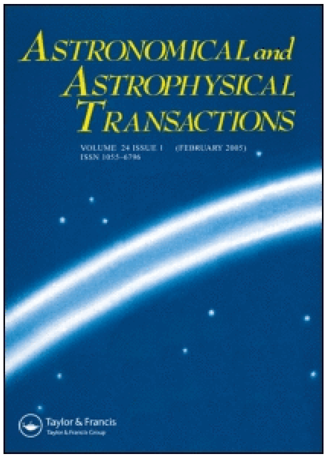

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ORBITAL TRENDS IN SYNCHRONOUS BINARY MOTION IN THE CONTEXT OF ENERGY MINIMIZATION

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The evolution of a natural system is characterized by a tendency to minimize its energy, and tidal evolution favours synchronism of spin and orbital motion. In this context we study the orbital evolutionary trends in synchronous binary motion, using a condition which favours minimization of energy of the system and reverts from the point mass approximation. Considering binaries to be subsystems of a microcanonical ensemble, we find that after tidal capture the equilibrium thermal distribution function favours high eccentricities.

Results are applied to extended bodies with constituents, such as stellar systems.

KEY WORDS Celestial mechanics, towards energy minimization, gravity softening, analytical methods

1 INTRODUCTION

Binaries are the most common stellar systems and they influence the dynamical evolution of stellar systems. However, the two celestial bodies are extended objects and not point masses; hence an important aspect of the two-body problem is that of taking into account the extended nature of the two components. Also as nature favours minimization of total energy of the system, one should use a condition which favours the minimization of energy and softens the gravity of the components on that basis. The transfer of angular momentum between orbital motion and spin (rotation) probably takes place from the epoch of formation of celestial bodies; for example a galaxy spins up due to tidal torquing of neighbouring galaxies (e.g., Peebles, 1969). Hence this interchange is an important mechanism related to statistical equilibrium and will determine the orbital characteristics of binary motion. In this context the orbital evolution of a binary system, under a condition which favours minimization of energy of the system, is of vital importance in dynamical astronomy, as it is likely to throw light on the expected orbital parameters on the basis of statistical mechanics, as well as on the preferred states; this also necessitates a generalized formulation of the problem, so as to be applicable to binaries in astronomy.

In a previous paper (Magalinsky and Chatterjee, 1997) we have studied the tidal evolution of a proto-planet as it condenses and orbits its primary, using a condition which favours the minimization of the energy of the system and reverts from the point mass approximation for the planet. Here, we extend the two-body problem, using a condition which favours energy minimization, as a consequence of which we revert from the point mass approximation; the rotations or spins of the bodies is taken into account, under synchronism with orbital motion. This is supported by the fact that tidal evolution favours synchronism, like the Earth–Moon system (e.g., Khentov, 1995). We analyse the expected distribution of the orbital parameters of the binary, on the basis of statistical mechanics, as perceived in the inertial frame corresponding to the centre of mass of the system and explore the preferred evolutionary trends.

2 THEORY AND ANALYSIS

2.1 Primitive Model

We consider the binary motion of two extended bodies of given masses $M_1 = M_2 = M$, and radius of gyration ρ_1 and ρ_2 (corresponding to their rotational inertia), softening the gravitational potential as,

$$\bar{U} = -\frac{GM^2}{R} \tag{1}$$

with

$$R^2 = r^2 + \rho_1^2 + \rho_2^2, \tag{2}$$

where r is the separation between the centres of the bodies and $\bar{U}$ is the mean value of U.

The internal energy is considered in the form of a flux (see Appendix I). The Lagrangian function of the system consists of its orbital, spin (or rotational) and potential energies, the mean value being

$$L = \frac{M}{4}(\dot{r}^2 + r^2\dot{\phi}^2) + \frac{M}{2}(\dot{\rho}_1^2 + \dot{\rho}_2^2) - \bar{U}. \tag{3}$$

The softening of the gravity of the masses is achieved as a consequence of the approximation of the Newtonian attraction by means of a rigorous inequality, which favours a tendency towards minimum energy, as the potential energy is bounded from above by its average value, i.e. $U \leq \bar{U}$ (see Appendix II). The total energy of the system (in the configuration space of (r, ρ)) defines the boundaries inside which the binary motion can take place, and we are interested in the case where the energy tends to a minimum.

The orbital and spin angular momentums, l and s, can be obtained using the relations,

$$\rho_1 = \rho\cos\psi \tag{4a}$$

$$\rho_2 = \rho\sin\psi \tag{4b}$$

whence

$$r^2\dot{\phi} = 2^{1/2}l \tag{5}$$

and

$$\rho^2\dot{\psi} = s \tag{6}$$

where ϕ and ψ correspond to the azimuthal angles for orbital and rotational motion, respectively.

In the inertial frame, corresponding to the centre of mass of the system, the Hamiltonian of the system is

$$H = \frac{P_r^2}{M} + \frac{P_\rho^2}{2M} + W \tag{7}$$

(P_r and P_ρ being the usual orbital and spin momentum respectively), where,

$$W = \frac{l^2}{2Mr^2} + \frac{s^2}{2M\rho^2} - \frac{GM^2}{R} \tag{8}$$

$$W = E \tag{9}$$

being the total energy of the system.

2.2 Equilibrium and Dimensionless Variables

The equilibrium values of r and ρ, denoted by r_0 and ρ_0 are obtained by using the condition which favours minimum energy, $\min_{r,\rho} W$; whence we get

$$r_0 = R_0\cos\beta \tag{10a}$$

$$\rho_0 = R_0\sin\beta \tag{10b}$$

where

$$R_0 = \frac{j^2}{GM^3} \tag{11}$$

$$j = l + s \tag{12}$$

$$\cos^2\beta = \frac{l}{j} \tag{13a}$$
$$\sin^2\beta = \frac{s}{j} \tag{13b}$$

j being the total angular momentum of the system, such that our definitions imply $\cos^2\beta$ is proportional to the fraction of the angular momentum shared by the spin and the orbit. From equations (10)–(13) it follows that the orbital and spin periods coincide as a result of the tidal evolution.

The minimum energy condition implies virialization, such that,

$$\min_{r,\rho} W = W_0 = -\frac{GM^2}{2R_0}. \tag{14}$$

At this stage, it is convenient to introduce the following dimensionless variables

$$\begin{gathered} r = R_0 x, \quad \rho = R_0 y, \quad R = R_0 z, \quad W = -W_0 w, \\ E = W_0 h \text{ (where } 0 < h < 1), \\ w = x^{-2}\cos^4\beta + y^{-2}\sin^4\beta - 2z^{-1} \end{gathered} \tag{15}$$

where

$$z^2 = x^2 + y^2$$

2.3 Regions of Movement

We analyse the regions in which binary motion is permissible, in the configurational space of separation, x, and (linear) size, y, using polar coordinates

$$x = z\cos\alpha \tag{16a}$$
$$y = z\sin\alpha. \tag{16b}$$

Thus

$$\eta = \frac{y}{x} = \tan\alpha \tag{17}$$

determines the relative separation of the components with respect to their size. The sizes of the galaxies are determined by the angle they subtend at the centre of mass of the system, the angle α; β being its limiting value (in time), i.e.

$$\lim_{t\to\infty} \alpha = \beta. \tag{18}$$

Using the energy condition (9), we obtain the limiting values of z:

$$z(\alpha) = z_{1,2} = \frac{1 \pm \varepsilon}{h} \tag{19}$$

with

$$\varepsilon^2 = 1 - h\Lambda^2 \tag{20}$$

$$\Lambda^2 = \left(\frac{\cos^2\beta}{\cos\alpha}\right)^2 + \left(\frac{\sin^2\beta}{\sin\alpha}\right)^2, \tag{21}$$

ε is the orbital eccentricity; its maximum value for a given energy (determined by the parameter h) is

$$\varepsilon_m^2 = 1 - 2h \tag{20a}$$

and corresponds to the limiting value (β) of α.

The circular orbit for $\varepsilon_m = 0$ defines the limiting value of α, α_1 and α_2, deduced as

$$\cos 2\alpha_{1,2} = h\cos 2\beta \pm [(1-h)(1-h\cos^2 2\beta)]^{1/2}. \tag{22}$$

Between these two angles, the following relationships exist:

$$\cos(\alpha_1 - \alpha_2) = h^{1/2} \tag{23a}$$

$$\cos(\alpha_1 + \alpha_2) = h^{1/2}\cos 2\beta \tag{23b}$$

$$\tan\alpha_1 \tan\alpha_2 = \tan^2\beta\,. \tag{24}$$

Denoting by α_0 and α' the median value of α and the half-width angle, respectively, we obtain,

$$\cos 2\alpha_0 = h^{1/2}\cos 2\beta \tag{25}$$

$$\cos 2\alpha' = h^{1/2}. \tag{26}$$

It is obvious from this analysis that the region of motion of the binary lies within the closed curve given by

$$z_{1,2} = (1 \pm \varepsilon) \tag{27}$$

for a given energy. Its radial extent corresponds to the limiting value β of α, and is deduced to be $2\varepsilon_m/h$. The centre of the region, defined by the curve, is displaced by a distance $z_0 = 1/h$ from the origin of the coordinates (x, y). The angular size of the region is given by

$$\alpha_1 - \alpha_2 = \arccos h^{1/2} = \arcsin\varepsilon_m. \tag{28}$$

Notice that the limiting size of the region does not depend upon the orbital and spin angular momentum, but is defined by the energy parameter h, or the maximum eccentricity ε_m (for a given energy).

2.4 Microcanonical Distributions of the Relevant Parameters of the Model

We consider the binary systems to be subsystems of a microcanonical ensemble. The probablity density of the variables of the model in a microcanonical ensemble is given by the microcanonical distribution of Gibbs, as

$$W(P_r, P_\rho; x, y; h, \beta) = \Omega^{-1}\delta(E - H) \tag{29}$$

(using the usual notation for the Dirac δ function and the density of states Ω). It follows that the distribution of the parameters in the configurational space (x, y) is homogeneous and is given by

$$W(x,y;h,\beta) = W(z,\alpha;h,\beta) = \text{const.} \tag{30}$$

such that the calculation of the unidimensional probabilities reduces to the calculation of the corresponding Jacobi determinants.

As the orbital eccentricity depends only upon α, the above equation can be integrated between the limits $z_{1,2}$, whence we obtain

$$W(\varepsilon;h,\beta) = \text{const.} \cdot \varepsilon \left| \frac{\mathrm{d}\alpha}{\mathrm{d}\varepsilon} \right| \tag{31}$$

with the normalization,

$$\int W(\varepsilon)\,\mathrm{d}\varepsilon = 1, \tag{32}$$

the explicit form being

$$W(\varepsilon) = \frac{\text{const.}\varepsilon^2(1-\varepsilon^2)^{-1}\sin 2\alpha(\cos^4\beta\sin^2\alpha + \sin^4\beta\cos^2\alpha}{\cos^4\beta\sin^4\alpha - \sin^4\beta\cos^4\alpha}. \tag{31a}$$

The analytical cases of interest would be: (i) spin is insignificant ($\beta = 0$, implying $s/l = 0$); (ii) spin is dominant ($\beta = \pi/2$, implying $l/s = 0$); (iii) spin and orbital angular momentum are equally important ($\beta = \pi/4$, implying $l/s = 1$). Nevertheless, we find that in each case the distribution is almost insensitive to the orbital and spin angular momentum. In each case, the distribution of the eccentricity, for a given energy, has the form,

$$W(\varepsilon;\varepsilon_m) = Q^{-1}\varepsilon^2(1-\varepsilon^2)^{-1}(\varepsilon_m^2 - \varepsilon^2)^{-1/2} \tag{33}$$

with

$$Q = \frac{\pi}{2}[(1-\varepsilon_m^2)^{-1/2} - 1].$$

Thus the equilibrium distribution function is a monotonically increasing function of eccentricity, limited by its maximum value for a given energy.

The distribution of the separation of the components, x, is obtained from equation (31) as

$$W(x) = \text{const.} \cdot y(x) \tag{34}$$

where $y(x)$ is a positive root of the energy condition (9). This equation can be resolved analytically for the case of insignificant spin, $s = 0$, to obtain

$$W(x) = \text{const.}\frac{x}{1+hx^2}[4x^2 - (1+hx^2)^2]^{1/2} \tag{35}$$

where $x_1 < x < x_2$, with $x_{1,2} = 1/(1 \mp \varepsilon_m)$, such that (for $s = 0$) the motion takes place within the ring defined by x_1 and x_2.

Proceeding along the same lines, the distribution of the sizes, y, is obtained from equation (31) as

$$W(y) = \text{const.} \cdot y(x). \tag{36}$$

From equation (17) we see that the relative size, η, depends only upon α such that we obtain

$$W(\eta) = \text{const.} \cdot \varepsilon(\alpha) \cos^2 \alpha \tag{37}$$

with the normalization

$$\int W(\eta)\, \mathrm{d}\eta = 1. \tag{38}$$

2.5 *Behavior of the System near Equilibrium*

We develop the energy in the vicinity of the equilibrium configuration (r_0, ρ_0), using the last relation of equation (15),

$$w = x^{-2} \cos^4 \beta + y^{-2} \sin^4 \beta - 2z^{-1}.$$

Assuming a small perturbation,

$$x \to x_0 + \xi, \quad y \to y_0 + \eta \tag{39}$$

whence

$$x_0 = \cos \beta, \quad y_0 = \sin \beta$$

we obtain, using the energy condition (9),

$$a_{11}\xi^2 + 2a_{12}\xi\eta + a_{22}\eta^2 = \varepsilon_m^2 \tag{40}$$

where

$$a_{11} = 1 + 3\sin^2 \beta, \quad a_{22} = 1 + 3\cos^2 \beta, \quad a_{12} = -3 \sin \beta \cos \beta.$$

This indicates that the motion is stable, as the perturbative motion is bounded by an ellipse about the point in question. The semi-major axis of this ellipse is inclined at an angle β to the x-axis, the lengths of the semi-major and semi-minor axes being $A = \varepsilon_m$ and $B = \varepsilon_m/2$, respectively.

The distributions of the reduced separations and sizes is obtained as

$$x = x_0 + a_{11}^{-1}[-a_{12}\eta \pm (a_{11}\varepsilon_m^2 - 4\eta^2)^{1/2}] \tag{41a}$$
$$y = y_0 + a_{22}^{-1}[-a_{12}\xi \pm (a_{22}\varepsilon_m^2 - 4\xi^2)^{1/2}]. \tag{41b}$$

The limiting values of these variables is

$$x_{1,2} = x_0 - \frac{a_{12}\eta_{1,2}}{a_{11}}, \text{ with } \eta_{1,2} = \pm\frac{a_{11}^{1/2}\varepsilon_m}{2}; \text{ and}$$
$$y_{1,2} = y_0 - \frac{a_{12}\xi_{1,2}}{a_{22}}, \text{ with } \xi_{1,2} = \pm\frac{a_{22}^{1/2}\varepsilon_m}{2} \tag{42}$$

for the semi-minor axis $B = \varepsilon_m/2$.

Developing the energy in terms of the perturbative displacements ξ and η, and calculating the characteristic determinant, we obtain the normal frequencies of oscillations as

$$\omega^4 - 3(2+\lambda)\omega^2 + 8 = 0 \tag{43}$$

where

$$\lambda = \frac{l}{j} = \cos^2\beta \tag{44}$$

defines the ratio of orbital to total angular momentum; whence,

$$\omega_{1,2}^2 = \frac{3(2+\lambda) \pm (4+36\lambda+9\lambda^2)^{1/2}}{2}. \tag{45}$$

These two normal frequencies of oscillations in size and shape correspond to the Lissageou figures in the xy-plane.

Note that all the statistical characteristics can be calculated by using the derived formulas in an elementary manner.

3 RESULTS

We have formulated a Hamiltonian treatment of the problem of the dynamics of two extended bodies (of equal mass) with two degrees of freedom, corresponding to their separation and effective size. The internal movement of the constituents of the bodies is taken into account by the hydrodynamic velocity corresponding to the mass distribution and the gravitational potential is considered in the form of a flux (see Appendix I). A study of the binary motion is conducted, taking into account the spins (rotations) of the two components under synchronism with the orbital motion, subject to a condition which favours energy minimization; we are led to an analytical treatment of the gravitational softening of the bodies, as a consequence of this condition (see Appendix II).

We determine the region within which the binary motion is confined and test its stability, finding it to be stable. We consider binaries to be subsystems in a microcanonical ensemble and determine the equilibrium distribution function. We find that for bound binary systems, the equilibrium distribution of the system parameters favours high eccentricities, such that, following tidal capture, binaries are characterized by a state preferring high eccentricities.

4 APPLICATIONS AND CONCLUSIONS

This aspect of binary motion has important consequences for extended bodies with internal constituents, such as stellar systems, where the vast majority of the encounters involve a marginally bound capture, after which we have a slow bound binary orbit. The subject is reviewed by Karachentsev (1990) where he reflects upon the statistical equilibrium of binary galaxies. Many of the binary galaxies have their rotation and orbital periods of the same order, suggesting that their synchronous evolution may have characteristic trends. The equilibrium distribution of the parameters of bound binary systems, considering them to form a microcanonical ensemble, under the condition favouring minimization of energy, is a monotonically increasing function of eccentricity.

This tendency of favouring high eccentricities was noticed in previous research work. N-body models of the expanding universe of Evrard and Yahil (1985), using an initial Poission distribution of particles with three-dimensional information on position and velocity, indicate that amongst half of the masses that double up to form pairs, about 75% are in highly eccentric orbits, which seem to have ceded from the Hubble expansion and fallen radially towards each other. This is also obvious for an initial homogeneous distribution of positions and velocities of the binary components in phase space, as proportionately more phase space volume is available for higher values of eccentricities (cf. Jeans, 1929).

Recently, a critical study of binary galaxies has been conducted by Chengalur *et al.* (1993, 1994, 1995, 1996); in which they noted that the initial orbits must have been highly radial, favouring a high eccentricity and low energy. They find a small median value for the velocity difference distribution for wide galaxy pairs in low-density regions, suggesting that as the pairs are bound they are very likely to be near turnarounds (apocentres) of their orbits, which is indicative of the pairs being in almost radial low-energy orbits. As suggested by them and Evrard and Yahil, galaxies which feel their mutual two-body force will fall radially towards each other in orbits of high eccentricities, immediately after separation from the Hubble expansion.

Chengalur *et al.* find that, as for wide pairs, for close pairs also the velocity difference distribution has a small median value; the low impact parameter and slow velocity suggests that the close pairs have orbits similar to wide ones and thus indicates (as proposed by them) that high eccentricity orbits do not circularize easily. Our results favour orbits of high eccentricities for tidal capture binary galaxies. Studies of the tidal evolution of a proto-planet as it condenses and orbits its primary indicate that the process of circularization of the orbits is a very slow and gradual one (Magalinsky and Chatterjee, 1997). Thus it is quite conceivable that both wide and close pairs are on high eccentricity radial orbits, and that the wide ones evolve to close ones.

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APPENDIX I. INTERNAL ENERGY AND VELOCITY DISTRIBUTION OF THE BINARY COMPONENTS

Let $D(\boldsymbol{r}_i, t)$ $(i = 1, 2)$ define the mass distributions of the two extended components. We assume that the distributions have the form

$$D(\boldsymbol{r}_i, t) \equiv M \frac{f(\boldsymbol{r}_i/\rho_i)}{\rho_i^3} \tag{AI.1}$$

where

$$\rho_i \equiv \rho_i(t)$$

such that the symmetry of the distribution does not change with time and its evolution reduces to a transformation of scale. We normalize this distribution according to

$$\int f(\xi)\,\mathrm{d}^3\xi = 1, \quad \int \xi f\,\mathrm{d}^3\xi = 0, \quad \int \xi^2 f\,\mathrm{d}^3\xi = 1, \tag{AI.2}$$

whence it is evident that ρ_i is the radius of gyration of the two bodies (corresponding to their moments of inertia about their spin axes).

We denote by $\boldsymbol{u}_i$ the hydrodynamic velocity at a point at a distance $\boldsymbol{r}_i$ from the centre of the body. According to the equation of continuity,

$$\frac{\partial D}{\partial t} + \operatorname{div}(\boldsymbol{D}\boldsymbol{u}) = 0. \tag{AI.3}$$

Obviously the flow field $\boldsymbol{u}_i(\boldsymbol{r}_i, t)$ corresponds to a potential flux; hence we obtain from (AI.1) and (AI.2),

$$\boldsymbol{u}_i = -\frac{\dot{\rho}_i}{\rho_i}\boldsymbol{r}_i, \tag{AI.4}$$

such that the relative velocity of two constituent elements of the two components (of the binary) is given by

$$\boldsymbol{u}_r = \dot{\boldsymbol{r}} + (\boldsymbol{u}_1 - \boldsymbol{u}_2). \tag{AI.5}$$

The kinetic energy of the relative motion is then given by

$$K_{\mathrm{rel}} = \frac{1}{4}\int D(\boldsymbol{r}_1)D(\boldsymbol{r}_2)\boldsymbol{u}_r^2\,\mathrm{d}^3\boldsymbol{r}_1\,\mathrm{d}^3\boldsymbol{r}_2 \tag{AI.6}$$

where we take into account the independent mass distributions of the components and use the reduced mass corresponding to the relative motion.

Taking into account the previous equations, (AI.6) leads to the (mean) Lagrangian in equation (3).

APPENDIX II. CONDITION FAVOURING MINIMIZATION OF ENERGY AND GRAVITY SOFTENING

The Newtonian interaction between the two components is given in terms of the potential energy as

$$U = -G \int D(\boldsymbol{r}_1) D(\boldsymbol{r}_2) |\boldsymbol{r} + \boldsymbol{r}_1 - \boldsymbol{r}_2|^{-1} \, \mathrm{d}^3\boldsymbol{r}_1 \, \mathrm{d}^3\boldsymbol{r}_2. \tag{AII.1}$$

According to (AI.1) and (AI.2), f is a distribution such that we can denominate the mean value of a quantity, $A(\xi)$, as

$$\int A(\xi) f(\xi) \, \mathrm{d}\xi = \langle A \rangle \tag{AII.2}$$

such that the mean value of U is given by

$$\bar{U} = -GM^2 \langle [(\boldsymbol{r} + \boldsymbol{r}_1 - \boldsymbol{r}_2)^2]^{-1/2} \rangle. \tag{AII.3}$$

Noting that U is a convex function with respect to its argument $(\boldsymbol{r} + \boldsymbol{r}_1 - \boldsymbol{r}_2)^2$, we can approximate U from above as $U_{\mathrm{ex}} \le U_{\mathrm{ap}}$, where U_{ex} and U_{ap} denote the exact and approximate values of U, respectively, such that

$$U_{\mathrm{ex}} \equiv U \le \bar{U} = -GM^2 \langle (\boldsymbol{r} + \boldsymbol{r}_1 - \boldsymbol{r}_2)^2 \rangle^{-1/2}. \tag{AII.4}$$

from which we obtain an approximation for U from above, as

$$U \le \bar{U} = -\frac{GM^2}{R} \tag{AII.5}$$

where

$$R^2 = r^2 + \rho_1^2 + \rho_2^2.$$

This approximation favours the minimization of energy.

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