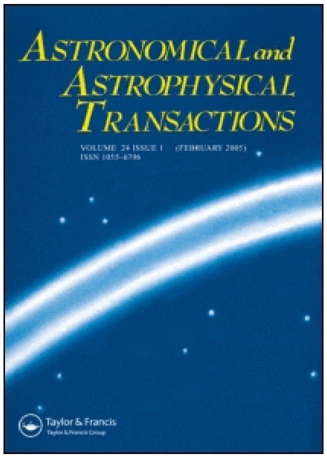

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NON-COMPACT ASTRONOMICAL OBJECTS AS MICROLENSES

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The microlensing of the distant stars by neutralino stars is considered. The neutralino sars have been considered in the recent paper of Gurevich and Zybin; moreover, it has been suggested that the stars should be regard as a major component of dark matter. The optics of these gravitational microlenses, namely the gravitational lens equation, its solutions, increasing of images and critical and caustic curves are analysed by using a clear approximation. The set of model parameters is considered.

KEY WORDS Gravitational lenses, dark matter, gravitational microlensing

1 INTRODUCTION

The first results of observations of microlensing which were presented in the papers of three groups (Alcock *et al.* (1993), Aubourg *et al.* (1993), Udalski *et al.* (1993)) discovered a phenomenon, predicted in the papers of Byalko (1969) and Paczynsky (1986). The character of a gravitational microlens is unknown till now, although the most widespread hypothesis assumes that they are compact dark objects such as brown dwarfs. Nevertheless, they could be other objects, in particular, dark objects consisting of the supersymmetrical weakly interacting particles (neutralino) as discussed in the papers of Gurevich and Zybin (1995) and Gurevich *et al.* (1996). The authors evolution of the Universe and to be stable during cosmological timescales.

Microlensing of a distant star by a neutralino star is considered in this paper.

We consider microlensing by a star in the framework of a rough model which is rather clear and we obtain analytical expressions for the results. Of course, a more exact model of the gravitational field of a neutralino star may be considered; nevertheless, we think that the qualitative estimation of the effect was considered correctly.

Geometric optics is used in the model which will be considered below but effects connected with diffraction and mutual interference of the images and analysed in the papers of Zakharov (1992, 1993a,b, 1994a,b); Zakharov and Mandzhos (1993) and Blair and Sazhin (1993) will not be taken into account.

2 MAIN ASSUMPTIONS AND EXPRESSIONS

We approximate the density of mass distribution of a neutralino star in the form

$$\rho_{\mathrm{NeS}}(r) = \rho_0 \frac{a_0^2}{r^2} \tag{1}$$

where r is the current value of the distance from the star's centre, ρ_0 is the mass density of a neutralino star at a distance a_0 from the centre, and a_0 is the radius of the neutralino star. The dependence is an approximation of the dependence which has been considered in the paper of Gurevich and Zybin (1995), namely

$$\rho_{\mathrm{NeS}}(r) = Kr^{-1.8}.$$

So, it is not difficult to compute the surface density mass, according to expression (1)

$$\Sigma(\boldsymbol{\xi}) = 2\rho_0 \int\limits_0^{\sqrt{a_0{}^2-\xi^2}} \frac{a_0^2}{\xi^2+h^2}\, dh = 2\rho_0 \frac{a_0^2}{\xi} \arctan \frac{\sqrt{a_0^2-\xi^2}}{\xi}. \tag{2}$$

In this case, if $a_0 \gg$, then $\Sigma(\boldsymbol{\xi}) \to \pi\rho_0 a_0^2/\xi$.

In that case the equation of a lens has the form

$$\boldsymbol{\eta} = \frac{D_s}{D_d}\boldsymbol{\xi} - D_{\mathrm{ds}}\hat{\boldsymbol{\alpha}}_{\mathrm{NeS}}(\boldsymbol{\xi}), \tag{3}$$

where D_s is the distance from the source to the observer, D_d is the distance from the gravitational lens to the observer, D_{ds} is the distance from the source to the gravitational lens, and the vectors $(\boldsymbol{\eta}, \boldsymbol{\xi})$ define a deflection on the plane of the source and the lens, respectively:

$$\hat{\boldsymbol{\alpha}}_{\mathrm{NeS}}(\boldsymbol{\xi}) = \int\limits_{R^2} d^2\xi' \frac{4G\Sigma(\boldsymbol{\xi}')}{c^2} \frac{\boldsymbol{\xi}-\boldsymbol{\xi}'}{|\boldsymbol{\xi}-\boldsymbol{\xi}'|^2}. \tag{4}$$

Note that the expression described by equation (1) (and by equation (2)) has two significant shortcomings: (1) there is a singularity for $r = 0$ (infinite density for the value). Nevertheless, one can see that in that case a mass of any finite volume is finite. (2) a second shortcoming of equation (1) is infinite mass of a neutralino star in the case if expression (1) is considered with an infinite value a_0. However, when the effect of the gravitational lens is analysed a mass concentrated at radius

$\xi' > \xi$ does not influence the gravitational lensing for the impact parameter ξ. We use the characteristic value of the radius a_0, corresponding to the microlens "mass" M; thus we obtain the lens equation in dimensionless form:

$$M = 4\pi\rho_0 a_0^3, \tag{5}$$

We introduce the dimensionless variables in following way

$$\boldsymbol{\xi} = \frac{\boldsymbol{\xi}}{\xi_0}, \quad \boldsymbol{y} = \frac{\boldsymbol{\eta}}{a_0},$$

where $\eta_0 = a_0 D_s/D_d$, $\Sigma_{\rm cr} = c^2 D_s/(4\pi G D_d D_{\rm ds})$

$$\hat{\alpha}(\boldsymbol{\xi}) = \frac{1}{\pi}\int\limits_{R^2} d^2x' k(\boldsymbol{x}') \frac{\boldsymbol{x} - \boldsymbol{x}'}{|\boldsymbol{x} - \boldsymbol{x}'|^2}.$$

and

$$k(\boldsymbol{x}) = \frac{\Sigma(a_0\boldsymbol{x})}{\Sigma_{\rm cr}}.$$

We define the surface mass densities for the neutralino star:

$$\Sigma(\boldsymbol{\xi}) = \pi\rho_0\frac{a_0^2}{\xi}. \tag{6}$$

Since we supposed that the surface density is an axially symmetric function then the equation of the gravitational lens may be written in the scalar form (see, for example, Schneider *et al.*, 1992)

$$y = x - \alpha(x) = x - \frac{m(x)}{x}, \tag{7}$$

where

$$m(x) = 2\int\limits_0^x x'\,dx' k(x').$$

We recall that we have the following expression for the function $k(x)$

$$k(x) = \frac{k_0}{x}, \tag{8}$$

where

$$k_0 = \frac{\pi\rho_0 a_0}{\Sigma_{\rm cr}} = \frac{M}{a_0^2}\frac{\pi G D}{c^2}, \tag{9}$$

$$D = \frac{D_d D_{\rm ds}}{D_s}. \tag{10}$$

Hence, the lens equation has the following form

$$y = x - R_0\frac{x}{|x|}, \tag{11}$$

where $R_0 = 2k_0$. If we normalize distances in the lens plane and in the source plane using R_0, namely if we introduce the variables $\hat{y} = y/R_0$, $\hat{x} = x/R_0$, then the lens equation has quite clear form

$$\hat{y} = \hat{x} - \frac{\hat{x}}{|\hat{x}|}. \tag{12}$$

The symbol $\hat{}$ will not be written below. It is easy to see that the equation of the lens in the dimensionless form is the same as the lens equation for the model of galactic mass distribution corresponding to an isothermal sphere (see, for example, Schneider *et al.*, 1992; Zakharov, 1997).

We recall some results concerning the lens equation (12). First we will consider the solution of the lens equation. Without loss of generality we may consider that $y > 0$, and if $y < 1$, the lens equation has two solutions $x_+ = y + 1$, $x_- = y - 1$. In the case when $y > 1$, there is only one root $x = y + 1$. Recall the definition of the magnification of the gravitational lens (Schneider *et al.*, 1992; Zakharov, 1997). The value μ, defined by the equation of the gravitational lens, is called the magnification of the gravitational lens, if μ is an inverted value of the determinant of the Jacobian matrix, namely if

$$A(\boldsymbol{x}) = \frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}, \tag{13}$$

or

$$A_{ij} = \frac{\partial y_i}{\partial x_j}, \tag{14}$$

so the magnification is defined by the following expression

$$\mu(\boldsymbol{x}) = \frac{1}{\det A(\boldsymbol{x})}. \tag{15}$$

Since we have the following expression for the determinant

$$\det A(\boldsymbol{x}) = 1 - \frac{1}{|x|}, \tag{16}$$

then the magnification is equal to

$$\mu = \frac{|x|}{|x| - 1}. \tag{17}$$

It is clear that in this case the critical curve has the equation $|x| = 1$ (i.e. a unit circle). Recall that the circular critical curves are called tangential (Schneider *et al.*, 1992; Zakharov, 1997). In that case the caustic curve degenerates into the point $y = 0$. It is not difficult to see how the sources are distorted by a gravitational lens. It is clear that the images are not distorted in the radial direction, but in the tangential direction they are expanded in according to the relation (17). We recall that there is a contraction of images of two times for a Schwarzschild lens (if $y \ll 1$) in the radial direction and there is a similar expansion (as for a neutralino star), in the tangential direction (Schneider *et al.*, 1992; Zakharov, 1997), which is equal ($\approx 1/y$). If we consider the case $y > 1$, then

$$\mu_P(y) = \mu(x_+) = \frac{y+1}{y} = 1 + \frac{1}{y}. \tag{18}$$

In case $0 < y < 1$, we have

$$\mu(x_+) = \frac{|x_+|}{|x_+| - 1} = \frac{y+1}{y} = 1 + \frac{1}{y}, \tag{19}$$

$$\mu(x_-) = \frac{|x_-|}{|x_-| - 1} = \frac{y-1}{y}. \tag{20}$$

Since $\mu(x_-) < 0$, the total magnification is defined by the expression

$$\mu_P(y) = \mu(x_+) + |\mu(x_-)| = \frac{2}{y}. \tag{21}$$

Recall that in the case when the gravitational microlens is a point gravitating body (Schwarzschild lens) then the magnification is defined by the following expression (see, for example, Zakharov, 1995b)

$$\mu_P(y) = \frac{y^2+2}{y\sqrt{y^2+4}}. \tag{22}$$

Thus, the difference between the magnification of a Schwarzschild lens and neutralino star is an essential factor which distinguishes these objects.

We consider two asymptotics to show the difference between the magnification factors for the two cases. First we write μ_{tot} for a neutralino star:

$$\mu(y) = \frac{2}{y}, \qquad y < 1, \tag{23}$$

$$\mu(y) = 1 + \frac{1}{y}, \quad y > 1. \tag{24}$$

which are simultaneously the precise equations being suitable for approximation. The same expressions for the magnification producing by a compact body (Schwarzschild lens) are:

$$\mu(y) = \frac{1}{y}, \qquad y \ll 1, \tag{25}$$

$$\mu(y) = 1 + \frac{2}{y^4}, \quad y \gg 1. \tag{26}$$

3 THE MAGNIFICATION FOR DIFFERENT PARAMETERS R_0

We consider the magnification for different parameters R_0. We recall that the gravitational lens equation is correct for small values of the parameters y, x only.

We write the gravitational lens equation for large values of the parameters supposing that the density of neutralino star mass is defined by the expression (6). In that case the gravitational lens equation is

$$\begin{aligned} \boldsymbol{y} &= \boldsymbol{x} - \frac{\boldsymbol{x}}{|\boldsymbol{x}|}, \qquad &&\text{for } |\boldsymbol{x}| \le \frac{1}{R_0}, \\ \boldsymbol{y} &= \boldsymbol{x} - \frac{1}{R_0}\frac{\boldsymbol{x}}{|\boldsymbol{x}|^2}, &&\text{for } |\boldsymbol{x}| > \frac{1}{R_0}. \end{aligned} \tag{27}$$

We consider three different sets for the parameter R_0.

Case I. Let

$$\frac{1}{R_0} \ge 2.$$

Case Ia. If

$$0 < y < 1,$$

then the gravitational lens equation has two solutions, both of which correspond to the values of the impact parameter, which has smaller absolute value than the size of the neutralino star, i.e. the corresponding light rays pass through the neutralino star, and the following solutions of the gravitational lens equation are (we consider that the axes are chosen so as $y > 0$, as in all other cases which will be considered below)

$$\begin{aligned} x_+^{\mathrm{NeS}} &= 1 + y, \\ x_-^{\mathrm{NeS}} &= y - 1, \end{aligned}$$

We recall that in that case we get from expressions (19), (20)

$$\begin{aligned} \mu_+^{\mathrm{NeS}} &= 1 + \frac{1}{y}, \\ |\mu_-^{\mathrm{NeS}}| &= \frac{1}{y} - 1, \end{aligned}$$

In that case the total magnification with due regard for the signs of the magnifications are defined by the expression (21).

Case Ib. If

$$1 \le y \le \frac{1}{R_0} - 1,$$

then the gravitational lens equation has one solution corresponding to the value of the impact parameter, which has smaller absolute value than the radius of the neutralino star, i.e. the corresponding light ray passes through the neutralino star, namely there is the following solution of the gravitational lens equation

$$x_+^{\mathrm{NeS}} = 1 + y.$$

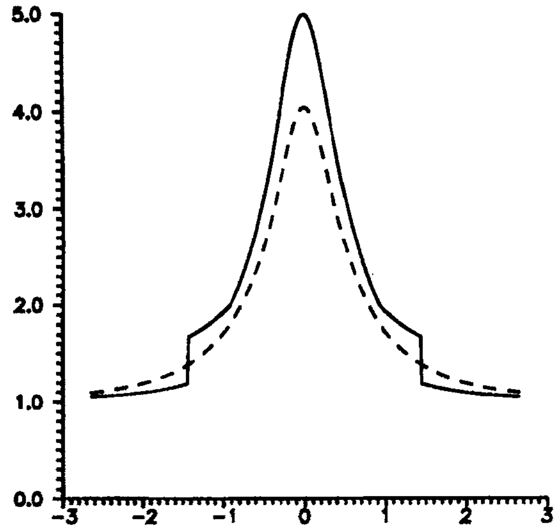

Figure 1 The light curve for a non-compact body (solid line) and for a Schwarzschild lens (dashed line), for $R_0 = 0.4$ (*case I*).

The magnification corresponding this solution is μ_+^{NeS}.

Case Ic. If

$$\frac{1}{R_0} - 1 \leq y,$$

then the gravitational lens equation has one solution corresponding to the value of the impact parameter, which has greater absolute value than the size of the neutralino star i.e. the corresponding light ray passes outside the neutralino star, namely there is a solution

$$x_+^S = \frac{y + \sqrt{y^2 + 4/R_0}}{2}.$$

The corresponding magnification is

$$\mu_+^S = \frac{1}{4}\left(\frac{y}{\sqrt{y^2 + 4/R_0}} + \frac{\sqrt{y^2 + 4/R_0}}{y} + 2\right).$$

Case II. Let

$$1 < \frac{1}{R_0} < 2.$$

Case IIa. If

$$0 < y < \frac{1}{R_0} - 1,$$

then the gravitational lens equation has two solutions and both corresponding impact parameters have smaller absolute value than the size of the neutralino star,

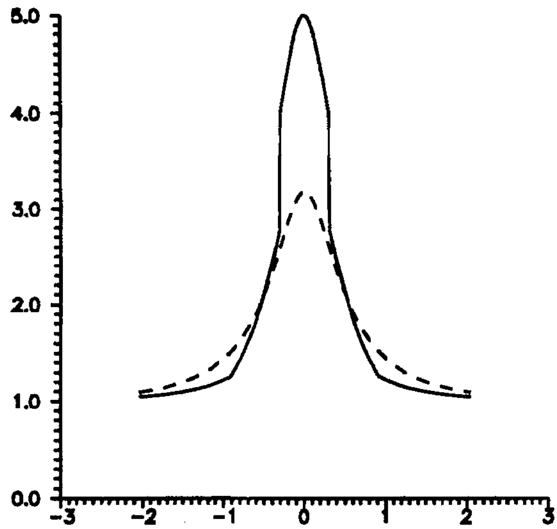

Figure 2 The light curve for a non-compact body (solid line) and for a Schwarzschild lens (dashed line), for $1/R_0 = 1.5$ (*case II*).

i.e. the corresponding light rays pass through the neutralino star, namely there are the following solutions of the gravitational lens equation

$$x_+^{\mathrm{NeS}} = 1 + y,$$
$$x_+^{\mathrm{NeS}} = y - 1.$$

We recall that in this case we get from expressions (19), (20)

$$\begin{aligned} \mu_+^{\mathrm{NeS}} &= 1 + \frac{1}{y}, \\ |\mu_-^{\mathrm{NeS}}| &= \frac{1}{y} - 1, \end{aligned}$$

The total magnification is defined by relation (21) in this case.

Case IIb. If

$$\frac{1}{R_0} - 1 \leq y \leq 1,$$

then the gravitational lens equation has two solutions, one of which corresponds to the impact parameter which has greater absolute value than the size of the neutralino star, i.e. the corresponding light ray passes outside a neutralino star, namely there is a solution

$$x_+^S = \frac{y + \sqrt{y^2 + 4/R_0}}{2}.$$

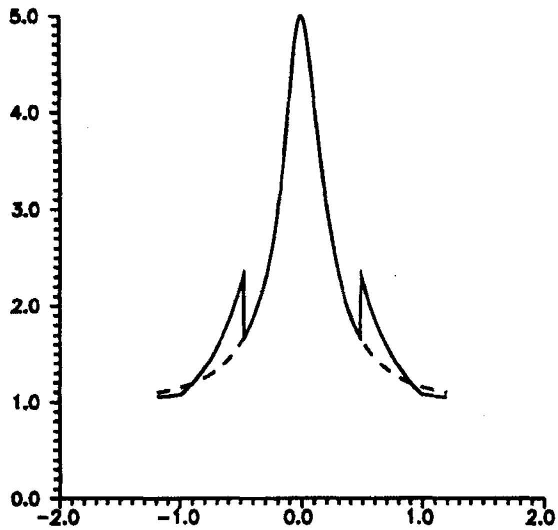

Figure 3 The light curve for a non-compact body (solid line) and for a Schwarzschild lens (dashed line), for $R_0 = 2$ (*case III*).

The magnification factor corresponding to the solution is μ_+^S. The other solution corresponds to the impact parameter, which has smaller absolute value than the size of the neutralino star, i.e. the corresponding light ray passes inside the neutralino star, namely there is a solution of the gravitational lens equation

$$x_-^{\mathrm{NeS}} = 1 - y.$$

The magnification factor corresponding to this solution is μ_-^{NeS}.

Case IIc. If

$$1 < y,$$

then the gravitational lens equation has one solution corresponding to the impact parameter, which has greater absolute value than the radius of the neutralino star, i.e. the corresponding light ray passes outside the neutralino star, namely there is a solution x_+^S. The magnification corresponding to the solution is μ_+^S.

Case III. Let

$$0 < \frac{1}{R_0} < 1.$$

Case IIIa. If

$$0 < y < 1 - \frac{1}{R_0},$$

the gravitational lens equation has two solutions, both of them corresponding to the impact parameters, which have greater absolute values than the size of the neutralino star, i.e. the corresponding light rays pass outside a neutralino star, namely there is a solution

$$x_+^S = \frac{y + \sqrt{y^2 + 4/R_0}}{2}.$$

The magnification is μ_+^S for the solution. The other solution is also similar to the solution of the Schwarzschild lens equation

$$x_-^S = \frac{y - \sqrt{y^2 + 4/R_0}}{2}.$$

The magnification corresponding to the solution is

$$|\mu_-^S| = \frac{1}{4}\left(\frac{y}{\sqrt{y^2 + 4/R_0}} + \frac{\sqrt{y^2 + 4/R_0}}{y} - 2\right).$$

Case IIIb. If

$$\frac{1}{R_0} - 1 \leq y \leq 1,$$

then the gravitational lens equation has two solutions, one of which corresponds to the impact parameter which has smaller absolute value than the radius of the neutralino star, i.e. the corresponding light ray passes outside the neutralino star, namely there is a solution of the gravitational lens equation

$$x_+^S = \frac{y + \sqrt{y^2 + 4/R_0}}{2}.$$

The magnification is μ_+^S. The other solution corresponds to the impact parameter which has smaller absolute value than the radius of the neutralino star, i.e. the corresponding light ray passes through the neutralino star, namely there is a solution of the gravitational lens equation

$$x_-^{\mathrm{NeS}} = 1 - y.$$

The magnification corresponding to the solution is μ_-^{NeS}.

Case IIIc. If

$$y > 1,$$

then the gravitational lens equation has one solution which corresponds to the value of the impact parameter, which has greater absolute value than the size of the neutralino star, i.e. the corresponding light ray passes outside the neutralino star, namely there is a solution x_+^S. The magnification which corresponds the solution is μ_+^S.

So, the magnification of an non-compact body (neutralino star) has no differences from the amplification coefficient of the Schwarzschild lens only in case IIIa. In all other cases in principle it is possible to distinguish these astronomical objects.

In Figures 1–3 the light curves for a non-compact body and a Schwarzschild lens for cases I–III are shown. One can note that the light curve for an non-compact body has a discontinuity in the case when the image is x_+, corresponding to the impact parameter which has absolute value being to the radius of the neutralino star. This is very easy to understand since the lens equation has at this point a discontinuous derivative, therefore also has a discontinuity for the general case. This is the result of the influence of two of our assumptions; the first one is that $a_0 \gg \xi$ and therefore $\Sigma(\xi) \approx 1/\xi$ (the factor $\arctan \frac{\sqrt{a_0 - \xi^2}}{\xi}$ is considered to be equal to $\pi/2$). This assumption becomes rather rough for $\xi \approx a_0$. The second assumptions is the approximation of a point source. It is clear that in this case if we suppose that one of the assumptions is incorrect then the light curve will be continuous for a non-compact object. Nevertheless, the model is rather clear, so we can investigate it in detail and thereby define the limits of its usage. A more detailed discussion of microlensing by non-compact bodies is in the papers by Zakharov and Sazhin (1996a,b, 1997a). In particular, if we consider the influence of a mass distribution of our Galaxy to the microlensing model, then it is possible to get caustic curves which are similar to astroids (Zakharov and Sazhin, 1997a,b), therefore there are light curves which have two peaks corresponding to the intersection of the caustic curve by the source (we calculate a magnification near a fold singularity using expressions of Schneider *et al.* (1992) and expressions of Zakharov (1995a) for a cusp singularity).

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