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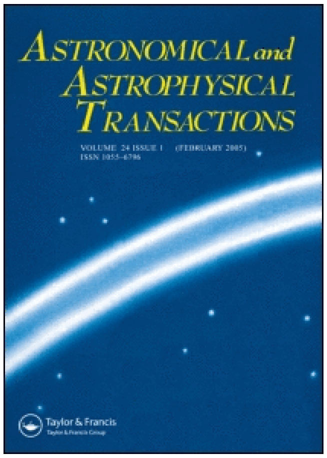

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Timing of gamma-ray pulsars: Search in seven-parametric space

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TIMING OF GAMMA-RAY PULSARS: SEARCH IN SEVEN-PARAMETRIC SPACE

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Timing of the Geminga gamma-ray pulsar is done using the data of COS B and EGRET. It is shown, that errors in angular coordinates of sources similar to Geminga strongly influence the determination of $\ddot{\nu}$, so that at angular precision less then 10^{-3} arc sec determination of the value of $\ddot{\nu}$ by means of this criterion gives an error of more than 100%. Attemps have been made to improve the coordinates of the gamma-ray pulsar using timing analysis. In addition to searching for ν, $\dot{\nu}$ and $\ddot{\nu}$, a technique is first developed permitting the search of two angular coordinates, the absolute speed value and the direction of the proper motion. In that way, timing of gamma-ray pulsars gives an amount of information compatible with radio pulsars, but using real data gives much poorer precision. In gamma-ray sources with rare pulses the periodicity criteria are quite different from those in the radio region. Data on the coordinates and proper motion of Geminga, obtained from timing studies, do not contradict, within the errors, its identification with a G'' star and its proper motion. These errors are larger than those in optical measurements, but are smaller than the corresponding errors in X-ray and γ-ray data. Estimation of the gamma-ray pulsar coordinates and its proper motion could be obtained independently of its optical or radio component, and are available in their absence.

KEY WORDS Gamma ray sources, pulsars, timing

1 INTRODUCTION

The discovery of Geminga as a 'true' pulsar, but without visible radio emission, gave additional evidence to the idea that hard gamma-ray emission ($E \geq 30$ MeV) is an inherent property of pulsar radiation. Before this only the Vela pulsar, the strongest hard gamma-ray source, and the young Crab pulsar gave hints of this possibility. Observations of the sky, made by EGRET on CGRO (Thompson *et al.*, 1992; 1994), have shown that only young pulsars with an age not exceeding several tens of thousand of years and, possibly, millisecond pulsars (Verbunt *et al.*, 1996), give an observable flux in gamma radiation. What is not yet clear, is the mechanism of this gamma radiation and whether it has a threshold character, or if there is a gradual decrease of hard gamma-ray flux with age.

The existence of the Geminga pulsar indicates that there could be other gamma-ray pulsars with no radio emission, which are exhibited in EGRET observations as ordinary point-like sources (see e.g. Mukherjee *et al.*, 1995; Brazier *et al.*, 1996).

Determination of pulsations in a hard gamma-ray source is a very difficult problem, connected with the rarity of arriving quanta $\delta t \gg P$, and the small total number of quanta. When the value of the period is known from other observations (radio or X-ray), timing analysis gives the possibility of reproducing this periodicity also in the gamma region (Bignami and Caraveo, 1992; Mayer-Hasselwander *et al.*, 1994). When there is no information about the period, it could, in principle, be found from pure gamma data (Gurin *et al.*, 1988a), but this could take an enormous amount of computer time and has not been fully used in practice.

The position of pulsars with no radio emission on the sky cannot be established precisely; the best positions obtained from X-ray observations are between 3″ (Einstein) and 5″ (ROSAT) for the 90% level (Becker *et al.*, 1993; Becker, 1995). Optical identification of Geminga with a very faint $> 25^{m}.5$ object has been done in Bignami *et al.* (1987) and later measurements of its proper motion (Bignami *et al.*, 1993) and parallax (Caraveo *et al.*, 1996) can be considered as evidence for the reality of this identification.

Timing of Geminga in the hard gamma region based on COS-B (Bignami and Caraveo, 1992; Hermsen *et al.*, 1992) and EGRET (Mayer-Hasselwander *et al.*, 1994) data gave an anomalously high braking index $n = \nu\ddot{\nu}/\dot{\nu}^2 \sim 10$–$30$, corresponding to a very high second derivative $\ddot{\nu}$. While there is the possibility that it is connected with the poor precision of $\ddot{\nu}$ determination, it is worth investigating other explanations. It was suggested by Bisnovatyi-Kogan and Postnov (1993) that the high value of $n(\ddot{\nu})$ results from errors in its coordinates, leading to an incorrect barycentre reduction procedure, which spoils the timing procedure. This problem is well known for pulsar timing, where the error in coordinates gives a one-year periodic deviation from the smooth curve in the pulse arriving time, which permits us to improve the pulsar position to an amazing precision of the order and even better than VLBI observations (Harrison *et al.*, 1993; Taylor, 1992).

Here we describe a method for the investigation of the timing of gamma pulsars, represented by periodic objects with rare pulses, which gives the possibility of determing seven parameters of a gamma pulsar: frequency ν, its two derivatives $\dot{\nu}$ and $\ddot{\nu}$, angular coordinates α and δ of the source, absolute value v, and the direction of the velocity of the proper motion, characterized by the angle θ. When registered pulses are rare, so that their time separation δt is much larger than the period P, the method of investigation is quite different from that used for radio pulsars. On the artificial sample of data, where properties simulate Geminga, but in contrary, have a very narrow light curve (δ-function), no systematic errors and no false quanta, we have managed to determine the coordinates and proper motion parameters with very high precision. This precision is decreasing when we go to pulses with a finite width, in the presence of background and systematic errors.

Application of this method to a real data sample of Geminga from COS-B and EGRET was not so successful, because of the smooth light curve, the 'non-perfectness' of the data, and possible glitches. We present here results for the most probable

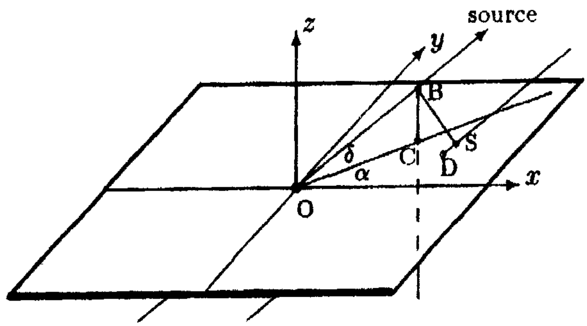

Figure 1 The barycentric correction.

position and proper motion characteristics, determined by using periodicity criteria, which are not in contradiction with more precise optical data, and have a better coordinate precision than γ-ray or X-ray data.

2 BARYCENTRIC CORRECTIONS: TAKING ACCOUNT OF ANGULAR COORDINATE CORRECTION AND PROPER MOTION

For timing analysis all data must be presented in the same coordinate system, which as a rule is connected with the barycentre of the Solar System. Consider first the situation, when the angular coordinates of a source α and δ are known exactly. In Fig. 1 xyz is a coordinate system that remains at rest with respect to distant stars. The point O is the barycentre of the Solar System. The space probe is at the point S with Cartesian coordinates (x_0, y_0, z_0). The direction to the source is defined by a straight line with coordinate angles α and δ, where angle α is measured in the xy-plane counterclockwise from the positive direction of the x-axis. The angle δ is measured in the plane perpendicular to the xy-plane. SB is the perpendicular from the space probe (point S) on to a line from the barycentre to the source. BC is the perpendicular from the point B to the xy-plane, and SD is a line parallel to OB; points C and D belong to the xy-plane. The unit vector pointing from the barycentre to the source has Cartesian coordinates $(\cos\alpha\cos\delta, \sin\alpha\cos\delta, \sin\delta)$. The scalar product of this vector and the radius-vector of the space probe is the length of the segment OB:

$$OB = x_0\cos\delta\cos\alpha + y_0\cos\delta\sin\alpha + z_0\sin\delta\,.$$

The time interval during which light travels the length OB is a barycentre correction, if a source is far enough and SB is part of a flat wavefront. This time interval ΔT, which must be added to the moment of each event at the point S to obtain a corresponding barycentre moment, is determined as

$$\delta T = \frac{1}{c}(x_0 \cos\delta \cos\alpha + y_0 \cos\delta \sin\alpha + z_0 \sin\delta)\,. \tag{1}$$

In a common choice α and δ coincide with right ascension and declination, when xy is the Earth's equatorial plane for some fixed epoch and the x-axis points to the Spring equinox at the same epoch. The procedure for calculating ΔT with precise account of Earth and satellite motion is described by Mayer-Hasselwander (1985).

Suppose that coordinates α and δ are not known exactly, with corresponding errors $\mathrm{d}\alpha$ and $\mathrm{d}\delta$. Then for small errors we may find from (1) the corresponding barycentre corrections in the linear approximation

$$\begin{aligned}\delta T &= \frac{\partial \Delta T}{\partial \alpha}\,\mathrm{d}\alpha + \frac{\partial \Delta T}{\partial \delta}\,\mathrm{d}\delta \\ &= \frac{1}{c}(-x_0 \cos\delta \sin\alpha + y_0 \cos\delta \cos\alpha)\,\mathrm{d}\alpha \\ &+ \frac{1}{c}(-x_0 \sin\delta \cos\alpha - y_0 \sin\delta \sin\alpha + z_0 \cos\delta)\,\mathrm{d}\delta.\end{aligned} \tag{2}$$

Consider for simplicity the case when the space probe orbit around the Sun is circular and, consequently, its angular velocity ω is a constant. Then $x_0 = R\cos\omega t$, $y_0 = R\sin\omega t$, $z_0 = 0$ and

$$\Delta T = \frac{R}{c}(\cos\omega t \cos\delta \cos\alpha + \sin\omega t \cos\delta \sin\alpha) = \frac{R}{c}\cos\delta\cos(\omega t - \alpha)\,, \tag{3}$$

where R is a radius of its orbit. The correction (2) is then reduced to

$$\delta T = \frac{R}{c}[\cos\delta \sin(\omega t - \alpha)\,\mathrm{d}\alpha - \sin\delta\cos(\omega t - \alpha)\,\mathrm{d}\delta]\,. \tag{4}$$

Assume that only first and second derivatives of ν are essential, and the frequency of the signal may be represented by

$$\nu = \nu_0 + \dot{\nu} t + \frac{\ddot{\nu}_0^2}{2} t^2\,, \tag{5}$$

where the index '0' is referred to the epoch $t = 0$. Define the current arrival time of photons from the source, measured on the satellite, as $\tilde{t}$. When the source coordinates are known exactly, the barycentre arrival time t is found as

$$t = \tilde{t} + \Delta T\,, \tag{6}$$

and having barycentre arrival times it is possible to find ν_0, $\dot{\nu}_0$ and $\ddot{\nu}_0$ using the criteria from Gurin *et al.* (1992, 1996).

When the source coordinates are not known exactly and their possible errors are $\mathrm{d}\alpha$ and $\mathrm{d}\delta$, the error in the barycentre correction is determined by (4). In order to estimate the input of these errors on the timing characteristics let us compare the phases of the arriving signal calculated from measurements ϕ' (with errors) and in true barycentre time ϕ, so that

$$\phi' = \int_0^{t'} \nu' \, \mathrm{d}t' \,, \quad \phi = \int_0^{t} \nu \, \mathrm{d}t \,. \tag{7}$$

Here t' is the time calculated from (6), and ν' is the frequency found after barycentre corrections, containing errors. Times t and t' correspond to the same event, so we may rewrite ϕ' in true barycentre coordinates as

$$\phi' = \int_0^{t} \nu \left[1 + \frac{\mathrm{d}\delta T}{\mathrm{d}t}\right] \mathrm{d}t \,. \tag{8}$$

Using (4) and (5) in (8) we obtain, after integration and taking account of (4), (5):

$$\begin{aligned} \phi' &= \mathrm{const} + \int_0^t \nu \, \mathrm{d}t + \nu \delta T - \int_0^t \delta T (\dot{\nu}_0 + \ddot{\nu}_0 t) \, \mathrm{d}t \\ &= \mathrm{const} + \int_0^t \nu \, \mathrm{d}t + \left(\nu - \frac{\ddot{\nu}_0}{\omega^2}\right) \delta T - (\dot{\nu}_0 + \ddot{\nu}_0 t) \int_0^t \delta T \, \mathrm{d}t \,. \end{aligned} \tag{9}$$

Here and below the relations

$$\delta \ddot{T} = -\omega^2 \delta T \,, \quad \int \left(\int \delta T \, \mathrm{d}t \right) \mathrm{d}t = -\frac{\delta T}{\omega^2} \,, \quad \frac{\mathrm{d}\delta T}{\mathrm{d}t} = -\omega^2 \int \delta T \, \mathrm{d}t \tag{10}$$

are used, and it follows from (4) that

$$\int \delta T \, \mathrm{d}t = -\frac{R}{c\omega} [\cos\delta \cos(\omega t - \alpha) \, \mathrm{d}\alpha + \sin\delta \sin(\omega t - \alpha) \, \mathrm{d}\delta] \,. \tag{11}$$

Differentiating (9) we obtain the input of the angular coordinate errors into the values of frequency and its derivatives

$$\begin{aligned} \nu' &= \frac{\mathrm{d}\phi'}{\mathrm{d}t} = \nu \left(1 + \frac{\mathrm{d}\delta T}{\mathrm{d}t}\right) , \\ \dot{\nu}' &= \frac{\mathrm{d}^2\phi'}{\mathrm{d}t^2} = \dot{\nu} \left(1 + \frac{\mathrm{d}\delta T}{\mathrm{d}t}\right) - \nu \omega^2 \delta T \,, \\ \ddot{\nu}' &= \frac{\mathrm{d}^3\phi'}{\mathrm{d}t^3} = \ddot{\nu} + (\ddot{\nu} - \nu\omega^2) \frac{\mathrm{d}\delta T}{\mathrm{d}t} - 2\dot{\nu}\omega^2 \delta T \,, \end{aligned} \tag{12}$$

where

$$\dot{\nu} = \dot{\nu}_0 + \ddot{\nu}_0 t, \quad \ddot{\nu} = \ddot{\nu}_0, \tag{13}$$

and the current values ν', $\dot{\nu}'$ and $\ddot{\nu}'$ are connected with the corresponding values at $t = 0$ as

$$\begin{aligned} \nu' &= \nu_0' + \dot{\nu}_0' t + \ddot{\nu}_0' \frac{t^2}{2}, \\ \nu_0' &= \nu' - \dot{\nu}' t + \ddot{\nu}_0' \frac{t^2}{2}, \\ \dot{\nu}_0' &= \dot{\nu}' - \ddot{\nu}' t, \\ \ddot{\nu}_0' &= \ddot{\nu}'. \end{aligned} \tag{14}$$

A detailed variant of previous calculations can also be found in Bisnovatyi-Kogan *et al.* (1997).

In the presence of the proper motion of the source the errors $d\alpha$ and $d\delta$ change linearly in a first approximation as

$$d\alpha = d\alpha_0 + v_\alpha t, \quad d\delta = d\delta_0 + v_\delta t. \tag{15}$$

This leads to further complication of the formulas (9)–(12). Note that in simulations we deal not with these formulae, but directly with arrival times of photons $\tilde{t}_i$, t_i' and t_i. In the problem of the timing of radio pulsars the precision of observational data is very high, so the appearance of the periodical one-year component gives a direct indication of the errors in the angular coordinates of the pulsar, and gives in the opportunity to improve them (Taylor, 1992; Harrison *et al.*, 1993), and to determine the proper motion. In periodic sources with rare pulses the duality of the data is much worse and other methods, based on criteria from Gurin *et al.* (1988a, b), must be used.

3 MATHEMATICAL SIMULATION

For checking the possibility of using the criteria mentioned above for the determination of corrections to the angular coordinates and proper motion, in addition to frequency and its two derivatives, an artificial sample of data was produced. A pulse shape was taken as a δ-function with the frequency of the signal changing in time according to (5), which corresponds to a phase dependence

$$\phi = \phi_0 + \nu_0 t + \dot{\nu}_0 \frac{t^2}{2} + \ddot{\nu}_0 \frac{t^3}{6}. \tag{16}$$

We need to find time moments, corresponding to phase values $\phi_i = 2\pi i$. Two sets of input parameters were considered. The time $t = 0$ is related to a point of the orbit, where $\alpha = 0$.

(i) $$\phi_0 = 0,\ \nu_0 = 4\ \mathrm{s}^{-1},\ \dot{\nu}_0 = -2 \times 10^{-8}\ \mathrm{s}^{-2}, \\ \ddot{\nu}_0 = 3 \times 10^{-16}\ \mathrm{s}^{-3},\ \omega = 6.060171 \times 10^{-6}\ \mathrm{s}^{-1};$$

(ii) $$\phi_0 = 0,\ \nu_0 = 4\ \mathrm{s}^{-1},\ \dot{\nu}_0 = -2 \times 10^{-13}\ \mathrm{s}^{-2}, \\ \ddot{\nu}_0 = 3 \times 10^{-26}\ \mathrm{s}^{-3},\ \omega = 1.991063802 \times 10^{-7}\ \mathrm{s}^{-1}. \tag{17}$$

The source parameters are chosen,to satisfy the relation $\nu_0\ddot{\nu}_0/\dot{\nu}_0^2 = 3$, supposed to be valid for ejecting pulsars (Manchester and Taylor, 1977). Because of the low reliability of $\ddot{\nu}$ detection in gamma observations, some authors (Mattox *et al.*, 1996) set it equal to zero. The modelling year duration $(2\pi/\omega)$ is equal to 12 days in the first case; and has a true value of 365.2422 days in the second, when the moment $t = 0$ corresponds to 21 March. Note that in the second case the values of ν_0 and $\dot{\nu}_0$ are chosen very close to that of Geminga (Hermsen *et al.*, 1992).

One possible way to find t_i is to use the Burmann–Lagrange expression, which links the Taylor coefficients of direct and inverse functions. We have used instead a procedure, valid for a general law of the phase dependence $\phi(t)$, based on a Taylor expansion formula

$$t_i = \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{\mathrm{d}^n t}{\mathrm{d}\phi^n}\right)_0 \phi_i^n \equiv \sum_{n=0}^{\infty} \frac{A_n}{n!}\phi_i^n\,. \tag{18}$$

To find the coefficients $A_n = (\mathrm{d}^n t/\mathrm{d}\phi^n)_0$, $n \geq 1$, we use the evident equality

$$\dot{\phi}\frac{\mathrm{d}t}{\mathrm{d}\phi} = 1, \tag{19}$$

which gives

$$A_1 = \frac{\mathrm{d}t}{\mathrm{d}\phi_0} = \frac{1}{\dot{\phi}_0}\,. \tag{20}$$

Differentiating (19) $(n-1)$ times with respect to t we obtain a relation, linear in A_n, which permits us to express A_n as a function of A_m, $m \leq n-1$, and $\overset{[m]}{\phi}$, $\overset{[n]}{\phi}$. As an example, after five differentiations we get

$$\begin{aligned} A_1\overset{[6]}{\phi}_0 + 6A_2\overset{[5]}{\phi}_0\dot{\phi}_0 + 15A_2\overset{[4]}{\phi}_0\ddot{\phi}_0 + 10A_2\dddot{\phi}_0^2 + 15A_3\overset{[4]}{\phi}_0\dot{\phi}_0^2 & \\ +60A_3\dot{\phi}_0\ddot{\phi}_0\dddot{\phi}_0 + 15A_3\ddot{\phi}_0^3 + 20A_4\dot{\phi}_0^3\dddot{\phi}_0 + 45A_4\dot{\phi}_0^2\ddot{\phi}_0^2 & \\ +15A_5\dot{\phi}_0^4\ddot{\phi}_0 + A_6\dot{\phi}_0^6 &= 0, \end{aligned} \tag{21}$$

where the index '0' indicates time $t = 0$. For (16) with $\overset{[m]}{\phi}_0 = 0$ at $m \geq 4$ we have $\dot{\phi}_0 = \nu_0$, $\ddot{\phi}_0 = \dot{\nu}_0$, $\dddot{\phi}_0 = \ddot{\nu}_0$ and from (21) we get an equation for A_6

$$\begin{aligned} 10A_2\ddot{\nu}_0^2 + 60A_3\nu_0\dot{\nu}_0\ddot{\nu}_0 + 15A_3\dot{\nu}_0^3 + 20A_4\nu_0^3\ddot{\nu}_0 & \\ +45A_4\nu_0^2\dot{\nu}_0^2 + 15A_5\nu_0^4\dot{\nu}_0 + A_6\nu_0^6 &= 0. \end{aligned} \tag{22}$$

The first criterion K_1 of periodicity (Gurin *et al.*, 1988a, b; 1992; Brazier, 1994) was used to investigate the periodicity properties of series of pulses. For the purpose of testing, short intervals of 'observation' were taken in a different parts of the year. The error in the coordinates was taken equal to 5 arc seconds in absolute value $\sqrt{\mathrm{d}\alpha^2 + \mathrm{d}\delta^2}$, but the deviations from the initial point were taken in eight different

Table 1. Properties of a source with angular coordinates and timing characteristics close to Geminga from (17), case (ii), calculated using criterion K_1. Barycentre corrections have been done using angular coordinates with indicated errors.

Source coordinates: $\alpha = 6^h 30^m 00^s$, $\delta = +17°30'00''$

Month, day	*Errors*		*Theoretical values*		
	$d\alpha$, $''$	$d\delta$, $''$	$\tilde{\nu}_0$, s^{-1}	$\tilde{\dot{\nu}}_0$, 10^{-13} s^{-2}	$\tilde{\ddot{\nu}}_0$, 10^{-13} s^{-2}
May, 20	5.00	0.00	3.99999999748	−1.9738	−28600
May, 20	2.24	2.24	3.99999999744	−1.9879	−9620
May, 20	0.00	5.00	3.99999999681	−1.9992	7140
May, 20	−2.24	2.24	3.99999999970	−2.0114	16000
May, 20	−5.00	0.00	4.00000000252	−2.0262	28600
May, 20	−2.24	−2.24	4.00000000256	−2.0121	9620
May, 20	0.00	−5.00	4.00000000319	−2.0008	−7140
May, 20	2.24	−2.24	4.00000000030	−1.9989	−16000
July, 10	5.00	0.00	3.99999999621	−1.9695	−35700
Aug., 20	5.00	0.00	4.00000000544	−1.9852	−22300
Aug., 20	0.00	−5.00	4.00000001020	−2.0155	9110
Nov., 20	5.00	0.00	4.00000008067	−2.0725	29150
Nov., 20	−2.24	2.24	3.99999996211	−1.9631	−16200

directions, separated by 45°. The values of ν_0', $\dot{\nu}_0'$, $\ddot{\nu}_0'$ that have been detected by the K_1 criterion coincide in both cases, with a very high precision with theoretical ones from (12)–(14); see Table 1. The strong influence of the errors on the determination of $\ddot{\nu}'$ by using the criterion may be seen from Table 1. While the error in $\ddot{\nu}'$ is almost linearly proportional to the error in $\sqrt{d\alpha^2 + d\delta^2}$, it is evident that at an angular error larger then 10^{-3} arc sec direct determination of $\ddot{\nu}'$ by the criterion becomes impossible. This may be a reason for the high braking index of Geminga (Hermsen *et al.*, 1992; Bisnovatyi-Kogan and Postnov, 1993).

Let us now formulate the problem of the timing of a gamma pulsar, which gives the possibility of a search of its timing properties together with angular coordinates and the proper motion. Assume that the gamma pulsar simulated by computer is emitting signals with a frequency changing according to (5), satisfying the condition $\nu_0\ddot{\nu}_0/\dot{\nu}_0^2 = 3$.

Let the signal registered on the probe be reduced to the barycentre time, using the source coordinates α_0 and δ_0 (base point), which contain errors $d\alpha_0$ and $d\delta_0$, respectively. We suspect also a proper motion of the source defined by the following parameters: at the moment $t_0 = 0$ the source has coordinates $\alpha_0 + d\alpha_0$ and $\delta_0 + d\delta_0$ and a velocity modulus $v_< = \text{const}$. The velocity direction is defined by an angle θ, which is counted clockwise from the positive α-axis direction. The current source coordinates are consequently

$$\alpha = \alpha_0 + d\alpha_0 + v_< t\cos\theta, \quad \delta = \delta_0 + d\delta_0 + v_< t\sin\theta, \tag{23}$$

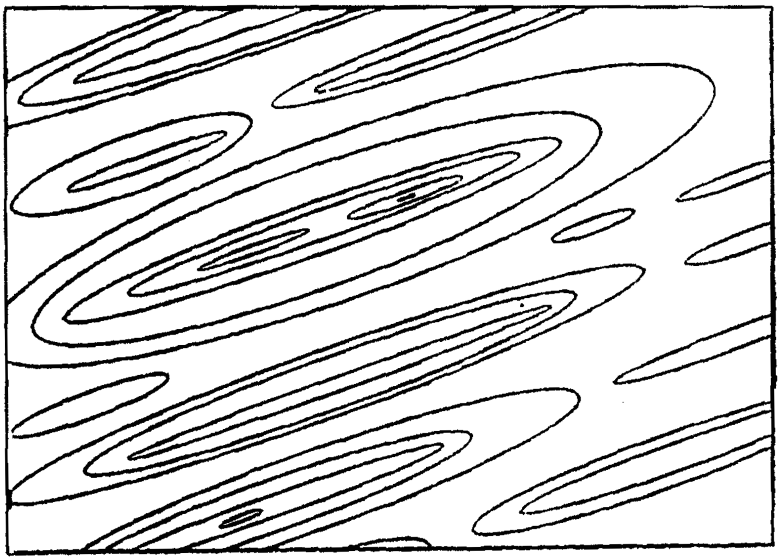

Figure 2 The absolute maximum structure.

Thus, we have seven parameters that need to be found self-consistently:

ν_0 – frequency of the source signal;

$\dot{\nu}_0$ – first derivative of the frequency;

$\ddot{\nu}_0$ – second derivative of the frequency;

$d\alpha_0$ – shift in right ascension from the base point (α_0, δ_0) at epoch t_0;

$d\delta_0$ – shift in declination from the base point (α_0, δ_0) at epoch t_0;

$v_<$ – proper angular velocity of the source in celestial coordinates;

θ – direction of velocity $v_<$, counted clockwise from the positive α-axis direction;

with the additional restriction $\nu_0\ddot{\nu}_0/\dot{\nu}_0^2 = 3$ following from the model of the pulsar radiation (Manchester and Taylor, 1977).

In data simulation we fix the parameters ν_0,$\dot{\nu}_0$, $\ddot{\nu}_0$ as (ii) in (17), find the true α and δ from (23) with $d\alpha_0 = 2''$, $d\delta_0 = 3''$, $v_< = 0''.2$ per year, $\theta = 60°$, and create the simulated data, i.e. the sequence of time moments of the pulses. Time moments (true barycentre) found for a source from (19)–(22) are then recalculated for the probe using the correct coordinates and (23). Now to a set of time moments 'registered' by a probe from the source with subscribed coordinates α_0 and δ_0, containing errors, we apply the algorithm for searching the periodic signal to extract all seven parameters from the simulated data set. Namely, we consider a number

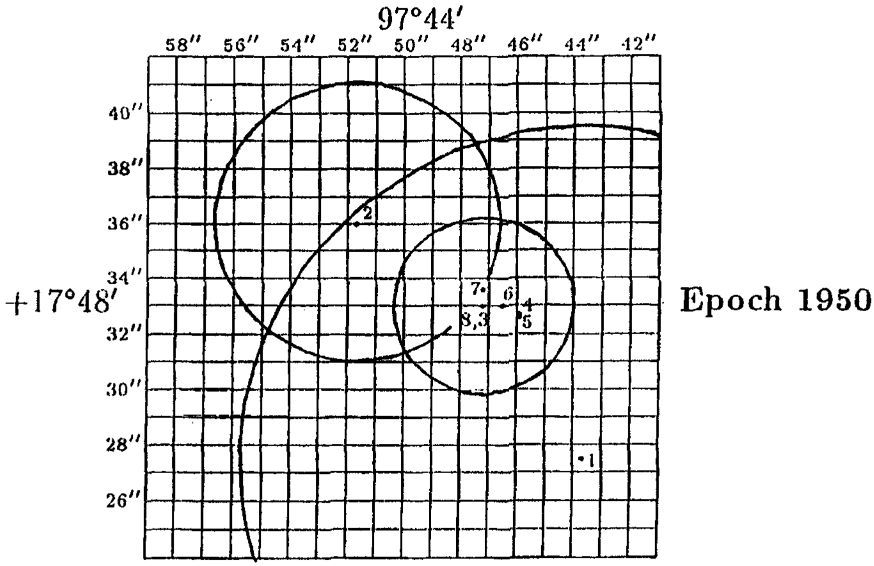

Figure 3 The Geminga position.

of different sets of the seven mentioned parameters. For each set we evaluate the supposed errors, introduced in the data due to errors in the position and in the proper motion of the object. After that we subtract these supposed errors from the data. Then, assuming the data free of errors the periodicity criterion value was calculated. Recall that if we assume the data free of errors, it means that the phases of the pulses must obey the simple relation (16). The criterion reaches its absolute maximum only for an exact set of parameters. There appear a number of local (or false) maxima. Figure 2 demonstrates the typical structure of the criterion depending on two parameters: ν and $\dot{\nu}$, when the other five are fixed (see also Mattox *et al.*, 1996). Because the height of a 'false' maximum is close to the 'true' one, it is very difficult to separate the absolute maximum among a series of local ones.

We are looking for an absolute maximum, using a grid in four-dimensional space ($d\alpha_0$, $d\delta_0, v_<, \theta$), which was defined in the following way: $d\alpha_0$ varies from $-5''$ step $0\overset{''}{.}5$ to $5''$, $d\delta_0$ varies from $-5''$ step $0\overset{''}{.}5$ to $5''$, $v_<$ varies from $0''$/year step $0\overset{''}{.}05$/year to $0\overset{''}{.}5$/year and θ varies from $0°$ step $10°$ to $350°$. Three other parameters $(\nu_0, \dot{\nu}_0, \ddot{\nu}_0)$ were detected jointly for each point of the above grid, using the grid $6 \times 6 \times 6$ with 12 times consecutively diminishing steps for all three axes. The maximum, detected at the previous step was placed to the centre of the grid and all the scale multiplied by a factor of 0.4. This procedure was repeated 12 times, so that the last grid steps are $0.4^{12} \approx 1.7 \times 10^{-5}$ times as small as the first ones. All seven parameters chosen for modelling were found with precision limited only by the computational grid connected with the power of the computer, using criterion K_1. So, for a clean set of data the proposed procedure of searching is

Table 2. Geminga position from different measurements.

		Geminga position, epoch 1950		
No.	α	δ	*Error*	*Comments*
1	$97°44'43''.7$	$+17°48'27''.5$	$12''$	ROSAT PSPC, 1991, Sep. 19–21, (Becker, 1995)
2	$97°44'51''.7$	$+17°48'36''.0$	$\approx 5''$	ROSAT HRI, 1991, Mar. 19, (Becker, 1995)
3	$97°44'47''.2$	$+17°48'33''.0$	$3''.2$	Einstein, 1981, Mar. 18, (Becker, 1995)
4	$97°44'45''.9$	$+17°48'32''.7$	$0''.46$	G'' star, 1984, (Bignami *et al.*, 1993)
5	$97°44'45''.9$	$+17°48'32''.6$	$0''.5$	G'' star, 1986, Feb. 3, (Becker, 1995)
6	$97°44'46''.5$	$+17°48'33''.0$	$0''.68$	G'' star, 1987, (Bignami *et al.*, 1993)
7	$97°44'47''.2$	$+17°48'33''.6$	$0''.16$	G'' star, 1992, (Bignami *et al.*, 1993)
8	$97°44'47''.2$	$+17°48'33''.0$	$3''.0$	Einstein, 1981, Mar. 18 (Bignami *et al.*, 1983)

working effectively. The situation becomes much more controversial when we apply it to real data existing at the moment.

4 APPLICATION TO GEMINGA

4.1 Analysis of COS-B Data

The COS-B mission operated from August, 1975 till April 1982, and had observed Geminga in five shifts (Mayer-Hasselwander, 1985; Bignami and Caraveo, 1992). Their numbers are 00, 14, 39, 54 and 64. Some measurements of the Geminga position, reduced to the epoch 1950.0, are summarized in Table 2 and are plotted in Fig. 3.

From COS-B data (Hermsen *et al.*, 1992) it was found that: $\nu = 4.217$ Hz, $\dot{\nu} = -1.952 \times 10^{-13}$ Hz s^{-1}, and a large value of $\ddot{\nu} = (28 \pm 16) \times 10^{-26}$ Hz s^{-2}, corresponding to a braking index $n = 31 \pm 18$. Barycentre corrections have been done for standing Geminga with coordinates No. 3, 8 in Table 2. Quanta section used in our analysis has been done in two different ways:

1. all the quanta within a circle of $r = 5°$ around the Geminga position; the interval 54 was excluded because of low reliability; it is a total of 1505 quanta;

2. all the quanta with energy $E > 50$ MeV laying in the circle $r = 12.5 \times E^{-0.16}$, where E is measured in MeV and r in degrees (Buccheri *et al*, 1983); a total of 1883 quanta.

The second selection is close to that used in Bignami and Caraveo (1992). The problem of the quanta selection criterion is very delicate one because it is practically impossible, for a single quantum, to decide which it was really radiated by Geminga or belongs to the background. Both these mentioned selections are noisy, but the first one is worse.

Geminga was considered as a moving object. The base coordinates that are used for initial barycentre reduction are the position measured by the Einstein satellite in 1981 (see Table 2). Geminga motion was defined by its velocity, direction and initial position at the epoch 1979, March, 14.0 (Hermsen *et al.*, 1992). The obtained barycentre time moments for each quantum were additionally reduced to the barycentre, using expression (2). The motion of the probe defined by $x_0(t)$, $y_0(t)$, $z_0(t)$ was taken from databases of COS-B (Mayer-Hasselwander, 1985) or EGRET. The object coordinates were calculated by this procedure separately for each quantum according to the supposed object motion.

The results using the second selection from those mentioned above, are not very certain. The criterion appears to exhibit a gently sloping maximum at the following model parameters: velocity $v_< = 0''\!.2$–$0''\!.3$ per year, direction $\theta = 40°$–$60°$ and initial coordinate offsets $d\alpha_0$ and $d\delta_0$ at this epoch are $-2''$ for both the α- and δ-axes, but the uncertainty here is high and may reach $2''$ for both coordinates. As to the periodicity parameters, they are in good agreement with Hermsen *et al.* (1992) except the second derivative $\ddot{\nu}_0$, which is a bit smaller but lies within the error box of standing Geminga. The motion of Geminga, obtained in our investigation does not contradict the motion of the G'' star (Bignami *et al.*, 1993).

Unfortunately, this solution is not unique, and there are a number of other maxima of approximately the same height. When we search for a global maximum in seven-dimensional space it is more difficult to detect 'the main maximum' among a series of other local maxima. For example there is an accessory maximum at $v_< = 0''\!.1$–$0''\!.2$ per year, $\theta = 340°$–$360°$ and very badly detected initial offsets (it can only be said that they are both negative). The periodicity parameters here are approximately the same as there above, but $\ddot{\nu}_0$ appears to be negative.

4.2 Analysis of EGRET Data

The EGRET experiment has been operating since April 1991. There are nine periods of observations where Geminga was not far from the centre of the field of view (less than 30°, the standard requirement). The following sessions were used for data investigations: 2, 3, 4, 5, 10, 21, 2130, 2210, 3100. The standard procedure for the barycentre correction was used (Pence, 1995a, b), but we have used coordinates of the object from Bignami *et al.* (1983), line 8 in Table 2, different from those, indicated in the EGRET database. We have used the same Geminga coordinates for both satellites, GOS-B and EGRET. They are the best fit Einstein position, but in the last case we were to reduce them to the epoch 2000.0, because it is used in the appropriate barycentre reduction routines. Reduced coordinates are the following:

$$\alpha_{2000} = 98°28'30''\!.90 = 98^\circ\!.47525, \quad \delta_{2000} = +17°46'11''\!.6 = +17^\circ\!.76989, \qquad (24)$$

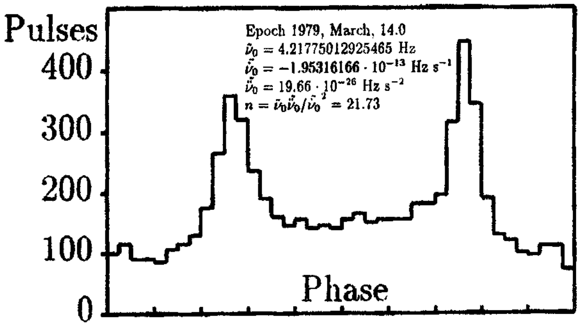

Figure 4 Geminga 40-bin light curve from EGRET data, for standing Geminga with cooordinates No. 8 from Table 2 and standard quanta selection.

And in the EGRET database the coordinates are:

$$\alpha_{2000} = 98^\circ\!.48 = 98^\circ 28' 48'', \quad \delta_{2000} = +17^\circ\!.77 = +17^\circ 46' 12''.$$

Other authors (Caraveo *et al.*, 1996; Mattox *et al.*, 1996) use the coordinates close to (24); they differ by less than $1''$ from the centre of the Einstein error box (lines 3, 8 in the Table 2).

We have used a number of techniques for quanta selection and have compared the results. The selections used are the following:

1. all the quanta of energy $E > 70$ MeV lying in the circle $r = 5.85 \times (E/100)^{-0.534}$, where E is measured in MeV and r in degrees (Thompson *et al.*, 1993); this is called the standard selection; a total of 6751 quanta;

2. all the quanta of energy $E > 1500$ MeV in the circle $r = 2^\circ$; a total of 365 quanta;

3. all the quanta of energy $E > 2000$ MeV in the circle $r = 2^\circ$; a total of 223 quanta;

4. all the quanta of energy $E > 2000$ MeV in the circle $r = 0^\circ\!.5$ round the Geminga position; a total of 100 quanta;

5. all the quanta of energy $E > 3000$ MeV in the circle $r = 0^\circ\!.5$ round the Geminga position; a total of only 51 quanta.

The criterion value for unmoving Geminga, resting in Einstein's HRI position, is $K_1 = 0.0572$ and the parameters of periodicity are $\nu_0' = 4.21775012925$ Hz, $\dot{\nu}_0' = -1.95312 \times 10^{-13}$ Hz s^{-1}, $\ddot{\nu}_0' = (20 \pm 12) \times 10^{-26}$ Hz s^{-2}. An appropriate light curve is shown in Fig. 4. The results of a search in seven-dimensional space for different selections are:

1. There is a gently sloping maximum in the criterion value at the following parameters: $\nu_0 = 4.2177501295$ Hz, $\dot{\nu}_0 = -1.9532 \times 10^{-13}$ Hz s^{-1}, $\ddot{\nu}_0 = (22 \pm 13) \times 10^{-26}$ Hz s^{-2}, $v_< = 0''.3$–$0''.4$ per year, $\theta = 40°$–$60°$, $d\alpha_0$ and $d\delta_0$ are defined with very low precision and both lie in the interval $-2''$ to $0''$. The criterion value is $K_1 = 0.05732$. The second derivative $\ddot{\nu}$ here is of rather large value, so that the braking index is approximately equal to 25.

2. There is a gentle maximum in the criterion value at the following parameters: $\nu_0 = 4.2177501273$ Hz, $\dot{\nu}_0 = -1.952711 \times 10^{-13}$ Hz s^{-1}, $\ddot{\nu}_0 \approx (0 \pm 10) \times 10^{-26}$ Hz s^{-2}, $v_< = 0''.3$ per year or more, $\theta = 40°$–$60°$, $d\alpha_0 = 0''$–$(-2'')$, $d\delta_0 = 0''$–$(-1'')$ and the criterion value is $K_1 = 0.22357$. The second derivative here is very close to zero and the braking index is, respectively, also low and could be close to its theoretical value.

3. There is a gentle maximum in the criterion value at approximately the following parameters: $\nu_0 = 4.2177501261$ Hz, $\dot{\nu}_0 = -1.952611 \times 10^{-13}$ Hz s^{-1}, $\ddot{\nu}_0 = (-2.4 \pm 10) \times 10^{-26}$ Hz s^{-2}, $v_< = 0''.3$–$0''.4$ per year, $\theta = 40°$–$80°$, $d\alpha_0 = 0''$–$(-2'')$, $d\delta_0 = -1''$–$(-3'')$ and the criterion value is $K_1 = 0.25535$. It was impossible to determine the parameters with higher precision.

4. There is a gentle maximum in the criterion value at the following parameters: $\nu_0 = 4.2177501344$ Hz, $\dot{\nu}_0 = -1.954477 \times 10^{-13}$ Hz s^{-1}, $\ddot{\nu}_0 = (68 \pm 40) \times 10^{-26}$ Hz s^{-2}, $v_< = 0''.3$–$0''.4$ per year or more, $\theta = 220°$–$260°$, $d\alpha_0$ and $d\delta_0$ are negative and the criterion value is $K_1 = 0.3072$. Because of the very poor statistics this selection as well as the following one can be considered as a test only. The line of motion here is the same as in the previous cases, but the direction is opposite.

5. There is a relatively good maximum despite the very poor statistics at the following parameters: $\nu_0 = 4.2177501220$ Hz, $\dot{\nu}_0 = -1.95282 \times 10^{-13}$ Hz s^{-1}, $\ddot{\nu}_0 = (12 \pm 10) \times 10^{-26}$ Hz s^{-2}, $v_< = 0''.2$–$0''.3$ per year, $\theta = 20°$–$60°$, $d\alpha_0 = 0''$–$(-2'')$, $d\delta_0 \approx 0''$ and the criterion value is $K_1 = 0.44441$. The reliability of this result is not high, but a large criterion value indicates that we may deal with real motion of the object.

4.3 Analysis of Combined COS-B and EGRET Data

We have combined COS-B and EGRET data with the following selection criteria:

COS-B: $E > 50$ MeV and the standard conditions for $r : r = 12.5 \times E^{-0.16}$ (Buccheri *et al.*, 1983);

EGRET: $E > 70$ MeV and the standard conditions for $r : r = 5.85 \times (E/100)^{-0.534}$ (Thompson *et al.*, 1993).

There are a total of $1883 + 6751 = 8634$ quanta.

The combined series is not self-contradicting. There is a gentle maximum in the criterion value at the following parameters: $\nu_0 = 4.21775012323 \pm 0.00000000025$ Hz, $\dot{\nu}_0 = (-1.952554 \pm 0.000025) \times 10^{-13}$ Hz s^{-1}, $\ddot{\nu}_0 = (-2.5 \pm 10) \times 10^{-26}$ Hz s^{-2}, $v_< = 0''.5$–$0''.6$ per year, $\theta = 55°$–$65°$, $d\alpha_0 = -2''$–$(-4'')$, $d\delta_0 = 1''$–$2''$ and the criterion value is $K_1 = 0.04937$.

Note that the 1σ errors above were obtained by approximate estimations.

5 DISCUSSION

According to our investigations the coordinates and motion of Geminga obtained from the timing of the gamma pulsar is in a satisfactory agreement with the motion of the G'' star when COS-B or EGRET data are used separately. The parameters following from the combined data set are in much worse agreement. There are two possible reasons of this. First there could exist a systematic error between the data of two probes; and second, the period could behave non-monotonically between the years 1982 and 1991, and the period jump (pulsar glitch) of the order of $\Delta P/P \geq 10^{-10}$ could already spoil the parameters obtained by the criteria.

The value of the second derivatives $\ddot{\nu}_0$ does not coincicle with the theoretical one; however it is lower than in the previous investigations, but there are weighty reasons to explain this phenomenon. The second derivative is a very sensitive variable, and even a $0''.001$ error in angular coordinates changes $\ddot{\nu}_0$ by a value comparable with the result (see Table 1). If there was a jump in the pulsar period, it may also cause an incorrect value of a variable. The possibility of improving the angular resolution by timing is strongly limited by the small number of quanta and the existence of considerable background.

As a result of application to Geminga of the developed method of timing of gamma pulsars we have found that the determination of the true value of $\ddot{\nu}_0$ is possible only at very high precision (better then $0''.001$) of angular localization. With good statistics of gamma pulsars corresponding improvements would become possible from timing analysis. New types of gamma ray telescopes based on very wide aperture ($\geq 2.5\pi$ steradian) and higher threshold of a few hundred MeV (Bisnovatyi-Kogan and Leikov, 1993; Leikov and Bisnovatyi-Kogan, 1994) would permit us to get higher angular resolution (~ 1 arc min) reducing the influence of the background, and get ~ 100 times better statistics due to continuous monitoring of larger parts of the sky in this region.

For the existing data of Geminga from COS-B and EGRET it was found, using only gamma-ray data, that the criterion value reaches its maximum at a non-zero value of the proper motion. The coordinates of the source were confirmed with a precision $\sim 2''$ which is better than that which follows from X ray data, but of course is worse than the precision obtained in optical observations.

Observations of radio pulsars have shown that their optical and X-ray luminosity is decreasing with time much more rapidly than radio and hard gamma radiation. So at increasing sensitivity we expect the discovery of tens of new gamma-ray pulsars similar to Geminga, perhaps without X-ray and optical counterparts. For such objects the method of timing of gamma-ray pulsars developed above would be the main and perhaps the only means of investigation of such sources by data processing.

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