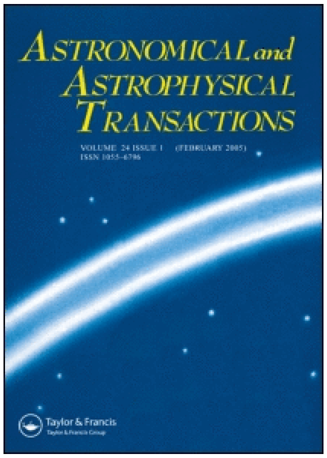

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 December 1999
To cite this Article: Sapar, A. and Sapar, L. (1999) 'Formation of resonance spectral line profiles in stellar wind: Theory and observations', Astronomical & Astrophysical Transactions, 18:3, 475 - 485
To link to this article: DOI: 10.1080/10556799908203004
URL: http://dx.doi.org/10.1080/10556799908203004

Astronomical and Astrophysical Transactions, 1999,
Vol. 18, pp. 475–485
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FORMATION OF RESONANCE SPECTRAL LINE PROFILES IN STELLAR WIND: THEORY AND OBSERVATIONS

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(Received March 10, 1998)

The theory of the formation of resonance spectral line profiles in the stellar wind of hot stars has been elaborated taking into account the presence of microturbulent motions. In this case the local frequency redistribution in the line profile function is incomplete. This circumstance generates dominating backscattering of photons and drastic changes in the resonance line profiles, including the presence of wide dark plateaux in the central region of the profile. It also modifies essentially the dynamics of the radiatively driven stellar wind. The resonance spectral line profiles, their variability and DACs in them have been analysed based on IUE spectra of a set of B supergiants and on the theoretical concepts.

KEY WORDS Early-type stars, mass loss in stars, line profiles, scattering, turbulence, ultraviolet stars

1 INTRODUCTION

The formation of resonance spectral line profiles in stellar wind is a sophisticated problem and it is crucial for the quantitative analysis of stellar wind dynamics. The theory and computer codes for the study of resonance spectral line profiles have been elaborated in various approximations, based on different assumptions and input physics (Castor *et al.*, 1975; Pauldrach *et al.*, 1986; Owocki, 1992). In this study we confine ourselves to isotropic and stationary stellar winds, thus ignoring stellar rotation and magnetic fields. Our treatment is most adequate for stellar winds of hot supergiants, the IUE spectra of which we have studied. To interpret the peculiarities of observed stellar resonance line profiles and stellar wind dynamics, we assume besides the thermal motion also the presence of strong supersonic turbulence in the stellar wind. We do not discuss the mechanisms of its generation in the present paper.

2 SCATTERING WITH COMPLETE FREQUENCY REDISTRIBUTION

The thickness of the absorbing stellar wind layer in the z-direction is given by (see Sapar and Sapar, 1990)

$$\frac{\partial v_z}{\partial z}\Delta z = v_D, \quad \frac{\partial v_z}{\partial z} = \frac{v}{r}C_\mu, \quad C_\mu = 1 + \mu^2\left(\frac{r}{v}\frac{dv}{dr} - 1\right) \tag{1}$$

where v_D is the mean Doppler velocity and r is the distance from the stellar centre. The optical depth along the light ray is $\tau_\mu = \sigma n \Delta z$. Taking into account that $K_\tau = \sigma v_D$ is a constant which does not depend on temperature, we can write

$$\tau_\mu = K_\tau n / \frac{\partial v_z}{\partial z}. \tag{2}$$

In the case of complete redistribution in the line profile (the Sobolev approximation) the source function at ν is constant throughout the interacting layer. From the equation of radiative equilibrium it follows that the source function S is to be found from

$$\sigma \int_{-1}^{1} \beta_\mu (S - I_*) d\mu + \kappa(S - B) = 0 \tag{3}$$

where the expression for the escape probability β_μ in the given direction μ is specified by

$$\beta_\mu = \frac{1}{\tau_\mu}(1 - e^{-\tau_\mu}). \tag{4}$$

For brevity we have omitted the index ν. If stellar limb darkening is ignored then equation (3) reduces to

$$(1-\varepsilon)\beta S + \varepsilon S = (1-\varepsilon)\beta_* I_* + \varepsilon B, \quad \varepsilon = \frac{\kappa}{\kappa + \sigma} \tag{5}$$

where

$$\beta = \int_0^1 \beta_\mu \, d\mu, \quad \beta_* = \frac{1}{2}\int_{\mu_l}^1 \beta_\mu \, d\mu. \tag{6}$$

In this formula μ_l is the value of μ for the stellar limb. The emergent intensity in the given direction μ can be represented by $I_\nu = S\tau_\mu\beta_\mu$. Expressing the Doppler frequency shift by

$$\Delta\nu = \nu - \nu_0 = \frac{v\mu}{c}\nu_0 \tag{7}$$

we fix the frequency shifts $\Delta\nu$ corresponding to the stellar disk centre, i.e. at $\mu = 1$ and $v(r_i)$. To find the emission line profile we can go from integrating over the impact parameters $a = r\sqrt{1-\mu^2}$ to integration over r. Thus we get for the accelerating stellar wind the emission contribution of the line profile in the form

$$L_\nu^e = \int I_\nu a\, da = \int_{r_i}^{\infty} S\tau_0 \beta_\nu r\, dr. \tag{8}$$

The last integral can be easily computed.

For the red wing we specify the Doppler shifts by $\mu = -1$ and use the corresponding negative values of the frequency shift index i. In this case the screening effect of the stellar disk is to be accounted for by changing the lower integration limit. The contribution by the penetrating stellar radiation into the line profile is given by

$$L_\nu^a = \int_0^{r_*} I_\nu^*(1-\beta_*) a\, da. \tag{9}$$

The line profile function can finally be found by

$$f_\nu = \frac{L_\nu^e + L_\nu^a}{L_\nu^*}, \quad L_\nu^* = \int_0^{r_*} I_\nu^* a\, da. \tag{10}$$

For the line profile of a doublet the calculation of its blue component profile is to be carried out in the same way as for a single spectral line. Thereafter absorption of its scattered radiation in the overlapping red component and its contribution to the source function of the red doublet component are to be taken into account.

3 PARTIALLY COHERENT SCATTERING

The presence of partial thermalization ($\varepsilon \neq 0$) changes drastically the line profiles, but it is important only for very dense WR stellar winds. For the study of the stellar wind of O, B and A stars we take $\varepsilon = 0$ which seems to be a good approximation. The presence of strong resonance lines of multiply ionized atoms, which appear only at high temperatures, makes evident the presence of turbulence and shock waves in stellar winds. In the case of partial frequency redistribution due to microturbulence and in a low-density stellar wind if the elastic collision rate during the age of the excited state in the resonance line transition is too small, the frequency redistribution is incomplete in multiple scattering chains. In this case the problem is much more complicated than in the Sobolev approximation, because the source function at a given frequency is not constant throughout the scattering layer. For partial frequency redistribution we get

$$S_{t_1} = S_0 + S' t_1 = S_0 + S' q_\mu t_\mu, \quad q_\mu = \frac{r\, dv}{v\, dr} / C_\mu \tag{11}$$

where S_0 is the value of the source function at the outer boundary of the scattering layer and t_μ is the current optical depth in the given direction. We have found expressions for S_0 and S' for different cases of incomplete frequency redistribution due to the presence of turbulence. The results will be published in a separate paper. Thus in the case of pure scattering with partial frequency redistribution we obtain

$$\int_{-1}^{1} \beta_\mu (S_0 + S' q_\mu t_\mu - I_*)\, d\mu = 0. \tag{12}$$

From here it follows that

$$\beta S_0 + \gamma S' = \beta_* I_*, \quad \gamma = \int_{-1}^{1} q_\mu t_\mu \beta_\mu \, d\mu. \tag{13}$$

4 RADIATIVE ACCELERATION

The dynamics of stellar wind is described by the momentum conservation equation

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{d(p+P)}{dr} - \frac{GM}{r^2}, \tag{14}$$

where the radiation pressure is

$$P = \int_0^\infty \int_{-1}^{1} \Pi_\nu \, d\nu d\mu, \quad \Pi_\nu = \frac{2\pi}{c}\mu^2 I_\nu. \tag{15}$$

Having found the source function, the next task is to determine the radiative force in the spectral lines of the stellar wind. We start from the general expression of the radiative force

$$\rho g_{\mathrm{rad}} = -\frac{dP}{dr} \tag{16}$$

where P is given by (15). Using this in (16) we can write

$$\rho g_{\mathrm{rad}} = -\int_0^\infty \int_{-1}^{1} \frac{d\Pi_\mu}{dr}\, d\mu d\nu = -\int_0^\infty \int_0^1 \left(\frac{d\Pi_\mu^+}{dr} + \frac{d\Pi_\mu^-}{dr}\right) d\mu d\nu. \tag{17}$$

Here we denote for the upwards propagating radiation the index "+" and for the downwards propagating radiation the index "–". Using instead of the infinitessimal differences in the derivatives the finite differences specified by the boundary values of the scattering layer we can write

$$\rho g_{\mathrm{rad}} = -\int_0^\infty \int_0^1 \left(\frac{\Delta\Pi_\mu^+}{\Delta r} + \frac{\Delta\Pi_\mu^-}{\Delta r}\right) d\mu d\nu. \tag{18}$$

Eliminating Δr by the expression $1/\Delta r = \frac{1}{v_D}\frac{dv}{dr}$, we get

$$\rho g_{\mathrm{rad}} = -\frac{2\pi}{cv_D}\frac{dv}{dr}\int\limits_0^\infty\int\limits_0^1 (\Delta I_\mu^+ + \Delta I_\mu^-)\mu^2\,d\mu d\nu. \tag{19}$$

In this expression

$$\Delta I_\mu^+ = -I_\mu^* \tau_\mu \beta_\mu + I_\mu^+, \quad \Delta I_\mu^- = -I_\mu^-. \tag{20}$$

Thus, finally

$$\rho g_{\mathrm{rad}} = \frac{2\pi}{cv_D}\frac{dv}{dr}\int\limits_0^\infty\int\limits_0^1 (I_\mu^* \tau_\mu \beta_\mu - I_\mu^+ + I_\mu^-)\mu^2\,d\mu d\nu. \tag{21}$$

Taking into account that $S_t = S^0 + S' q_\mu t_\mu$, the emerging scattered intensity at the outer surface of the scattering layer takes the form

$$I_\mu^+ = \int\limits_0^{\tau_\mu} S_t \exp(-t_\mu)\,dt_\mu = S_0 \tau_\mu \beta_\mu + S' q_\mu \tau_\mu (\beta_\mu - \exp(-\tau_\mu)), \tag{22}$$

and the downwards propagating intensity at the inner surface is

$$I_\mu^- = \int\limits_0^{\tau_\mu} S_t \exp(-\tau_\mu + t_\mu) dt_\mu = S_0 \tau_\mu \beta_\mu + S' q_\mu (\tau_\mu - 1 + \exp(-\tau_\mu)). \tag{23}$$

Using these expressions we find from (21)

$$\rho g_{\mathrm{rad}} = \frac{2\pi}{cv_D}\frac{dv}{dr}\int\limits_0^\infty\int\limits_0^1 (I_\mu^* \beta_\mu + S' q_\mu (1 + \exp(-\tau_\mu) - 2\beta_\mu))\mu^2\,d\mu d\nu. \tag{24}$$

As we see, the terms with S_0 cancel but the ones with S' give a definite contribution to the radiative acceleration of the stellar wind.

5 STELLAR WIND DYNAMICS

Let us formulate now a simple approximation formula for stellar wind dynamics. We start from the expression of radiative acceleration in its usual form

$$\rho g_{\mathrm{rad}} = \frac{4\pi}{c}\int\limits_0^\infty k_\nu H_\nu\,d\nu, \quad k_\nu = \kappa_\nu + \sigma_\nu. \tag{25}$$

The mean stellar flux in a spectral line l of the stellar wind is

$$H_l = H_l^* W_H \tag{26}$$

where the dilution for the flux is $W_H = \frac{r_*^2}{r^2}$ and H_l^* is the radiative flux at the stellar surface. In a strong resonance line its contribution to the radiative acceleration can be estimated using the radial gradient of the frequency of the scattering photons in the spectral line. So we obtain

$$\rho g_{\mathrm{rad}}^l = \frac{4\pi}{c} H_l \frac{d\nu_l}{dr}, \quad \frac{d\nu_l}{dr} = \nu_l \frac{dv}{c\,dr}. \tag{27}$$

Thus finally

$$g_{\mathrm{rad}}^l = 4\pi \frac{r_*^2}{\rho r^2} \frac{H_l^* \nu_l}{c^2} \frac{dv}{dr} = \frac{L_l \nu_l}{\dot{M} c^2} v \frac{dv}{dr} \tag{28}$$

where the stellar luminosity at the current frequency of the spectral line is $L_l = 4\pi r_*^2 4\pi H_l^*$. The total radiative acceleration in strong spectral lines is given by

$$g_L = \sum_l g_{\mathrm{rad}}^l = \frac{n_L L}{\dot{M} c^2} v \frac{dv}{dr} = n_L \alpha v \frac{dv}{dr}. \tag{29}$$

The effective number of lines accelerating the stellar wind in this formula is

$$n_L = \sum_l H_l^* \nu_l / (\sigma_B T_{\mathrm{eff}}^4) \tag{30}$$

and $\alpha = L/\dot{M}c^2$ where $L = 4\pi r_*^2 \sigma_B T_{\mathrm{eff}}^4$. The acceleration due to Thomson scattering on free electrons is $g_e = \frac{\Gamma GM}{r^2}$, where $\Gamma = \frac{\sigma_e n_e L}{4\pi p c G M}$. As a result of recoil during backscattering the general radiative acceleration in (29) must be about doubled. On the other hand, radiative acceleration in strong resonance lines drops when Δv reaches the value of the doublet component difference. The contribution of weak spectral lines to g_{rad} does not depend on velocity and it is to be found directly from (25). The behaviour of moderate spectral lines lies statistically between the two extreme cases. However, they do not change the situation drastically. For the gas pressure acceleration $g_p = -\frac{dp}{\rho\, dr}$, using the formulae $n = \sum_i n_i$, $\rho = \sum_i n_i m_i = n\bar{m}$, we can approximately write

$$g_p = -v_T^2 \frac{d \ln p}{dr}, \quad v_T^2 = \frac{kT}{\bar{m}}. \tag{31}$$

Now we can write the equation of wind dynamics in the form

$$\frac{dv^2}{2dr} = -v_T^2 \frac{d \ln p}{dr} + \alpha_L \frac{dv^2}{2dr} + \frac{\Gamma GM}{r^2} - \frac{GM}{r^2}, \quad \alpha_L(r) = n_L \alpha \tag{32}$$

from which

$$(1 - \alpha_L)\frac{v^2}{2} + v_T^2 \ln p - (1 - \Gamma)\frac{GM}{r} = \mathrm{const.} \tag{33}$$

The density n can be eliminated by $\dot{M} = 4\pi r^2 n\bar{m}v$. In the supersonic region we can ignore the gas pressure term. Thus we get

$$(1-\alpha_L)\frac{v^2}{2} - (1-\Gamma)\frac{GM}{r} = (1-\alpha_L)\frac{v_\infty^2}{2} = \text{const.} \tag{34}$$

from which we obtain

$$v^2 = v_\infty^2 + \frac{1-\Gamma}{1-\alpha_L}\frac{GM}{r}, \quad \alpha_L \gg 1. \tag{35}$$

Using (35) also in order to specify the sound velocity v_s at the sound point r_s and eliminating $\frac{(1-\Gamma)}{(1-\alpha_L)}$ we get

$$v^2 = v_\infty^2(1 - \frac{r_s}{r}) + \frac{v_s^2 r_s}{r}. \tag{36}$$

This expression is correct only in the supersonic region and gives the wind acceleration. At small velocities α_L tends to zero for which the correct solution can be found only using successive approximations. The most sophisticated is the sonic point neighbourhood for which self-consistency can be obtained by successive approximations. In the subsonic zone we in fact have a leakage regime: the upper layers are blown away and lower layers have to compensate for this loss. In the zone of small wind velocities we can calculate the corrections to the static model atmospheres by perturbation theory. Thus we obtain for the subsonic zone

$$v\frac{dv}{dr} = -\frac{d\Delta p}{\rho_0\,dr} + \frac{\Delta\rho}{\rho_0^2}\frac{dp_0}{dr} \tag{37}$$

and

$$\dot{M} = 4\pi r^2(\rho_0 + \Delta\rho)v. \tag{38}$$

At small velocities we can find v ignoring $\Delta\rho$ in the formula for $\dot{M}$ and thereafter solving equations (37) and (38).

We have studied stellar wind dynamics above using simplifications for radiative acceleration which correspond to complete frequency redistribution in spectral lines. As we have shown above, due to the dominant backscattering of photons in the case of a lack of complete frequency redistribution in resonance spectral lines the radiative acceleration can be up to doubled, increasing about twice the energy transfer to the stellar wind, i.e.

$$2\dot{M}_0(v_e^2 + v_0^2) \approx \dot{M}_n(v_e^2 + v_n^2).$$

In this expression v_e^2 is the escape velocity, $\dot{M}_0$ and $\dot{M}_n$ are, correspondingly, the mass loss rate according to traditional theory and to our treatment. As a result of backscattering there appear large changes in the line profiles. Namely, there will appear wide dark plateaux in the blue wing of the spectral line and more energy will be swept into the red wing. Due to additional radiation in the red spectral wing the maximum of the P Cygni profile will shift somewhat toward the red.

Further we study spectra of some B supergiants showing the presence of high-velocity supersonic motions in them and some evidences of dominant backscattering in resonance line profiles.

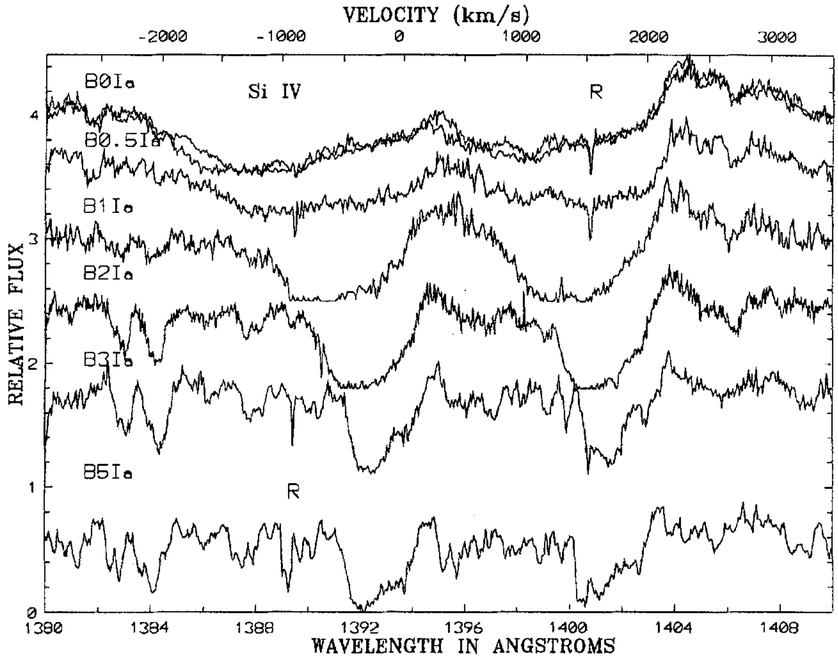

Figure 1 Si IV resonance doublet line profiles (λ 1393.76Å, 1402.77Å) for ϵ Ori (B0Ia, SWP 3402, 3483), κ Ori (B0.5Ia, SWP 3484), κ Cas (B1Ia, SWP 3480), χ^2 Ori (B2Ia, SWP 6471), 24 CMa (B3Ia, SWP 3469) and η CMa (B5Ia, SWP 3388) from IUE high resolution spectra. The dependence of the terminal velocity on the spectral class is clearly visible. R, contaminated reseau mark feature. Two exposures of ϵ Ori demonstrate the change of line profile.

6 OBSERVATIONS

High-resolution ultraviolet spectra of O and B stars show the presence of stellar winds; strong resonance lines have P-Cygni type profiles and these profiles are variable. These variations characterize the presence of fixed-position discrete absorption components (DACs) superimposed on the wide resonance spectrum line profile. The DACs migrate in the blue edge region of the absorption trough and in the region of line bottoms in the unsaturated P Cygni type resonance spectral lines, and their intensity changes. In order to study their migration, a continuous time series of high resolution and large signal-to-noise ratio observations are needed. Very often resonance line profiles of hot supergiants show a slanting slope of the blue wing of the profile and a very wide central dark plateau.

We have studied a number of high dispersion IUE spectra of selected supergiants of different B subclasses, namely ϵ Ori (B0Ia), κ Ori (B0.5Ia), κ Cas (B1Ia), χ^2 Ori (B2Ia), 24 CMa or o^2 CMa (B3Ia) and η CMa (B5Ia). We have analysed mainly the resonance line profiles of N V (λ 1238.81Å, λ 1242.50Å), C II (λ 1334.532Å, λ 1335.708Å), Si IV (λ 1393.76Å, λ 1402.77Å), C IV (λ 1548.20Å, λ 1550.77Å), and Al III (λ 1854.72Å, λ 1862.79Å), trying to find from these the main characteristics

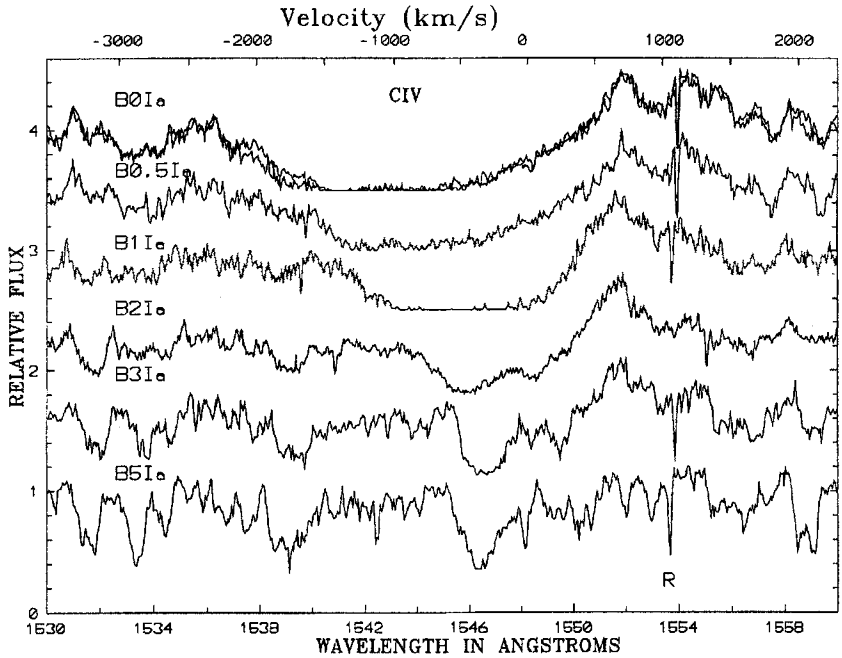

Figure 2 C IV resonance doublet line profiles (λ 1548.20Å, 1550.77Å) for ϵ Ori (B0Ia, SWP 3402, 3483), κ Ori (B0.5Ia, SWP 3484), κ Cas (B1Ia, SWP 3480), χ^2 Ori (B2Ia, SWP 6471), 24 CMa (B3Ia, SWP 3469) and η CMa (B5Ia, SWP 3388) from IUE high resolution spectra. The dependence of the extendness of the dark absorption trough on the spectral class is visible. *R*, contaminated reseau mark feature. Two exposures of ϵ Ori demonstrate the variability of the violet edge line profile.

of the stellar wind, to study the resonance spectral line profiles and their variability and the presence of fixed-position DACs in the spectra.

In Figures 1 and 2 we give Si IV and C IV resonance doublet line profiles for the above-mentioned B stars. From the spectral curves we see how the structure of the profile and the terminal velocities depend on the spectral class. The line profiles of late B subclasses are deeper and narrower and these stars have smaller terminal velocities. So, the terminal velocity of the stellar wind in η CMa is about -600 km s^{-1}, but the velocity in ϵ Ori (B0Ia) exceeds -2100 km s^{-1}. Beginning from spectral class B2 and earlier the Si IV and C IV resonance line profiles of the supergiants have wide saturated profiles with dark plateaux. So, the merged doublet profile of C IV in κ Cas has a dark plateau in the blue component from 0 to -1000 km s^{-1} and an emission feature in the red wing. The features can be explained by the presence of optically thick supersonic turbulence. In this case almost all radiation will be backscattered. A wide blue slope gives additional evidence for the presence of high-speed turbulence in the optically thin outer layers of the stellar wind. The velocities of the blue edge of Si IV and C IV resonance lines are variable. For example, the shifts of velocity of the blue edge of these resonance lines for 24

CMa reaches 200 km s^{-1}, which can be ascribed to variations of the turbulence velocity in the stellar wind.

For the B supergiants studied by us the presence of variability in the stellar wind is typical. We do not have a good series of continuous observations for all the studied stars. Only using for ϵ Ori a 10 exposure series, made during the time interval from January 28th to February 6th 1987, we can conclude that there are short-time variations lasting some days and that this time interval is a typical age for migrating DACs. The long-time variations exceed about twice the daily variations. The above-mentioned series of spectra give evidence of rapid change of DACs. The merged NV resonance doublet line profiles of ϵ Ori are unsaturated and have a strong red-wing elevation of P Cygni type, pointing out that it originates in the extended wind. However, the structure of this line in the spectrum recorded on September 9th 1979 differs drastically from the rest and it has an enigmatic run of the profile which can be treated as the presence of photospheric lines shifted to a velocity of −1200 km s^{-1} in both components of the N V resonance doublet. This behaviour somewhat reminds us of the observed extraordinarily deep and highly blue-shifted absorption events in the H_α line of B supergiants described as high-velocity absorption (HVA) (Kaufer *et al.*, 1996a, b). The variation of the spectrum of ϵ Ori is a good specimen to study large-scaled clumpy structures and shells in the stellar wind of hot supergiants and to discuss their generation and dynamics. The terminal velocity of the stellar wind is rather constant but the blue slope of all studied resonance lines varies essentially and its shape cannot be explained even by the theory of extended turbulent stellar wind. This holds for the wide dark plateaux of the resonance lines and for the observed DACs and their variations, giving evidence that the keyword to these problems is the instabilities, which originate both from the stellar atmospheres and the stellar wind, generating clumpiness and shell structures both in the velocity and density distributions.

7 CONCLUSIONS

The present study is primarily devoted to the analysis of radiation scattering in the resonance line profiles of stellar wind and of its dynamical action on the stellar wind. The presence of supersonic turbulence has been assumed and it has been shown how the dominant backscattering in this case modifies the line profiles, generating wide dark plateaux and approximately doubling the radiative acceleration. Physically dominant backscattering is due to a lack of complete frequency redistribution in the resonance spectral line profile functions due to scattering in turbulent stellar wind layers.

Using the high-resolution spectra of IUE for a sample set of B supergiants we have studied resonance line profiles, their peculiarities and time variations. The study suggests that turbulence phenomena and dominating backscattering are essential both in spectral line profile formation and in stellar wind dynamics.

Acknowledgement

We thank the Estonian Science Foundation for support of this work by grant ESF 2629.

References

Castor, J. I., Abbott, D. C., and Klein, R. I. (1975) *Astrophys. J.* **195**, 157.
Kaufer, A., Stahl, O., Wolf, B., Gäng, Th., Gummersbach, C. A., Kovács, J., Mandel, H., and Szeifert, Th. (1996a) *Astron. Astrophys.* **305**, 887.
Kaufer, A., Stahl, O., Wolf, B. *et al.* (1996b) *Astron. Astrophys.* **314**, 559.
Owocki, S. P. (1992) In: U. Huber and S. Jeffrey (eds.), *Atmospheres of Early-Type Stars*, Springer, Berlin, 393.
Pauldrach, A. W. A., Puls, J., and Kudritzki, R. P. P. (1986) *Astron. Astrophys.* **164**, 86.
Sapar, A. and Sapar, L. (1990) *Tartu Astrofüüs. Obs. Publ.* **53**, 52.