and the north–south direction corresponds to the orientation of the polar axis LRS to Hudson Bay (to the geographical point 63°3 N, 93°4 W).

As a result of the analysis and interpretation of the obtained values of the dynamic characteristics (1)–(11), some interesting regularities of the plate motion were established.

2 DYNAMIC REGULARITIES OF THE PLATE MOTION

The most important regularity reflects the phenomenon of ordering of the pole positions of the dynamic vector characteristics $\boldsymbol{q}_\sigma$, $\boldsymbol{G}_\sigma$ with respect to the three great circles $\mathrm{E_L}$, O^*_L and M^*_L on the Earth's spherical surface. The planes of these three great circles are mutually orthogonal ($\mathrm{E_L}$ is the lithosphere equator, O^*_L and M^*_L are meridians of the LRS, corresponding to longitudes 45° E and 135° E). The intersections of these planes define the new reference system $Ox_\mathrm{G}y_\mathrm{G}z_\mathrm{G}$ which we will call the principal geodynamic reference system (PGRS). We will describe the above mentioned phenomenon in greater detail.

1. The poles of the vector $\boldsymbol{q}_\sigma$ of the momentum of the plate motion in accordance with the HS2-NUVEL1 theory are situated along three mutually orthogonal great circles $\mathrm{E_L}$, O^*_L and M^*_L on the Earth's surface.

1.1 The poles of the momentum vectors $\boldsymbol{q}_\sigma$ of the plates NA, PH, PA, AN, SoA, NoA, CA, EA are located along the lithosphere equator.

1.2 The poles of the momentum vectors $\boldsymbol{q}_\sigma$ of the plates PH, CO, SoA, NoA, CA are located along the meridian O^*_L of the LRS.

1.3 The poles of the momentum vector $\boldsymbol{q}_\sigma$ of the plates AU, IN, AR, AF are located along the meridian M_L^* of the LRS.

1.4 The poles of the vectors $\boldsymbol{q}_\sigma$ are situated in the equatorial zone of the LRS and un its northern hemisphere.

The last property reflects the global tendency of the plate motion – the displacements of their centres to the northern hemisphere. The positions of the poles of the vectors $\boldsymbol{G}_\sigma$ are characterized by analogous properties and regularities.

2. The poles of the angular momentum vectors $\boldsymbol{G}_\sigma$ of the plate motions in the HS2-NUVEL1 theory are situated along the three great circles E_L, O_L^* and M_L^*.

2.1 The poles of the vectors $\boldsymbol{G}_\sigma$ of the three main plates PA, SoA and PH are situated in the region of the southern pole of the polar axis of the LRS and for the NA plate the pole of the vector $\boldsymbol{G}_\sigma$ is situated in the vicinity of the northern pole.

2.2 The poles of the vectors $\boldsymbol{G}_\sigma$ of the plates IN, AF, AH, AN, CO, CA, AR are situated in the vicinity of the lithosphere equator (on the lithosphere belt).

2.3 The poles of the vectors $\boldsymbol{G}_\sigma$ of the plates EA, AU, PH, NA are situated in the vicinity of the meridian O_L^*.

2.4 The poles of the vectors $\boldsymbol{G}_\sigma$ of the plates AR, NoA, SoA are situated along the meridian M_L^*.

2.5 The poles of the vectors $\boldsymbol{G}_\sigma$ are situated essentially in the vicinity of the lithosphere equator and in the southern hemisphere of the LRS.

The last property reflects the other global tendency in the plate motion – their westward drift or global rotation of the lithosphere as a single body in the western direction with respect to LRS. Figure 1 illustrates these properties.

3. The poles of the vectors $\boldsymbol{q}_\sigma$ and $\boldsymbol{G}_\sigma$ are located at angular distance $\alpha_\sigma = 90°$. This means that the vectors $\boldsymbol{q}_\sigma$, $\boldsymbol{G}_\sigma$ for every plate are orthogonal

$$(\boldsymbol{q}_\sigma, \boldsymbol{G}_\sigma) = 0. \tag{12}$$

The values of α_σ from Table 5 confirm this regularity.

Let $\boldsymbol{G}_{C\sigma}^{(r)}$ be the angular momentum vector of the motion of the plate P_σ with respect to the Cartesian reference system $C_\sigma xyz$ with the origin at the mass centre of this plate and with axes which are parallel to the corresponding axes of the main reference system $Oxyz$. The plates move in accordance with the HS2-NUVEL1 theory. Let $\boldsymbol{G}_{C\sigma}$ be the angular momentum of the plate P_σ with the condition that its mass is concentrated at the mass centre C_σ:

$$\boldsymbol{G}_{C\sigma} = \boldsymbol{r}_{C\sigma} \times \boldsymbol{q}_\sigma,$$

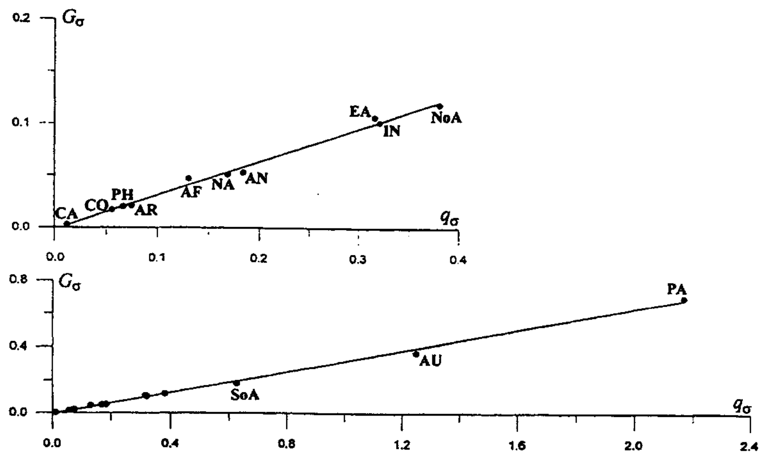

Figure 2 Values of the modulus of the vectors $\boldsymbol{G}_\sigma$, $\boldsymbol{q}_\sigma$.

where $\boldsymbol{r}_{C\sigma}$ is the radius-vector of the mass centre of the plate P_σ, and $\boldsymbol{q}_\sigma$ is the momentum vector.

In accordance with the principle of mechanics

$$\boldsymbol{G}_\sigma = \boldsymbol{G}_{C\sigma} + \boldsymbol{G}^{(r)}_{C\sigma} = \boldsymbol{G}^{(r)}_{C\sigma} + \boldsymbol{r}_{C\sigma} \times \boldsymbol{q}_\sigma, \tag{13}$$

and now taking into account relations (12) and (13) we obtain the formula:

$$(\boldsymbol{G}^{(r)}_{C\sigma} \cdot \boldsymbol{q}_\sigma) = 0. \tag{14}$$

3.1 The vectors of the momentum of the plate P_σ and its angular momentum are mutually orthogonal.

3.2 The vectors $\boldsymbol{G}_\sigma$, $\boldsymbol{G}_{C\sigma}$, $\boldsymbol{G}^{(r)}_{C\sigma}$, $\boldsymbol{r}_{C\sigma}$, $\boldsymbol{\Omega}_\sigma$ are coplanar and are situated in the plane which is orthogonal to the vector $\boldsymbol{q}_\sigma$.

The ratios of the modulus of the momentum and angular momentum vectors for every plate are presented in Table 5. This allows us to formulate the following conclusion.

4. The ratio of the modulus of the momentum and angular momentum vector for the motion of every plate with respect to the mantle reference system is equal to radius of the Earth's surface,

$$\frac{q_\sigma}{G_\sigma} = \frac{1}{R_\oplus} \tag{15}$$

where $R_\oplus$ is the mean radius of the Earth.

Hence, a relation similar to the third Keplerian law for a planetary system of celestial bodies follows.

Table 6. Model and calculated values G_n.

Plate	n	G_n *(law)*	G_n *(theory)*	q_n *(law)*	q_n *(theory)*
CA	−2	0.003	0.003	0.01	0.01
CO, PH, AR	1	0.022	0.02	0.08	0.07
AF, NA, AN	2	0.05	0.05	0.16	0.16
EA, IN, NoA	3	0.09	0.11	0.31	0.34
SoA	4	0.18	0.18	0.63	0.63
AU	5	0.36	0.37	1.25	1.25
PA	6	0.72	0.70	2.51	2.17

4.1 The ratio of the modulus of the momentum vectors for any two plates is equal to the ratio of the modulus of the angular momentum vectors of the same plates:

$$\frac{G_i}{G_j} = \frac{q_i}{q_j}, \quad (i, j \in 1, 2, \ldots, N = 14). \tag{16}$$

The dots in Fig. 2 denote the modulus values (q_σ, G_σ) for the main and medium plates on the coordinate plane q, G. These dots are located along a straight line with definite and fixed inclination. The upper graph of Fig. 2 is the initial portion of the lower general graph on an enlarged scale. This figure gives a geometric illustration for the above-mentioned regularity (16). The progressive increment of the intervals between dots (or their groups) along the drawn straight line is noteworthy. This permits us to obtain the following empirical formula for the modulus of the plate angular momenta:

$$G_n = 2^n G_0, \quad G_0 = 0.0112 \times 10^{-2} \ (\mathrm{C}\ ^\circ\ \mathrm{my}^{-1}). \tag{17}$$

The theoretical values of the modulus of the angular momenta, calculated with formula (17), and their modulus obtained on the basis of the data of Table 5, are presented in Table 6.

Taking into account the unavoidable errors contained in the calculated parameters, we can say that the accuracy of relationship (17) is high and this is, probably, not accidental.

In view of property (16), an analogous relation holds for the modulus of the plate momentum:

$$q_n = 2^n q_0, \quad q_0 = 0.0392 mR \times 10^{-3} \ (^\circ\ \mathrm{my}^{-1}). \tag{18}$$

The values q_n, calculated with formula (18) for definite numbers n, is in good agreement with the model values of q_n for individual plates or their groups (for groups of plates the definite mean values q_n are used).

The empirical relations (17), (18) by their form resemble the Titius–Bode law of planetary distances (Neto, 1976).

The regularities established indicate that in the processes of fragmentation, integration of the plates, and their evolution on geological intervals of time, the modulus of the vectors $\boldsymbol{G}_\sigma$ and $\boldsymbol{q}_\sigma$ of the plate motion with respect to the mantle reference system have values close to definite discrete values.

Relationships (17), (18) mean that the modulus of the angular momenta of the plates are rationally commensurable with some standard value of the angular momentum

$$G_{\mathrm{PA}} = 2G_{\mathrm{AU}} = 4G_{\mathrm{SoA}} = 8G_{(\mathrm{EA,\ IN,\ NoA})} = 16G_{(\mathrm{AF,\ NA,\ AN})} = 32G_{(\mathrm{CO,\ PH,\ AR})}.$$

Analogous relations obviously hold for the modulus of plate moments:

$$q_{\mathrm{PA}} = 2q_{\mathrm{AU}} = 4q_{\mathrm{SoA}} = 8q_{(\mathrm{EA,\ IN,\ NoA})} = 16G_{(\mathrm{AF,\ NA,\ AN})} = 32G_{(\mathrm{CO,\ PH,\ AR})}.$$

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References

Barkin, Yu. V. (1995a) *Vestnik Mosk State University, Ser. 3. Phys Astron.* **36**, No. 6, 89 (in Russian).
Barkin, Yu. V. (1995b) *Proceedings of the International Conference 'Earth Rotation Reference Systems in Geodynamics and Solar Systems' (Warsaw, Poland, 18-20 September, 1995)*, SRC, RAS, Warsaw, Poland, p. 159.
Barkin, Yu. V. (1996a) *Abstracts First All-Russian conference on 'Interaction in Lithosphere–Hydrosphere–Atmosphere System' (Moscow, 28–29 November, 1996)*, MSU, p. 22 (in Russian).
Barkin, Yu. V. (1996b) *Abstracts Russian Conference on 'Modern Problems and Methods of Astrometry and Geodynamics' (St. Peterburg, 23–27 October, 1996)*, IPA RAN, p. 130.
Barkin, Yu. V. (1996c) *Vestnik Mosk State University, Ser. 3. Phys Astron.* **37**, No. 1, 79 (in Russian).
Barkin, Yu. V. (1997) *Eos Trans. AGU* **78**, No. 46, Fall Meet. Suppl., F145.
Barkin, Yu. V. (in press, a) *Astron. Astrophys. Trans.*
Barkin, Yu. V. (in press, b) *Astron. Astrophys. Trans.*
Gripp, A. E. and Gordon, R.G. (1990) *Geophys. Res. Lett.* **17**, 1109.
Neto, M. M. (1976) *Titius-Bode law. History and Theory*, Mir, Moscow.