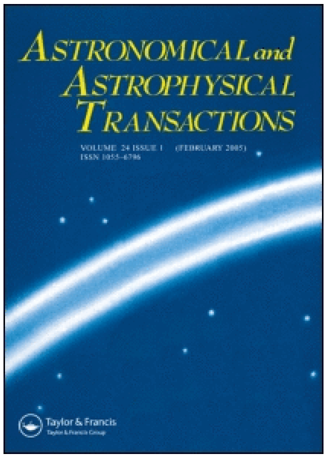

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Damping of gravitational waves and density perturbations in the early Universe

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DAMPING OF GRAVITATIONAL WAVES AND DENSITY PERTURBATIONS IN THE EARLY UNIVERSE

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Since the discovery of the large angular scale anisotropies in the microwave background radiation, the behaviour of cosmological perturbations (especially, density perturbations and gravitational waves) has been of great interest. In this study, after a detailed and rigorous treatment of the behaviour of gravitational waves in viscous cosmic media, we conclude that the damping of cosmological gravitational waves of long wavelengths is negligible for most cases of physical interest. A preliminary analysis suggests that similar results hold for density perturbations in the long wavelength limit. Therefore, long wavelength cosmological perturbations have not been practically affected by viscous processes, and are good probes of the very early Universe.

KEY WORDS Shear viscosity, damping, viscous cosmic media, amplitude of gravitational waves, density perturbations

1 INTRODUCTION

A likely explanation of the observed large angular scale anisotropy of the cosmological microwave background radiation (CMBR) is cosmological perturbations of quantum mechanical origin (Grishchuk, 1993), mainly gravitational waves (Grishchuk, 1994). However, one could argue that these perturbations are not responsible for the CMBR anisotropy if they could be washed out by viscous processes. As a first step, one should investigate whether, in a classical approach, the amplitude of such perturbations decreases significantly or not. This is the point of this study. We have dealt mainly with gravitational waves in several viscous cosmic media, possessing only shear viscosity. One could refer to some standard sources, like Hawking (1966), Weinberg (1972) and Grishchuk and Polnarev (1980). We adopted a technique previously used by Weinberg (1972). Several particular models for viscous cosmic media were from Mendez *et al.* (1997).

The assumptions used in the study of the early Universe can be found in standard textbooks, for example Weinberg (1972). Our study indicated that the zero chemical potential approximation should be reconsidered by the end of the radiation dominated era, in order to get a correct decoupling time between matter and radiation, and the hydrogen and baryon abundances should be taken into account. We neglect the bulk viscosity.

Information concerning transport phenomena can be found in Landau and Lifshitz (1966), Tabor (1970) and Reif (1965). We will only include the information necessary for our study. By "viscosity" we will mean specifically shear viscosity. If τ_i is the mean free time between collisions, $\epsilon_{t.i.}$ is the thermal energy density of the particles of the i species responsible for momentum transfers, then the shear viscosity coefficient is

$$\xi = \theta \sum_i \epsilon_{t.i} \tau_i \tag{1}$$

where θ is a numerical constant.

Elementary information required about local thermodynamical equilibrium, dissipation and decoupling are presented in Table 1 (Weinberg, 1972; Kolb and Turner, 1990). We should mention though that these conditions are a consequence of the constancy of the entropy per comoving volume element. We define a characteristic Hubble time $T_H \equiv \mathcal{R}/\dot{\mathcal{R}}$ were $\mathcal{R}$ is the scale factor. η is conformal time, a dot is differentiation with respect to t, while a prime is with respect to η.

Table 1. Local thermal equilibrium, dissipation and decoupling

$\tau \ll T_H$	Perfect thermal equilibrium holds. Dissipation is negligible
$\tau \leq T_H$	Departures from thermal equilibrium begin. At the equality sign, decoupling occurs and this interaction is no longer realized. Dissipation becomes important, until of course decoupling occurs and the interaction ceases
$\tau > T_H$	Decoupling has occurred, and no dissipation takes place since the interaction has stopped

2 THE BEHAVIOUR OF GRAVITATIONAL WAVES IN A NON-VISCOUS MEDIUM

Lifshitz pointed out how the different types of perturbations can be constructed in the form of scalar, vector and tensor harmonics, corresponding to density perturbations, rotational perturbations and gravitational waves respectively (see, for example, Lifshitz and Khalatnikov, 1963). The perturbed Einstein equations are given in Weinberg (1972). For gravitational waves, h_i^j denotes the time-dependent part of the perturbations and q the constant wavenumber of the perturbation, related to the time-dependent wavelength $\lambda(t)$ by the relation $q \equiv \frac{2\pi\mathcal{R}}{\lambda}$. Since all the h_i^j components obey the same equations, we will ignore the indices in our notation and refer to a single component as h. This equation has been transformed and interpreted as a parametrically excited oscillator (Grishchuk, 1993). The $\mu(\eta)$ amplitude is related to the $h(\eta)$ amplitude by $h(\eta) = \mu(\eta)\mathcal{R}(\eta)^{-1}$. The $\mathcal{R}^{-1}$ variation of h reflects the adiabatic decrease of h. The perturbations interact with the background time-dependent gravitational field, which supplies energy to waves with wavelengths that satisfy the parametric amplification condition. In the case of gravitational waves, the interaction potential $U(\eta) \boxminus \frac{\mathcal{R}''(\eta)}{\mathcal{R}(\eta)}$ represents the background gravitational field. In order for this interaction to take place, the frequency of the wave must be comparable with that of the variations of the background field. Depending on the wavelengths of the perturbations, the behaviour of μ and h is as described in Table 2. Therefore, gravitational waves interact parametrically with the background gravitational field, and the ones longer than the Hubble radius are "superadiabatically" amplified. The quantum treatment of this phenomenon and its implications for the CMBR statistics and anisotropy is investigated in Grishchuk (1993).

Table 2. Amplitude of gravitational waves in the nonviscous case

$q \gg U(\eta)$	In this overbarrier region, the oscillatory solutions experience only adiabatic decrease, since no interaction with the barrier takes place. The expressions for the amplitudes are $\mu(\eta) = C_1 \exp\{\pm iq\eta\}$ and $h(\eta) = C_1\mathcal{R}^{-1}\exp\{\pm iq\eta\}$.
$q \ll U(\eta)$	In this underbarrier region, the solutions are practically constant. These long wavelengths do not suffer from adiabatic decrease: due to their interaction with the barrier they are amplified ("superadiabatic amplification"). The expressions for the amplitudes are $\mu(\eta) = C_b\mathcal{R} + C_a\mathcal{R}\int \mathcal{R}^{-2}\, d(\eta) \simeq C_b\mathcal{R}$ and $h(\eta) = C_b + C_a \int \mathcal{R}^{-2}\, d(\eta) \simeq C_b$.

3 THE BEHAVIOUR OF GRAVITATIONAL WAVES IN THE PRESENCE OF A SHEAR VISCOSITY

In order to put the equations of propagation for gravitational waves in a viscous medium in a form that reveals the underlying physics, we will first derive an expression for shear viscosity. We begin from equation (1). It is obvious that species with $\epsilon_{t.i}\tau_i$ several orders of magnitudes smaller than those of the other constituents of the fluid will not participate. We will make some assumptions. The first is that in the radiation dominated era, the thermal energy density of a species is approximately equal to its total energy density: $\epsilon_{t.i} \simeq \epsilon_i$. The second assumption is that the ϵ_i

of the different species are of the same order of magnitude. We define the number $\kappa_i \boxminus \epsilon_i/\epsilon$, where epsilon is the total energy density of the fluid. The different κ_i will be of the same order of magnitude, and will all be denoted by κ. Therefore, (1) reduces to

$$\xi = \theta\left(\sum_i \kappa_i \tau_i\right)\epsilon. \tag{2}$$

Obviously, the quantity $\bar{\tau} \equiv \sum_i \kappa_i \tau_i$ is the mean free time of the fluid, and will be at most of order of the largest τ_i. The third assumption is that the τ_i of the species contributing to ξ are of the same order of magnitude and equal to τ. Then, (1) becomes

$$\xi = \theta\psi\kappa\epsilon\tau. \tag{3}$$

The fourth assumption is that the number of species contributing to viscosity is small: $\psi \sim 1$.

Following Weinberg (1972), one finds that in the presence of a cosmic medium possessing shear viscosity the propagation equations for the gravitational wave amplitude is

$$\ddot{h} + (3T_H^{-1} + 6\theta\psi\kappa T_H^{-2}\tau)\dot{h} + \left(\frac{cq}{\mathcal{R}}\right)^2 h = 0 \tag{4}$$

where we have used (3) and expressed ϵ in terms of T_H from the unperturbed Einstein equations. Equation (4) shows that a further (above adiabatic damping) decrease of the amplitude arises due to shear viscosity. This equation in terms of conformal time and for the μ amplitude is

$$\mu'' + 6\theta\psi\kappa A^2 \frac{c\tau}{\mathcal{R}}\mu' + (q^2 - V(\eta))\mu = 0 \tag{5}$$

where $A \equiv \frac{\mathcal{R}'}{\mathcal{R}}$ and the time-dependent potential is $V(\eta) = U(\eta) + 6\theta\psi\kappa A^3 \frac{c\tau}{\mathcal{R}}$. We put (5) into the form of the Schroedinger equation by introducing the function m:

$$\mu = m \exp\{-\int 3\theta\psi\kappa A^2 \frac{c\tau}{\mathcal{R}} d(\eta)\} \tag{6}$$

Then, equation (5) becomes

$$m'' + (q^2 - Y(\eta))m = 0 \tag{7}$$

with $Y(\eta) = F' + F^2$ where $F = A + 3\theta\kappa A^2 \frac{c\tau}{\mathcal{R}}$. This is again the equation for a parametrically excited oscillator, but with a modified potential due to shear viscosity.

It is known that the dissipation is expected to be negligible when the mean free time between collisions is much less than the Hubble time. This is a consequence of the constancy of entropy within a comoving volume element, which yields the relation $\mathcal{R} \propto T^{-1}$. The expansion rate T_H^{-1} determines the rate of temperature change: particles that are in thermal equilibrium should have an interaction rate greater than the rate of temperature change. For $\tau \propto T^{-q}$, like the ones we will consider, when $\tau \geq T_H$, a particle will interact less than once, so this species will drop out of equilibrium.

When $\tau < T_H$, the viscous term participating in the cofactor of $\dot{h}$ in (4) is much less than the expansion term. Even in the case of non-negligible viscosity, the viscous term can never become larger than the expansion term, because otherwise the condition of thermal equilibrium (not necessarily perfect thermal equilibrium) (see Table 1) will be violated. This means that decoupling will occur and the dissipative mechanism under consideration will cease to function as such. Significant dissipation is expected to occur at those times when the viscous term becomes comparable to the expansion term: then, we have large departures from perfect thermal equilibrium.

Let $\hat{h}$ be a solution in the absence of viscosity and $\tilde{h}$ a solution of (4), that is, in the presence of viscosity. A measure of the dissipation can be presented as

$$Z = \frac{\hat{h} - \tilde{h}}{\hat{h}}. \tag{8}$$

We will now consider short and long wavelengths separately.

3.1 Solution for Short Wavelengths

A solution of (4) in this limiting case of short waves is (Weinberg, 1972):

$$\tilde{h} = \hat{h} \exp\{-\int 3\theta\psi\kappa T_H^{-2}\tau\, dt\}. \tag{9}$$

This solution is applicable under the condition $T_H \ll \frac{cq}{\mathcal{R}}$, that is, when the relevant waves are well inside the (time dependent) Hubble radius. This result can be obtained from (7) by neglecting the potential $Y(\eta)$. Combining (8) with (9), one derives for this limiting case the damping

$$Z = 1 - \exp\{-\int 3\theta\psi\kappa T_H^{-2}\tau\, dt\}. \tag{10}$$

There is no a priori reason for this damping to be much smaller than 1; nevertheless, it will be at most of order unity, otherwise, the condition of thermal equilibrium will be violated.

3.2 Solution for Long Wavelengths

This is the limit which, apparently, was not considered before for viscous matter. In this case, wavelengths are much longer than the Hubble radius in the era under consideration and, therefore, the q^2 term in (7) can be neglected. We have found a solution of equations (4) and (7) in the form

$$\tilde{h} = C_1 + C_2 \int \mathcal{R}^{-3}(\exp\{-2\int 3\theta\psi\kappa T_H^{-2}\tau\, dt\})\, dt. \tag{11}$$

The solution in the non-viscous case is $\hat{h} = C_1 + C_2 \int \mathcal{R}^{-3}\, dt$ so viscosity affects only the "decaying" second term, but not the constant ("growing") first term. Even in the absence of viscosity, the "decaying" term is much smaller than the "growing" one. Thus, since the "decaying" term affected by viscosity in $\tilde{h}$ is smaller, or, at most, equal to the "decaying" term in $\hat{h}$, it will also be much smaller than the (unaffected) constant term. Therefore,

$$\tilde{h} \approx C_1 \approx \hat{h}.$$

This means that the viscosity does not practically affect the amplitude of waves with wavelengths longer than the Hubble radius. Of course, the same holds for the wavelengths that are longer than the Hubble radius today.

This is a central result of our study, because these long wavelengths are responsible for the large angular scale CMBR anisotropy. In this case, we derive for the absorption the expression

$$Z = \frac{C_2(\int \mathcal{R}^{-3}\, dt - \int \mathcal{R}^{-3}(\exp\{-2\int 3\theta\psi\kappa T_H^{-2}\tau\, dt\})\, dt)}{C_1 + C_2\int \mathcal{R}^{-3}\, dt}. \tag{12}$$

As we have already argued, this is always much smaller than 1.

4 DAMPING OF SHORT WAVES IN VARIOUS VISCOUS COSMIC FLUIDS

To derive concrete numbers, our strategy is to compare τ with T_H and then calculate Z defined by (8). T_H was taken as $T_H = 2t$, and τ, T_H, and Z were expressed as functions of the temperature T. We are dealing with radiative fluids, where viscosity arises due to the failure of perfect thermal equilibrium between matter and highly relativistic particles, like photons and neutrinos. The mean free time was calculated as the inverse of the product of the velocity of light with the relevant cross-section, σ, and with the particle density n of the particles of matter which interact with the radiation: $\tau = (\sigma n c)^{-1}$. In the case of a quark–gluon plasma, the expression of the mean free time was taken from Thoma (1991). All the scenarios are realized in the radiation dominated era. t_{pl} denotes the Planck time, m_{pl}, m_p, m_e, m_μ the Planck, proton, electron, and muon masses, respectively, α the fine structure constant, k the Boltzmann constant, while L is the product of the reduced Hubble parameter with the baryon density parameter: $L = \Omega_B h_r^2$. h_r is a dimensionless number between 0.4 and 1 that represents the uncertainty to the observed value of T_H^{-1} due to systematic errors. Ω_B is the ratio of the baryon density of the Universe over the critical density. We have adopted for L the value 2.5×10^{-2}.

The time–temperature relation is given by

$$t = 0.3(g_*)^{-1/2}\left(\frac{m_{pl}c^2}{kT}\right)^2 t_{pl} \tag{13}$$

(see Kolb and Turner, 1990) where g_* denotes the relativistic degrees of freedom (number of effectively massless degrees of freedom, $mc^2 \ll kT$), and varies with time.

4.1 Quark–gluon Plasma

According to Mendez *et al.* (1997), the period of interest is $(10^{27} \geq T \geq 10^{24})$ K. Taking the expression of the mean free path from Thoma (1991), the mean free time of quarks is given by

$$\tau_q = 3.8 \times 10 \frac{m_{pl} c^2}{kT} t_{pl} \tag{14}$$

reducing to the expression

$$\tau_q = 1.44 \times 10^{-20} \frac{10^{10} K}{T} s. \tag{15}$$

Similarly, for gluons

$$\tau_g = 1.3 \times 10 \frac{m_{pl} c^2}{kT} t_{pl} \tag{16}$$

reducing to the expression

$$\tau_g = 4.83 \times 10^{-21} \frac{10^{10} K}{T} s. \tag{17}$$

The relation between time and temperature reduces to

$$t = 3.2 \times 10^{-1} \left(\frac{10^{10} K}{T} \right)^2 s. \tag{18}$$

g_* is 106.75 for the quark–gluon plasma era. By considering three generations of quarks and eight kinds of gluons, we found $\kappa = 10^{-2}$, $\psi = 14$ and used $\tau \sim 10^{-20} \frac{10^{10} K}{T} s$. We took $\theta = 4/15$ after consulting Thoma (1991). Then, $Z \sim 4.13 \times 10^{-5}$. Equation (3) gives a shear viscosity coefficient $\xi = 7 \times 10^{-25} K^{-3} T^3$ g cm^{-1} s^{-1}, while the viscosity coefficient resulting from a straightforward use of the expression of ξ found in Thoma (1991) is $\xi = 4 \times 10^{-24} K^{-3} T^3$ g cm^{-1} s^{-1}. There is good agreement (the ratio of the shear viscosity coefficient of Thoma (1991) to our shear viscosity coefficient is at most 5). The absorption due to this mechanism is small, but there is a significant departure of our result from the one given in Mendez *et al.* (1997), which is $Z \sim 10^{-7}$. This can be explained from the fact that in Mendez *et al.* (1997) the shear viscosity coefficient used is given as $\xi = 1.88 \times 10^{-26} K^3 T^3$ g cm^{-1}s^{-1}. For the derivation of the latter, Mendez *et al.* (1997) also quote Thoma (1991), and they use the same assumption as we do.

An interesting feature of this absorption mechanism is that the quantity $AT_H^{-2}\tau$ (thus, the shear viscosity coefficient) is proportional to T^3, decreasing with time! This is the only considered medium having this behaviour. Large deviations from perfect thermal equilibrium occur in the beginning of this era and not its end, unlike the other considered media. Those deviations decrease and the fluid approaches perfect thermal equilibrium in the course of time. Viscosity is important towards the beginning of this era, when the density is large and the mean free time is small.

As can be seen the density and the mean free time are the parameters to determine viscosity. In the course of time, the decrease of density is accompanied by the increase of the mean free time. Depending on the fluid, either one or the other primarily determine the viscosity. Since viscosity in the quark–gluon plasma is greater at large densities, one expects that the collisions that cause the transport of momentum do not involve great momentum transfer between particles, but frequently take place and cause the dissipation to occur.

4.2 Electron-Neutrino Mixture

Two kinds of neutrinos are of main concern for this model, the muon and the electron types, and their antineutrinos. Muon-type neutrinos decoupled when the muons annihilated at $T \simeq 1.2 \times 10^{11}$ K, since their reaction rate is sensitive to the presence of muons (Weinberg, 1972). Electron-type neutrinos decoupled later, when $T \simeq 10^{10}$ K (time of electron–positron annihilation) because their reaction rate is sensible to the presence of electrons. Concerning electron and muon-type neutrinos and antineutrinos, electrons, positrons and photons to contribute in the energy density of this fluid, $\kappa_\nu \equiv \kappa_{\nu_e} = \kappa_{\bar{\nu}_e} = \kappa_{\nu_\mu} = \kappa_{\bar{\nu}_\mu} = 10^{-1}$. $\theta = 4/15$, in agreement with Weinberg (1971).

The period of interest is $(10^{12} \geq T \geq 10^{10})$ K. Taking into account the cross-section of weak interactions and the electron density (de Groot *et al.*, 1980), the mean free time of electron-type neutrinos and antineutrinos is given by

$$\tau_{\nu_e} = 2.8 \times 10^{11} \left(\frac{m_{pl}}{m_p}\right)^4 \left(\frac{m_{pl}c^2}{kT}\right)^5 t_{pl} \tag{19}$$

reducing to

$$\tau_{\nu_e} = \left(1.47 \times 10 \frac{10^{10}K}{T}\right)^5 s. \tag{20}$$

For muon-type neutrinos and antineutrinos,

$$\tau_{\nu_\mu} = 3.9 \times 10^{10} \left(\frac{m_{pl}}{m_p}\right)^4 \left(\frac{m_{pl}c^2}{kT}\right)^5 \exp\left\{\frac{m_\mu c^2}{kT}\right\} t_{pl} \tag{21}$$

reducing to

$$\tau_{\nu_\mu} = 2.1 \times 10^{10} \left(\frac{10^{10}}{T}\right)^5 \exp\left\{\frac{1.23 \times 10^{12}K}{T}\right\}. \tag{22}$$

The time–temperature relation is

$$t = 1.1 \left(\frac{10^{10}K}{T}\right)^2 s. \tag{23}$$

The ratio $\tau_{\nu_\mu}/\tau_{\nu_e} = 1.41 \times \exp\{1.23 \times 10^{12}K/T\}$ increases rapidly in the period $(10^{12} \geq T \geq 1.2 \times 10^{11})$ K in a range of half up to three orders of magnitude. Therefore, we shall study the behaviour of the fluid in two subperiods:

4.2.1 Subperiod $(10^{12} \geq T \geq 1.2 \times 10^{11})$ *K*

Due to the rapid increase of $\tau_{\nu_\mu}/\tau_{\nu_e}$, it is more convenient to express the shear viscosity coefficient through (13) and compare the average mean free time

$$\bar{\tau} = 2\kappa_\nu \tau_{\nu_e} \left(1 + 1.41 \exp\left\{\frac{m_\mu c^2}{kT}\right\}\right) \tag{24}$$

with T_H. The contribution of muon-type neutrinos and antineutrinos to viscosity is greater than that of their electron-type counterparts. This is due to the faster growth of τ_{ν_μ} in this subperiod. The shear viscosity coefficient becomes $3.17 \times 10^{35} K(1 + 1.41 \exp\{1.23 \times 10^{12} K/T\}) T^{-1}$ g cm^{-1}s^{-1}.

4.2.2 Subperiod $(1.2 \times 10^{11} \geq T \geq 10^{10})$ *K*

From now on, only electron neutrinos and antineutrinos contribute to the shear viscosity. We use (3) with $\psi = 2$ and the mean free time τ_{ν_e}. The resulting shear viscosity is $3.17 \times 10^{35} K T^{-1}$ g cm^{-1}s^{-1}, in good agreement with the shear viscosity resulting from de Groot *et al.* (1980), which is $2.68 \times 10^{35} K T^{-1}$ g cm^{-1}s^{-1}.

Dissipation is $Z \sim 4.11 \times 10^{-2}$ from both periods. The muon neutrino and antineutrino contribution turns to be of the same order, but slightly larger than that of their electron-type counterparts. This is due to the faster growth of τ_{ν_μ} being compensated by the shorter time period of contribution of the muon-type neutrinos and antineutrinos to viscosity. We are not aware of anyone having reached a similar or a contradicting result. Neglecting the muon neutrino and antineutrino contribution leads to $Z_{\nu_e} \sim 6.19 \times 10^{-3}$.

Mendez *et al.* (1997) use for the shear viscosity coefficient the expression $\xi = 8.79 \times 10^{33} K T^{-1}$ g cm^{-1} s^{-1} (they refer to de Groot *et al.* (1980) as the source of their derived shear viscosity coefficient as well). This can explain a departure of two orders of magnitude between their result and ours, but cannot justify the actual departure of seven orders of magnitude. Indeed, $Z \sim 5 \times 10^{-9}$ in Mendez *et al.* (1997). These authors have not considered the contribution of the muon neutrinos, but this cannot explain the difference.

4.3 Thomson Scattering

The period of interest is $(10^9 \geq T \geq 3 \times 10^3)$ K (Padmanabham, 1993). The mean free time in Mendez *et al.* (1997) is a result of the zero chemical potential approximation (also applied to the previous mechanisms). Nevertheless, in these stages of the radiation dominated era, prior to recombination, this assumption seems to be not valid, and the hydrogen and baryon abundances should be taken into account. Indeed, setting this mean free time equal to the Hubble time, one will reach the result that decoupling occurred at some temperature between 10^9 K and 10^8 K. Following up to a point Kolb and Turner (1990) (the Thomson cross-section and number density were taken from Weinberg, 1972), we adopted the mean free time

$$\tau = \frac{C_1 S_1^{3/2} e^{S_2}}{-1 + (1 + C_2 S_3^{-3/2} e^{S_2})^{1/2}} t_{pl} \tag{25}$$

with $C_1 = 7.56\alpha^{-2}\left(\frac{m_e}{m_{pl}}\right)^{1/2}$, $C_2 = 10^{-7}$, $S_1 = \frac{m_{pl}c^2}{kT}$, $S_2 = \frac{(m_p + m_e - m_H)c^2}{kT}$, $S_3 = \frac{m_e c^2}{kT}$. (25) reduces to

$$\tau \approx \frac{4.2 \times 10^{-2} K^{3/2} (T)^{-3/2} \exp\left(\frac{1.6\times 10^5 \text{ K}}{T}\right)}{-1 + (1 + 2.2 \times 10^{-22} K^{-3/2} (T)^{3/2} \exp\left(\frac{1.6\times 10^5 \text{ K}}{T}\right)^{1/2}} s. \tag{26}$$

For $(3.4 \times 10^3 > T \geq 3 \times 10^3)$ K, our mean free time reduces to $\tau = 2.8 \times 10^9 K^{9/4} T^{-9/4} \exp\{7.9 \times 10^4 K T^{-1}\} s$, in good agreement to $\tau = 8.7 \times 10^9 K^{9/4} T^{-9/4} \exp\{8 \times 10^4 K T^{-1}\} s$ which holds for temperatures close to photon decoupling (Padmanabham, 1993).

The time–temperature relation reduces to

$$t \approx 2\left(\frac{10^{10} \text{ K}}{T}\right)^2 s. \tag{27}$$

Absorption is more significant towards the end of the period of consideration ($T < 3.6 \times 10^3$ K), when departures from perfect thermal equilibrium become large. The main contribution is from this subperiod: $\kappa = 7 \times 10^{-1}$, $\psi = 1$ and $\theta = 4/15$. We have calculated $Z \sim 7.14 \times 10^{-4}$. Our result agrees with that of Mendez *et al.* (1997), who give $Z \sim 2 \times 10^{-3}$. We cannot explain this agreement in results, since we have adopted completely different assumptions. We abandon the zero chemical potential approximation, and in Mendez *et al.* (1997) this mechanism is considered only up to 4×10^3 K. Their adoption of zero chemical approximation gives physically wrong results (decoupling of photons between 10^9 K and 10^8 K, and also a damping due to Thomson scattering six orders of magnitude greater than for Compton scattering, even though the latter is a more efficient thermalizing mechanism). Besides, their shear viscosity coefficient $8 \times 10^{-19} K^{-5/2} T^{5/2} \exp\{6.02 \times 10^9 K T^{-1}\}$ could never lead to a result such as theirs, because the exponential blows up towards the end of the era of applicability of Thomson scattering, even within the temperature range adopted in Mendez *et al.* (1997).

4.4 Compton Scattering

The applicability of this mechanism is within $(10^9 \geq T \geq 5.8 \times 10^4)$ K. Using the cross-section given in Padmanabham, the mean free time turns out to be

$$\tau = \frac{C_3 S_1^{5/2} e^{S_2}}{-1 + (1 + C_2 S_3^{-3/2} e^{S_2})^{1/2}} t_{pl} \tag{28}$$

where $C_3 = 7.56\alpha^{-2}\left(\frac{m_e}{m_{pl}}\right)^{3/2}$. This expression is greatly simplified to

$$\tau \approx 2 \times 10^{30} K^4 T^{-4} s \tag{29}$$

after a Taylor expansion of the square root in the denominator. This is in good agreement to $\tau = 2.17 \times 10^{31} K^4 T^{-4} s$ (Padmanabham, 1993). Our shear viscosity coefficient is $\xi = 5.6 \times 10^{14}$ g cm^{-1}s^{-1}, roughly two orders of magnitude smaller than the $\xi = 4.37 \times 10^{16}$ g cm^{-1}s^{-1} derived by using the mean free time of Padmanabham (1993). The result is $Z \sim 6 \times 10^{-2}$, presenting a departure of seven orders of magnitude from the $Z \sim 10^{-9}$ of Mendez *et al.* (1997). This can be explained by their use of $\xi = 1.72 \times 10^5$ g cm^{-1} s^{-1} (they quote Padmanabham, 1993).

The damping due to this mechanism is two orders of magnitude greater than that of Thomson scattering. This is in agreement with the fact that Compton scattering is a much more efficient thermalizing mechanism than Thomson scattering (Padmanabham, 1993).

5 DAMPING OF DENSITY PERTURBATIONS

The treatment of density perturbations in the non-viscous case (for example Grishchuk, 1994) shows that they interact with the background gravitational field in the same manner as gravitational waves. Since one expects that viscosity will always act against the deformination of the medium produced by a perturbation independently of the nature of the perturbation, one expects that density perturbations should have a behaviour similar to gravitational waves in the presence of viscosity as well. In fact, this should be true for a certain range of wavelengths, since we know that density perturbations of smaller wavelengths are washed away. But in the longer wavelength limit one could still expect that the behaviour of density perturbations and gravitational waves should be quite similar. Of course, this issue requires a more rigorous treatment, following the same steps we have already performed for gravitational waves.

6 DISCUSSION OF RESULTS

The main result of this study (Table 3) has been that gravitational waves which are longer than the Hubble radius today are not practically affected by dissipation, and, consequently, there has been no damping in their amplitude. Thus, these modes are good candidates for the production of the CMBR large angular scale anisotropy, although a study at the quantum level is required in order to investigate whether viscosity affects more delicate properties such as squeezing.

For shorter waves, the picture is different. These modes have been affected by damping more severely than the longer ones. The absorptions we calculated for shorter wavelengths are given in the table of results. Compton scattering is the most

Table 3. Table of results

Viscous cosmic medium	*Era of Application*	*Damping*
Quark–gluon plasma	$(10^{27} \geq T \geq 10^{24})$ K	$Z \sim 1.55 \times 10^{-4}$
Electron–neutrino mixture	$(10^{12} \geq T \geq 10^{10})$ K	$Z \sim 4.11 \times 10^{-2}$
Thomson scattering	$(10^{9} \geq T \geq 3.1 \times 10^{3})$ K	$Z \sim 7.14 \times 10^{-4}$
Compton scattering	$(10^{9} \geq T \geq 5.8 \times 10^{4})$ K	$Z \sim 6.14 \times 10^{-2}$

efficient damping mechanism, Thomson scattering and viscosity due to electron neutrino mixture are of comparable efficiency, while the effect of the quark–gluon plasma in the damping of gravitational waves is much smaller. In the electron–neutrino mixture, it seems that the muon-type neutrinos and antineutrinos play a slightly more important role than the electron-type ones (they both give the same order of magnitude damping, with the numerical coefficients slightly in favour of muon-type neutrinos and antineutrinos).

Thus, the more time a wave has been within the Hubble radius, the more damped will its amplitude be. The ones longer than today's Hubble radius are not affected at all.

There is a large departure of our results from those of Mendez *et al.* (1997), possibly due to numerical disagreements. Our estimates of damping are several orders of magnitude higher than those of Mendez *et al.* According to these authors, these results are upper limits of damping, and are given in Table 4. In the case of Thomson scattering, they use the zero chemical potential approximation, which as we have shown should be reconsidered, and apply this mechanism for a shorter period of time. In the case of other mechanisms, the source of our disagreement lies elsewhere. We have used the same references for the expressions of mean free paths, cross-sections and number densities, and the same assumptions. Their approach is much more complicated than ours: they set the Einstein equations in quasi-Maxwellian form and retain in their formulae the shear viscosity coefficient. Then, they solve the resulting differential equations either analytically or numerically. If one calculates the shear viscosity coefficients directly from the references, these turn out to be several orders of magnitude larger than those used in Mendez *et al.* (1997). These departures can explain the differences of our results with the results of Mendez *et al.* (1997).

Table 4. Table of results of Mendez *et al.* (1997)

Viscous cosmic medium	*Era of Application*	*Damping*
Quark–gluon plasma	$(10^{27} \geq T \geq 10^{24})$ K	$\sim 10^{-7}$
Electron–neutrino mixture	$(10^{12} \geq T \geq 10^{10})$ K	$\sim 5 \times 10^{-9}$
Thomson scattering	$(10^{9} \geq T \geq 4 \times 10^{3})$ K	$\sim 2 \times 10^{-3}$
Compton scattering	$(10^{9} \geq T \geq 5.8 \times 10^{4})$ K	$\sim 10^{-9}$

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