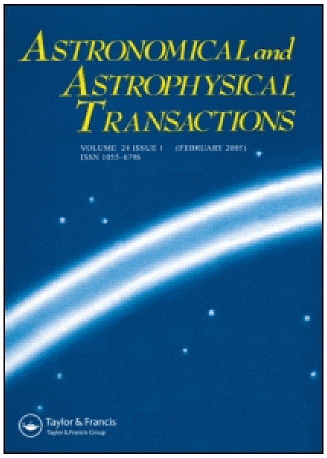

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THE SOLAR DYNAMO WAVE IN PARKER'S MIGRATORY DYNAMO

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A kinematic $\alpha\omega$-dynamo model of magnetic field generation in a thin convection shell with a non-uniform but sign-constant helicity profile for large dynamo numbers is considered in the framework of Parker's migratory dynamo. We obtained an asymptotic solution of equations governing the magnetic field with the form of an anharmonic travelling dynamo wave. This wave propagates over most latitudes of the solar hemisphere from high latitudes to the equator, and the amplitude of the magnetic field first increases and then decreases with propagation. However, over subpolar latitude the dynamo wave reverses; there the wave propagates polewards and decays with latitude. Butterfly diagrams are plotted and analysed; and these show that even a simple model may reveal some properties of the solar magnetic fields. There is an attractive opportunity to develop a more quantitatively precise model taking into account helioseismological data on the differential rotation and fitting the solar observational data on the magnetic field and turbulence, i.e. helicity.

KEY WORDS Solar dynamo, asymptotic solution

1 PARKER'S MIGRATORY DYNAMO

The subject of the present investigation is Parker's migratory dynamo. This model is a strong simplification of dynamo processes of the solar interior. Parker's migratory dynamo gives only a kinematic description of the solar dynamo wave and does not take into account non-linear processes. Nevertheless, it happens that this simple model yields for the value under consideration results which are comparable with observations.

Mean field magnetohydrodynamics in a thin differentially rotating convection shell for the axisymmetric case and extremely large dynamo numbers gives the following equations governing a dynamo wave (Parker, 1955):

$$\begin{aligned}\frac{\partial A}{\partial t} &= \alpha(\theta)B + \frac{\partial^2 A}{\partial \theta^2},\\ \frac{\partial B}{\partial t} &= -DG(\theta)\cos\theta\frac{\partial A}{\partial \theta} + \frac{\partial^2 B}{\partial \theta^2}.\end{aligned} \tag{1}$$

Here B is the azimuthal component of the mean magnetic field, A is proportional to the azimuthal component of the magnetic potential, D is the dimensionless dynamo number which characterizes the intensity of the sources of the magnetic field generation, θ is the latitude in the shell that is measured from the solar equator, $\alpha(\theta)$ is the mean helicity, and $G(\theta)$ the radial gradient of the angular rotation, which are normalized with respect to the maximum of their values, α_* and G_*, respectively.

The asymptotic solution of equations (1) for $|D| \gg 1$ has been obtained by Kuzanyan and Sokoloff (1995) using the WKB method. To leading order with respect to the small parameter $|D|^{-1/3}$, it has the following form:

$$\begin{pmatrix} A \\ |D|^{-2/3}B \end{pmatrix} = \exp(i|D|^{1/3}S + |D|^{2/3}\Gamma_0 t + \ldots)(\mathbf{f}_0 + \ldots), \tag{2}$$

where S and the vector $\mathbf{f}_0$ are complex functions of latitude θ. For the sake of definiteness, we use the simplest form of function $G(\theta)$: $G = \text{const} = 1$. We consider hereafter the case when $D < 0$, which yields a basically equatorward dynamo wave. The asymptotic solution under consideration should enable the dynamo wave to decay at locations remote from the maximum of the sources of generation. As shown by Kuzanyan and Sokoloff (1995) the function $\hat{\alpha}(\theta) = \alpha(\theta)\cos\theta$ appears in an asymptotic analysis of equations (1) and it corresponds to the source of generation of the magnetic field. So the solution should vanish in a region remote from the domain where $\hat{\alpha}$ is a maximum.

2 PROPERTIES OF THE SOLUTION

(1) While the function of sources of generation of the magnetic field is a maximum at the point θ_0, the maximum of the solution is situated at point $\theta_1 < \theta_0$ at which Im $S(\theta)$ is a minimum and for which

$$\hat{\alpha}_1 = \frac{\hat{\alpha}(\theta_1)}{\hat{\alpha}_*} = \frac{9\sqrt{3}}{16\sqrt{2}\sqrt{\sqrt{3}-1}} \approx 0.81,$$

where $\hat{\alpha}_* = \hat{\alpha}(\theta_0)$ is the maximum of the function $\hat{\alpha}(\theta)$ (the sources of generation). For $\alpha(\theta) = \sin\theta$ we have $\theta_0 = \pi/4 = 45°$ and $\theta_1 \approx 27°$ (see Figure 1). Note, that this shift is completely determined by the helicity $\alpha(\theta)$ and does not depend on the value of D.

(2) Over the main part of the northern solar hemisphere the dynamo wave propagates equatorwards. However, at point $\theta_2 > \theta_0$, for which

$$\hat{\alpha}_2 = \frac{\hat{\alpha}(\theta_2)}{\hat{\alpha}_*} = \frac{9\sqrt{6}}{64} \approx 0.35,$$

the quantity Re $S'(\theta)$ changes its sign, and the dynamo wave reverses to the pole. For $\alpha(\theta) = \sin\theta$ we have $\theta_2 \approx 80°$. This reversal is in fact observable over the Sun.

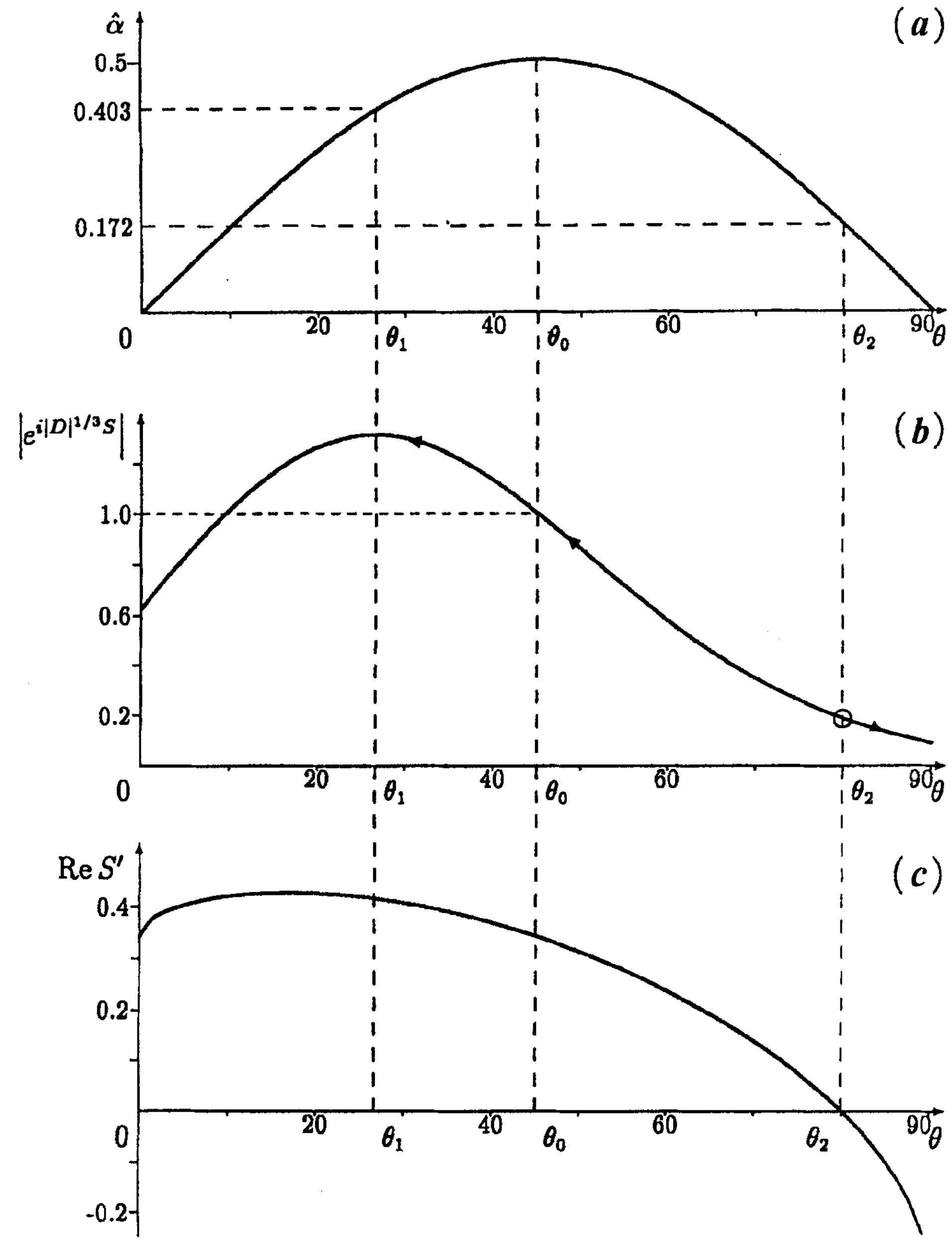

Figure 1 Asymptotic solution of equations (1). The following dependence upon latitude θ are shown: (a), function $\hat{\alpha}(\theta)$; (b), amplitude of the exponential term of the solution of equations (1) for $D = -10^3$; (c), real part of the wave number of the dynamo wave Re S'. The point of reversal of dynamo wave propagation is circled (b). To the left from this point the dynamo wave propagates equatorwards, and, to the right, polewards (for the northern Solar hemisphere). The direction of dynamo wave propagation is shown by the arrows (b). As a normalization condition Im $S(\theta_0) = 0$ is accepted.

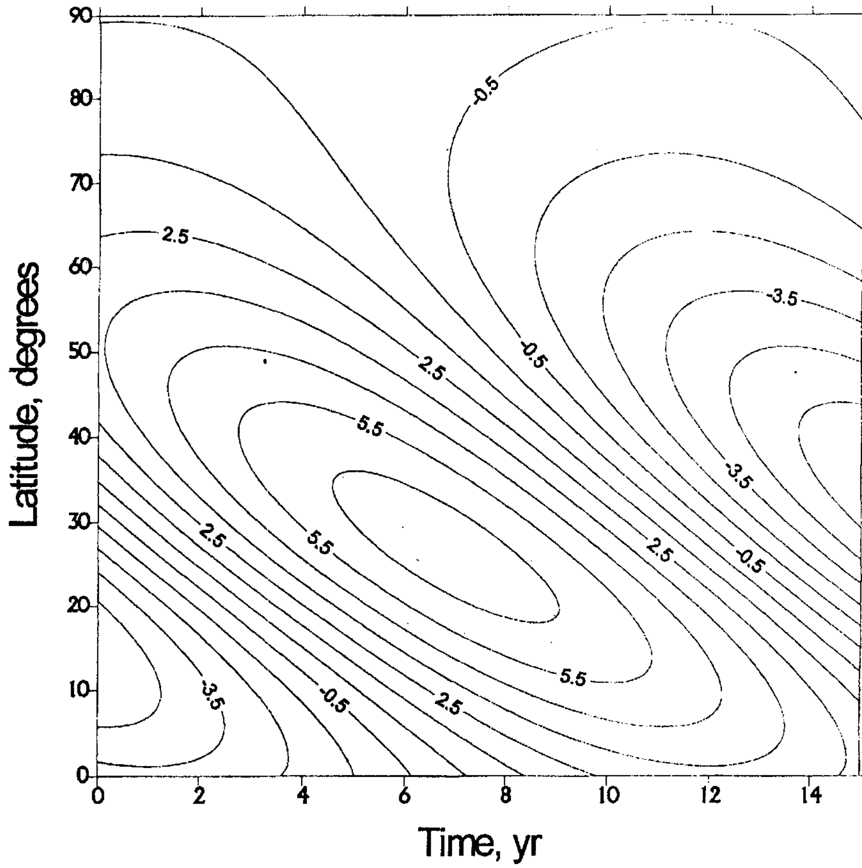

Figure 2 Values of the function $\mathrm{Re}\{\exp[i|D|^{1/3}S(\theta)+i|D|^{2/3}\mathrm{Im}\,\Gamma_0 t]\}$ measured in relative units over the solar cycle. The picture is qualitatively the same as the well-known Maunder butterfly diagrams. (Reproduced from Figure 2 of Kuzanyan, K. and Sokoloff, D. (1997) *Solar Physics* **172**, kind permission from Kluwer Academic Publishers.)

3 DYNAMO NUMBERS

The following definition for the dynamo number is used in equation (1):

$$|D| = R_0^4 \frac{\alpha_* G_*}{\beta^2},$$

where $R_0 \approx 7\times10^{10}$ cm is the solar radius and is β the turbulent magnetic diffisivity. For a crude estimation we use $G_* \approx \Omega/R_0$, where $\Omega \approx 2.7\times10^{-6}$ s^{-1} is the angular velocity of the Sun. Then the dynamo numbers is $|D| = R_0^3\alpha_*\Omega/\beta^2$.

Mixing length theory allows us to estimate the turbulent diffusivity as $\beta \sim 2\times10^{12}$ cm^2 s^{-1} and the mean helicity $\alpha_* \sim 10^2$ cm s^{-1}. Taking into account that for large $|D|$ the period of the solar cycle $T_0 \approx 22$ yr (observational value) according to Parker's migratory dynamo's approach is (in dimensional units)

$$T_0 = \frac{2\pi}{\mathrm{Im}\,\Gamma_0} |D|^{-2/3} \frac{R_0^2}{\beta}, \tag{3}$$

we accept $|D| = 10^3$ below.

4 BUTTERFLY DIAGRAM FOR THE ASYMPTOTIC SOLUTION

We calculate now the location and the half-width of the zone of increased value of the toroidal field (the zone of the dynamo wave's maximum) for different instants of time. We introduce the following function that is proportional to the magnitude of the toroidal magnetic field in the leading order of asymptotic expansion:

$$\mathrm{Re}\{\exp[i|D|^{1/3}S(\theta) + i|D|^{2/3}\mathrm{Im}\,\Gamma_0 t]\}, \tag{4}$$

where t is the dimensionless time measured in units of diffusion time R_0^2/β. This function is plotted in Figure 2. The picture seems to be comparable with the Maunder butterfly diagrams.

5 CONCLUSION

Thus, we considered the $\alpha\omega$-dynamo problem which includes inhomogenity in localization of the sources of magnetic field generation. We used a simple dependence of $\alpha(\theta)$, no meridional flow, no radial dependence, and a kinematic position of the problem. We also assumed the ω-effect to be constant (G = const = 1). Nevertheless, we obtained a qualitative picture of the dynamo wave propagation over the solar convection zone in one hemisphere. The constructed solution is quite similar to the Maunder butterfly diagrams. There are both growing equatorward migration of the magnetic activity and a poleward decaying branch.

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