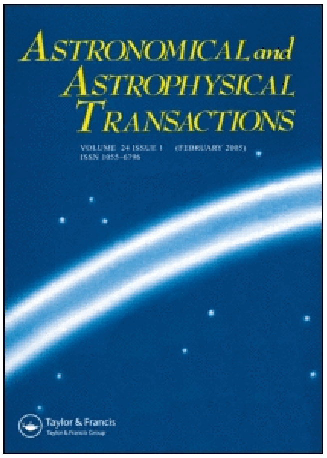

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Interstellar Medium

MHD-COLLAPSE OF PROTOSTELLAR CLOUDS

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The subject of this paper is the numerical simulation of two-dimensional gravitational MHD collapse of protostellar clouds. A new numerical method was constructed to solve the equations of a two-component mixture including the effects of ambipolar diffusion, non-stationary ionization, and heating (cooling). The method was implemented in a two-dimensional code that simulates the MHD-collapse on a Eulerian cylindrical grid. The code uses the explicit monotonic Osher scheme which gives a third-order accurate approximation in space and a second-order approximation in time. The method was tested on several problems, one of which, isothermal gravitational collapse of a rotating protostellar cloud, is discussed in some detail.

KEY WORDS Protostellar clouds, gravitational collapse, numerical methods

1 INTRODUCTION

The theory of star formation is based on numerical simulations of the gas dynamics of protostellar cloud contraction. This theory explains the formation of single stars with different masses (see Palla and Stahler, 1994). The observational data of the last decade show that at the present time star formation occurs in rotating magnetized interstellar clouds (Dudorov, 1990; Basu and Mouschovias, 1994). Estimates of rotation velocities show that the specific angular momentum of protostellar clouds is larger by 4–5 orders of magnitude than the specific angular momentum of single stars (Basu and Mouschovias, 1994). The same is true of magnetic fluxes. The problem of the angular momentum and magnetic flux in star-formation theory should be solved within a self-consistent approach. But first the problems of the evolution of the magnetic flux and that of the angular momentum are solved separately in order to determine what processes are important in each case.

The evolution of the fossil magnetic field was studied in several papers by A. Dudorov (see Dudorov, 1990). It was shown that forming stars can have a large fossil field. The evolution of rotating protostellar clouds has until now been studied under the assumption that the angular momentum is locally conserved (Burkert and Bodenheimer, 1993). The contraction of a rotating magnetized cloud is fundamentally different from this case because the magnetic field can transport angular momentum from the central parts of the contracting cloud to the ambient molecular cloud and to the cloud envelope.

Numerical computations in 1.5-dimensional approximation (Dudorov and Sazonov, 1987; Dudorov, 1990) show that the geometry and strength of the fossil magnetic field of young stars, as well as the efficiency of magnetic braking, depend mainly on the degree of defreezing of the protostellar magnetic field (see Dudorov, 1990). The degree of defreezing is determined by the efficiency of magnetic diffusional processes. Both ambipolar and ohmic diffusion of the magnetic field develop at the advanced stages of the collapse of protostellar clouds, when densities at the cloud centre and at its boundary differ by 5–6 orders of magnitude. At the same time the ionization fraction decreases to such extremely low values that the nonstationary recombinational decay of the plasma becomes an important factor.

The above-listed characteristics make it necessary to carry out numerical simulations of the interstellar cloud collapse within a multicomponent model. In this paper we show that it is possible to apply to this problem the modified Lax–Friedrix–Osher method (Chakravarthy and Osher, 1983; 1985) (see Section 3). The results of test computations are briefly discussed in Section 4.

2 STATEMENT OF THE PROBLEM

2.1 Basic Equations

Interstellar (protostellar) clouds consist of neutral (atoms, molecules and cosmic dust particles) and charged (electrons, ions and charged dust) components. In simulations of protostellar cloud collapse, the dust can be treated in a first approximation simply as a recombinational sink for electrons and ions, and the collapse dynamics can be described within a three-component model (electrons e, ions i, and neutrals n) (see Gershman *et al.*, 1984).

Practical simulation of the MHD-collapse of protostellar clouds within a three-component model requires a complex procedure to make the motion of the components self-consistent. However, the need for this procedure disappears if we describe the dynamics of MHD-collapse in a "diffusional" approximation (see Dudorov, 1990), in which neutrals are considered to be the main component of the interstellar cloud gas, while electrons and ions are viewed as a diffusional addition. Introducing, besides the traditionally defined mass variables, two "diffusional" variables: the ionization fraction

$$x = \frac{\rho_e + \rho_i}{\rho}, \quad \rho = \rho_e + \rho_i + \rho_n \tag{1}$$

and the velocity of ambipolar diffusion

$$\boldsymbol{v}_A = \boldsymbol{v}_p - \boldsymbol{v}_n, \tag{2}$$

where $\boldsymbol{v}_n$ and $\boldsymbol{v}_p$ are the mass velocities of neutrals and of the electron–ion plasma, and assuming that the ionization fraction is small $x \leq 10^{-5}$, we can transform the set of MHD equation for the three-component mixture to the following form:

$$\frac{\partial \rho}{\partial t} + \nabla_k(\rho v^k) = 0, \tag{3}$$

$$\frac{\partial(\rho v^i)}{\partial t} + \nabla_k(g^{ik}P + \rho v^i v^k - \sigma^{ik}) = -\nabla^i \Phi, \tag{4}$$

$$\frac{\partial(x\rho)}{\partial t} + \nabla_k(x\rho V^k) = 0, \tag{5}$$

$$\frac{\partial(x\rho V^i)}{\partial t} + \nabla_k[g^{ik}P_p + x\rho V^i V^k - \sigma^{ik}] = -x\rho\tau^{-1}v_A^i - x\rho\nabla^i\Phi + \rho V^i S, \tag{6}$$

$$\frac{\partial B^i}{\partial t} = [\nabla, [\boldsymbol{V}, \boldsymbol{B}] - \nu_m[\nabla, \boldsymbol{B}]]^i, \tag{7}$$

$$\frac{\partial}{\partial t}\left(\rho\varepsilon + \rho\frac{v^2}{2} + \rho\Phi + \frac{B^2}{8\pi}\right) + \nabla_k\left\{\rho v^k\left[\varepsilon + \frac{P}{\rho} + \Phi + \frac{v^2}{2}\right]\right.$$
$$\left. + [\boldsymbol{B}, [\boldsymbol{V}, \boldsymbol{B}] + \nu_m[\nabla, \boldsymbol{B}]]^k + q^k\right\} = \rho Q, \tag{8}$$

$$\Delta\Phi = 4\pi G\rho, \tag{9}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{10}$$

$$P = (\gamma - 1)\rho\varepsilon, \tag{11}$$

where

$$\sigma^{ik} = \frac{B^i B^k}{4\pi} - \delta^{ik}\frac{B^2}{8\pi}, \tag{12}$$

is the Maxwell tension tensor, S is the ion source function, $\boldsymbol{V} = \boldsymbol{v} + \boldsymbol{v}_A$, P_p is the pressure of the plasma component, τ^{-1} is the frequency of ion-neutral collisions, ν_m is the magnetic viscosity, q is the radiative flux, and Q is the cooling function. The set (3)–(8) was derived under the assumption that ions and electrons move with the same velocity due to the ambipolarity of the plasma.

2.2 The Initial and Boundary Conditions

The collapse of a rotating magnetized protostellar cloud can be studied in an axisymmetric approximation if the initial uniform magnetic field $\boldsymbol{B}$ is parallel to the vector of angular velocity of the cloud $\boldsymbol{\Omega} = [\boldsymbol{r}, \boldsymbol{v}]$. In this case the problem of collapse initiated by gravitational instability or by ambipolar diffusion can be solved on a two-dimensional computational domain ($0 \leq r \leq R$, $0 \leq z \leq Z$). The initial conditions for the collapse of a rigidly rotating spherically symmetric cloud with mass larger than the critical value for gravitational instability are: the initial values of density ρ_0 or radius R_0, and values of the parameters α, β, γ, which are the ratios of the initial values of, respectively, the internal, kinetic and magnetic energy to the absolute value of gravitational energy. Therefore, the initial conditions are given by:

$$\begin{array}{lll} \rho = \rho_0, & x = x_0, & \varepsilon = \varepsilon_0, \\ \boldsymbol{v} = 0, & \boldsymbol{v}_A = 0, & \boldsymbol{B} = \boldsymbol{B}_0. \end{array} \tag{13}$$

Conditions on the outer boundary are the same as for contraction of a self-gravitating cloud of a given size. The normal components of the mass velocity and the velocity of ambipolar diffusion should vanish, and the azimuthal components of the velocities should be continuous:

$$v_n = 0, \quad \frac{\partial v_r}{\partial n} = 0. \tag{14}$$

The magnetic field satisfies, on the outer boundary, the conditions of smoothness of the field lines:

$$\frac{\partial \boldsymbol{B}}{\partial n} = 0. \tag{15}$$

Similar boundary conditions are used for the density, internal energy and the ionization fraction.

The boundary conditions on the axis of symmetry and in the equatorial plane satisfy the requirements of axial and reflective symmetry. The boundary conditions for the mass velocity and the velocity of ambipolar diffusion are: on the rotation axis

$$v_r = v_\varphi = 0, \quad \frac{\partial v_z}{\partial r} = 0; \tag{16}$$

in the equatorial plane

$$\frac{\partial v_r}{\partial z} = \frac{\partial v_\varphi}{\partial z} = 0, \quad v_z = 0. \tag{17}$$

The magnetic field on the rotation axis satisfies the conditions:

$$B_r = B_\varphi = 0, \quad \frac{\partial B_z}{\partial r} = 0, \tag{18}$$

and in the equatorial plane it satisfies the condition of smoothness of the field lines (15).

3 NUMERICAL METHOD OF SOLUTION

3.1 The Specific Character of the Numerical Simulation of MHD-Collapse

When we simulate MHD-collapse, we must take into account the following specific features of the set of equations (3)–(11) with initial and boundary conditions (13)–(18):

(1) The strongly non-stationary character of collapse requires a numerical method which gives a high-order approximation in time.

(2) The strong non-homogeneity of the solution on the computational domain and a large decrease in the cloud size during the collapse are compatible only with implicit methods. This is due to the fact that the classic Courant–Friedrichs–Lewy (CFL) condition for explicit methods leads to time steps which are much smaller than the characteristic time of cloud evolution, if there are large variations of density, pressure and the Alfven speed from the cloud boundary to its centre. Explicit schemes do not allow simulation of advanced stages of the collapse.

(3) If the simulation uses a fixed grid, the dynamic concentration of gas toward the cloud centre during the collapse may lead to a situation when the whole process is concentrated in a few cells. Therefore, for high accuracy we should use not only high-order approximations in space, but also an adaptive (moving) grid whose cells concentrate at the centre of the computational domain.

3.2 The General Scheme of Computation

The set of equations (3)–(8) can be written in the following vector form:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial r} + \frac{\partial \boldsymbol{G}}{\partial z} = \boldsymbol{R}, \tag{19}$$

where $\boldsymbol{u}$ is the vector of conservative variables, $\boldsymbol{F}$ and $\boldsymbol{G}$ are the vectors of flux in the radial and vertical directions, and $\boldsymbol{R}$ is the vector of sources. The system of conservation equations (19) was solved numerically by the Lax–Friedrix–Osher method (Chakravarthy and Osher, 1985). The fluxes in the original numerical scheme (for simplicity we shall consider only the radial direction)

$$\frac{\partial \boldsymbol{u}_i}{\partial t} + \frac{\boldsymbol{H}_{i+1/2} - \boldsymbol{H}_{i-1/2}}{\Delta r_i} + \ldots = \boldsymbol{R}_i, \tag{20}$$

are calculated according to the following algorithm. First we find the fluxes using the Lax–Friedrix method:

$$\boldsymbol{H}^L_{i+1/2} = \frac{\boldsymbol{F}_i + \boldsymbol{F}_{i+1}}{2} - \frac{\lambda_{i+1/2}}{2}(\boldsymbol{u}_{i+1} - \boldsymbol{u}_i), \tag{21}$$

where

$$\lambda_{i+1/2} = \max_k \left(|\lambda^k_i|, |\lambda^k_{i+1}|\right); \tag{22}$$

λ^k – are the eigenvalues of the hyperbolicity matrix

$$A = \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{u}} \tag{23}$$

of the system (19). The Lax–Friedrix scheme (21) is a monotonic conservative scheme which gives a first-order approximation in space.

To increase the order of approximation of the scheme while preserving its monotonicity Osher suggested correcting fluxes (21) according to the following formula:

$$\begin{aligned}
\boldsymbol{H}_{i+1/2} &= \boldsymbol{H}^L_{i+1/2} - \frac{1-\phi}{4}\,\mathrm{minmod}\,(\boldsymbol{H}^-_{i+3/2}, \beta \boldsymbol{H}^-_{i+1/2}) \\
&- \frac{1+\phi}{4}\,\mathrm{minmod}\,(\boldsymbol{H}^-_{i+1/2}, \beta \boldsymbol{H}^-_{i+3/2}) \\
&+ \frac{1+\phi}{4}\,\mathrm{minmod}\,(\boldsymbol{H}^+_{i+1/2}, \beta \boldsymbol{H}^+_{i-3/2}) \\
&+ \frac{1-\phi}{4}\,\mathrm{minmod}\,(\boldsymbol{H}^+_{i-3/2}, \beta \boldsymbol{H}^+_{i+1/2}),
\end{aligned} \tag{24}$$

where ϕ and β $\left(1 < \beta < \frac{3-\phi}{1-\phi}\right)$ are the scheme parameters,

$$\boldsymbol{H}^-_{i+1/2} = \boldsymbol{H}_{i+1/2} - \boldsymbol{F}_i, \tag{25}$$

$$\boldsymbol{H}^+_{i+1/2} = \boldsymbol{F}_{i+1} - \boldsymbol{H}_{i+1/2}, \tag{26}$$

$$\mathrm{minmod}\,(x, y) = \mathrm{sign}\,(x) \max\left(0, \min\left(|x|, y\,\mathrm{sign}\,(x)\right)\right). \tag{27}$$

The scheme gives a third-order approximation in space if $\phi = 1/3$, and a second-order approximation otherwise. The stability condition for the Lax–Friedrix–Osher scheme can be written as:

$$\Delta t = \frac{4}{5 - \phi + \beta(1+\phi)} \min\left(\frac{\Delta r}{\lambda}\right). \tag{28}$$

An approximation of the time derivative in (19) can be obtained using, for example, a second-order Runge–Kutta method.

To apply the described numerical scheme to the set of equations (3)–(8) we need first to find the eigenvalues of the corresponding hyperbolicity matrix. After calculations we found the following set of eigenvalues:

$$\lambda_{1,2} = v_r, \quad \lambda_{3,4} = v_r \pm c, \quad \lambda_{5,6} = V_r \pm a_r,$$

$$\lambda_{7,8,9,10} = V_r \pm \frac{1}{2}\sqrt{c_p^2 + a^2 + 2a_r c_p} \pm \frac{1}{2}\sqrt{c_p^2 + a^2 - 2a_r c_p}, \tag{29}$$

where

$$\boldsymbol{a} = \frac{\boldsymbol{B}}{\sqrt{4\pi x \rho}}, \tag{30}$$

$$c = \sqrt{\gamma \frac{P}{\rho}}, \quad c_p = \sqrt{\gamma \frac{P_p}{\rho}}. \tag{31}$$

The z-part of the numerical scheme is constructed in the same way. We should note that for $x = 1$ formulae (29) are different from the formulae of an ideal MHD

(Koldoba *et al.*, 1992) because the set (3)–(8) was derived under the assumption that the ionization fraction is small.

The Poisson equation for the gravitational potential is solved by the Douglas–Rachford ADI method (see Godunov and Ryabenky, 1977), but our computations use a non-uniform grid, and therefore we cannot use the standard Douglas–Rachford algorithm to find the sequence of timesteps. Therefore we give an empirical iterational sequence (see Black and Bodenheimer, 1975).

4 TESTING OF NUMERICAL CODE

We tested our code on several problems with known (exact or approximate) analytical solutions:

(1) decay of an arbitrary discontinuity (Landau and Lifshitz, 1988);

(2) computation of the gravitational potential for various model density distributions;

(3) spherically symmetric isothermal collapse (Shu, 1977);

(4) isothermal collapse of a rotating cloud (Therebye *et al.*, 1984);

(5) isothermal collapse of a cloud with a frozen-in magnetic field (Galli and Shu, 1993).

Below are the results of one of the test computations: isothermal collapse of a rotating cloud with initial values of parameters $\alpha = 0.12$, $\beta = 0.03$ on a 100×100 grid. To avoid large gradients on the cloud boundary, the cloud density in the boundary region was taken in the form of an exponentially decreasing function. The cloud itself initially had a spherical form with radius $R_0 = 0.9$ and density $\rho_0 = 1$.

The results are given in Figures 1 and 2. Figure 1(a) shows the distribution of density for $t_1 = 0.8721 t_{ff}$. By this time due to the action of the centrifugal force the central part of the cloud becomes an oblate spheroid with the axis ratio approximately 0.75. By the time $t_2 = 1.0053 t_{ff}$ (see Figure 1(b)) the cloud becomes even more oblate with the axis ratio decreasing to < 0.5. The central density at this point increases to ≈ 65. The density distributions on the axis of summetry and in the equatorial plane for $t = 0$, $t = t_1$, $t = t_2$ are shown in Figures 2(a,b). Also shown is the curve $\rho \propto r^2$, which represents the self-similar solution for isothermal spherically symmetric collapse. On the axis of symmetry rotation produces a non-uniform density profile that is less steep than the curve r^2; in the equatorial plane the density profile becomes steeper with time.

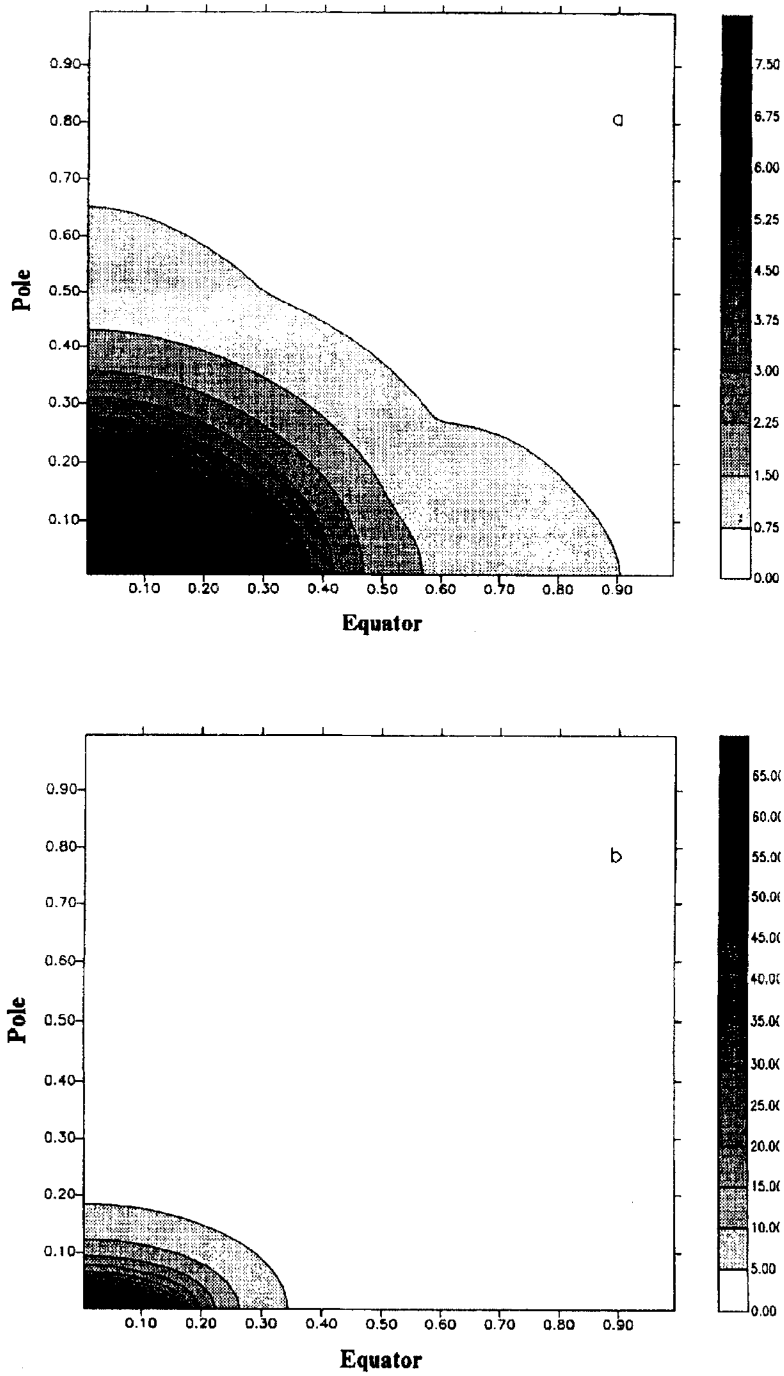

Figure 1 The contours of density distribution for $t = 0.8721t_{ff}$ (a) and $t = 1.0053t_{ff}$ (b).

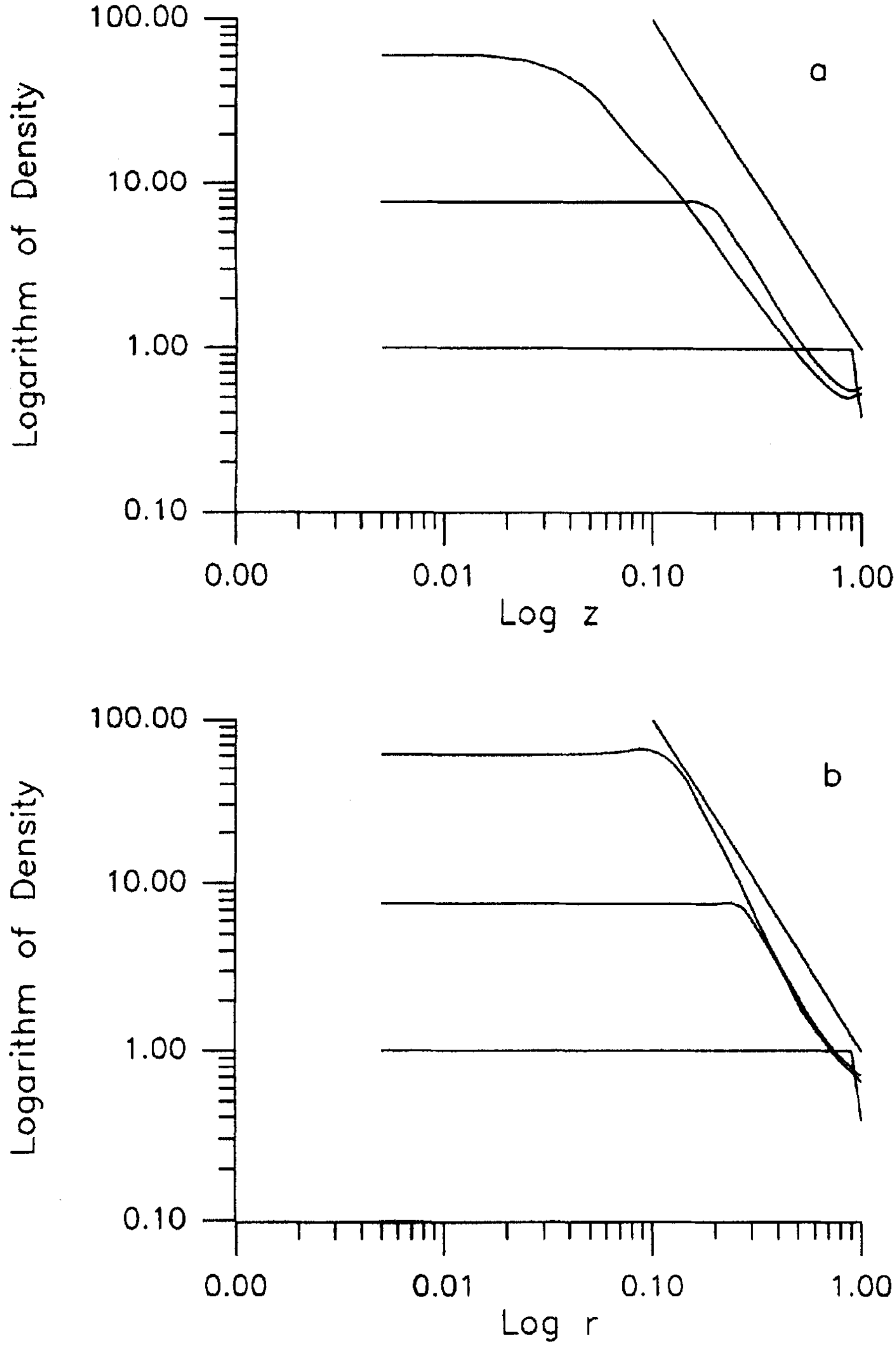

Figure 2 Density distributions on the axis of symmetry (a) and in the equatorial plane (b) for $t = 0.0$, $t = 0.8721 t_{ff}$ and $t = 1.0053 t_{ff}$. The slope of the straight line equals -2.

5 CONCLUSIONS

We have constructed and tested a two-dimensional MHD code for simulation of gravitational MHD collapse. The tests have shown that the code is accurate enough for this class of problem, but still an implicit code is desirable for simulation of advanced stages of the collapse.

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