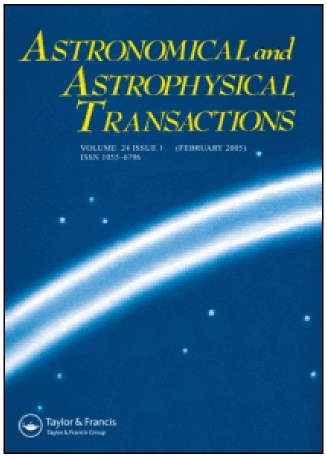

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THE MAGNETIC CYCLE OF THE SUN IN THE SPECTRAL REGION

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A combined spectral analysis of the Wolf number series of fixed and alternating sign was made over a time interval of $\sim$ 300 years and over its two parts. As a result of the analysis, the structure of the magnetic cycle spectrum has been determined more exactly, and the type of detecting, involved in the passage from the second to the first series, is discussed.

KEY WORDS Wolf number, magnetic cycle, spectral analysis, detecting

1 INTRODUCTION

Physical modelling and forecasting of solar activity requires an adequate analysis of original experimental data. The use of the Wolf numbers in the form of a sequence of the 11-year cycles of one sign is inconvenient for this task (Rivin, 1993–1995), because such a sequence is the result of detection of the basic solar cycle by the $T \simeq$ 22 year (magnetic) cycle through the selection of observational objects, namely the sunspot and their group numbers only. The present-day literature on the problem does not provide a unique spectrum of the initial processes on the Sun. Besides, it is not clear what factor is responsible for transformation of the basic cycle to the Wolf number variation. Below, we analyse the Wolf and Wolf–Anderson series and then check the possibility of using the linear and square detector models to describe the relationship between both series.

2 ORIGINAL DATA AND ANALYSIS TECHNIQUES

The study of the spectral composition of solar activity is to a large extent associated with the spectral analysis of the sign-constant series of Wolf numbers (W) (Attolini *et al.*, 1985; Berger *et al.*, 1990; Vasilyev, 1970; Kane and Trivedi, 1985; Knight *et al.*, 1979; Muzalevsky and Zhukov, 1989a; Rivin, 1989). The conclusions concerning different parts of the spectrum may differ significantly, depending on the analysis technique and the specific task of a study. Less attention has been given in the literature to the sign-variable series ($\tilde{W}$), which can be called the Wolf–Anderson series (Anderson, 1939; Vasilyev and Kandaurova, 1968; Cole, 1973; Muzalevsky and Zhukov, 1969b), because the idea of its construction belongs to Anderson.

The annual mean values of W for 1700–1994 are given in *Solar-Geophysical Data* (1993) and *Sunspot Hunbers: 1610–1985* (1987). The $\tilde{W}$ series is constructed by considering the even 11-year cycles positive and the odd cycles negative (Zurich numbers). The values of $\tilde{W}$ between the cycles, close to zero, are linearly interpolated, where it is necessary.

The spectral analysis has been performed by Fourier expansion. To obtain a precise spectral description, we have calculated fractional harmonics with a step, Δn, ranging from 0.1 to 1, which is analogous to adding zeros in a fast Fourier transformation.

According to Rivin (1993–1995), the magnetic cycles over the past $\sim$ 300 years is modulated in amplitude. The modulation has a complicated frequency composition and structure. One can see a trend and secular variation. Beginning with the cycle 10 started in 1856 an asymmetry of the even and odd cycles has been revealed in the magnetic cycle (the amplitude of the odd cycle is larger), and the rate of growth of the trend as well. To take into account these factors, the analysis was performed separately for the full $\tilde{W}$ and W series, their anterior (1700–1856) and posterior (1856–1994) parts.

3 SPECTRAL ANALYSIS OF THE WOLF–ANDERSON SERIES

The amplitude spectra of $\tilde{W}$ are given in Figure 1. We begin our treatement with the posterior part of the series where the accuracy of the data is higher.

This spectrum, determined at the interval with $N = 139$ years, consists of two parts: the central frequency band ($\Delta\omega \simeq 0.2$–0.4 rad/year), which corresponds to the magnetic cycle, and a higher-frequency band, that incorporates two groups of harmonics: with $\Delta T_1 \simeq 13$–14 years and with $\Delta T_2 \simeq 7$–8 years (see the hatching). Two satellite frequencies around the maximum of the magnetic cycle can be interpreted as amplitude modulation of the carrier frequency by a quasi-harmonic variation. As shown by the satellite deviation from the central frequency, the modulating period is $\sim$ 100 years. The amplitude ratio of the corresponding spectral frequencies gives the modulation depth $m \simeq 0.3$. At lower frequencies, any significant harmonics are absent.

Without taking into account higher frequencies, this spectrum can be described to a first approximation by the model:

$$y = (1 + m \sin \Omega t) \sin \omega t, \tag{1}$$

where $m = 0.3$, $T_\Omega = 100$ years, and $T_\omega = 21.8$ years. The spectrum for this model for $N = 139$ years is shown at the bottom of Figure 1. It can be used to estimate

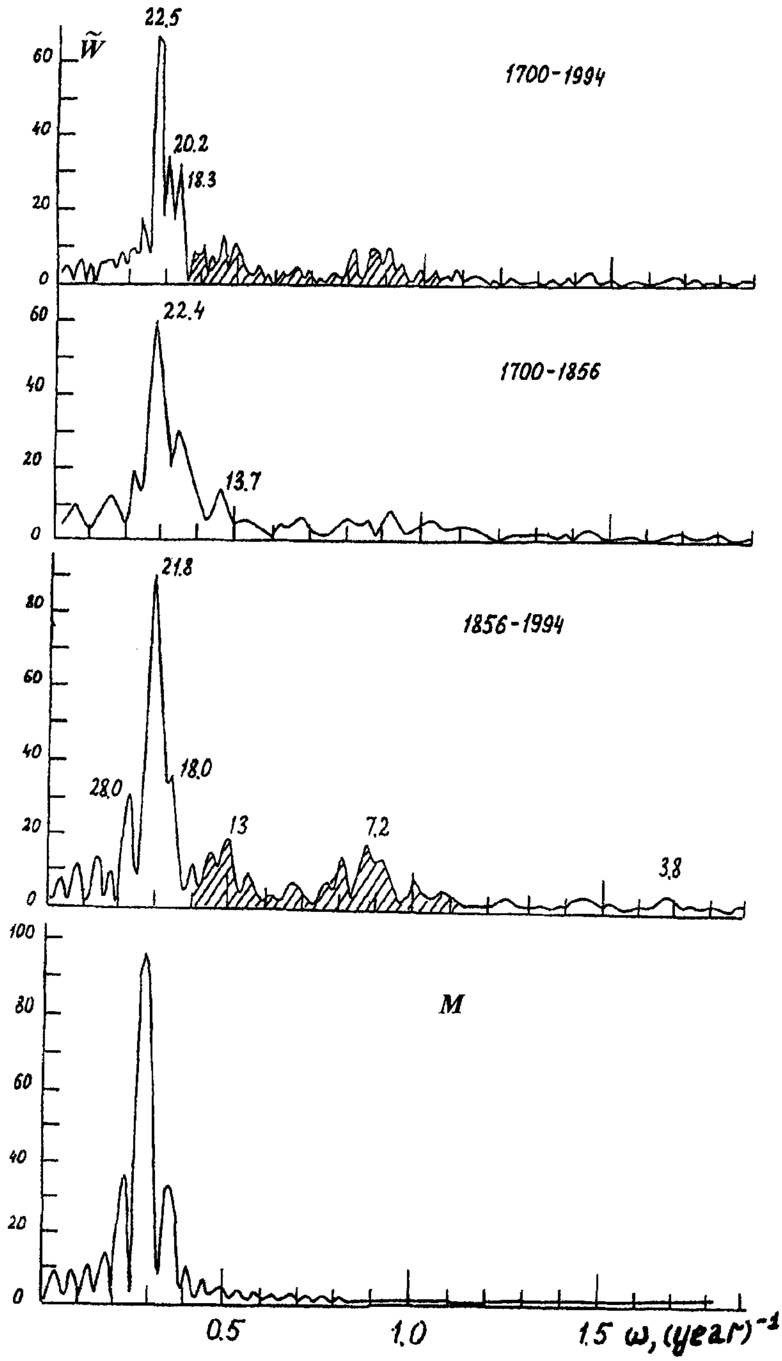

Figure 1 Amplitude spectra of the annual mean Wolf–Anderson series over various intervals using model (1)–M. Numerals at the peaks denote T in years.

the spectrum distortion, $\tilde{W}$, due to the limited expansion interval, as well as to isolate the features that are not described by the model.

Beginning with cycle 10, the central band of the $\tilde{W}$ spectrum displays an important feature that disagrees with the model. The right-hand satellite ($T \simeq 18$ years) merges with the central frequency band, whereas a similar merging in the left-hand satellite is absent. The merging is due to the complicated structure of the central maximum, i.e. the existence of two similar carrier frequencies. For example one can consider the spectrum of $\tilde{W}$ over a full interval of 295 years. In the region of the basic carrier frequency, there are two maxima (with $T \simeq 22$ years and $T \simeq 20$ years) and two satellites, that occur as a result of the growing expansion interval. For the same spectrum one can see that the amplitude of one central frequency ($T \simeq 22$ years) is about twice as large as the amplitude of the other.

In addition, the comparison of the model and $\tilde{W}$ spectra shows that the existence of the ΔT_1 and ΔT_2 bands does not depend on the analysis method, and that the spectrum harmonics with $6 > T > 30$ years are insignificant.

Now, let us turn to the other spectra in Figure 1. The spectrum of the first part of the $\tilde{W}$ series has the amplitude of the main maximum, ~ 1.5 times as small as the previous one. This may be attributed to its proximity to the Maunder minimum. The groups of harmonics with ΔT_1 and ΔT_2 are either faintly pronounced, or absent, and the maximum of the central carrier frequencies is shifted towards the harmonic with $T \simeq 22.4$ years. The other spectral characteristics are the same as above.

The total $\tilde{W}$ spectrum combines the properties of both parts of the series. However, as noted above, the resolution over all the frequency band increases due to a nearly doubled expansion interval. It is useful to note that the lower-frequency part of this spectrum ($\omega < 0.2$ rad/year) may also be explained as a result of errors in the original data and analysis method.

Thus, the structure of the $\tilde{W}$ spectrum over the past ~ 300 years can be described as follows:

(1) Two oscillations with close periods (~ 20 years) and different amplitudes (~ 30–90 units) have the greatest power. Their satellites are the result of amplitude modulation of the more powerful oscillation (or both of them) by a quasi-harmonic variation with $T_\Omega \simeq 100$ years. The mean modulation depth is $m \simeq 0.3$.

(2) Oscillations with $\Delta T_1 \simeq 13$–14 years and with $\Delta T_2 \simeq 7$–8 years and a spectral amplitude of ~ 17 units. These oscillations have now been isolated for the first time.

(3) The amplitudes in the lower-frequency part of the spectrum and the harmonics with $T < 6$ years differ from the model within 4 units. Because of such a small amplitude, these harmonics can be interpreted as an error in obtaining the spectra.

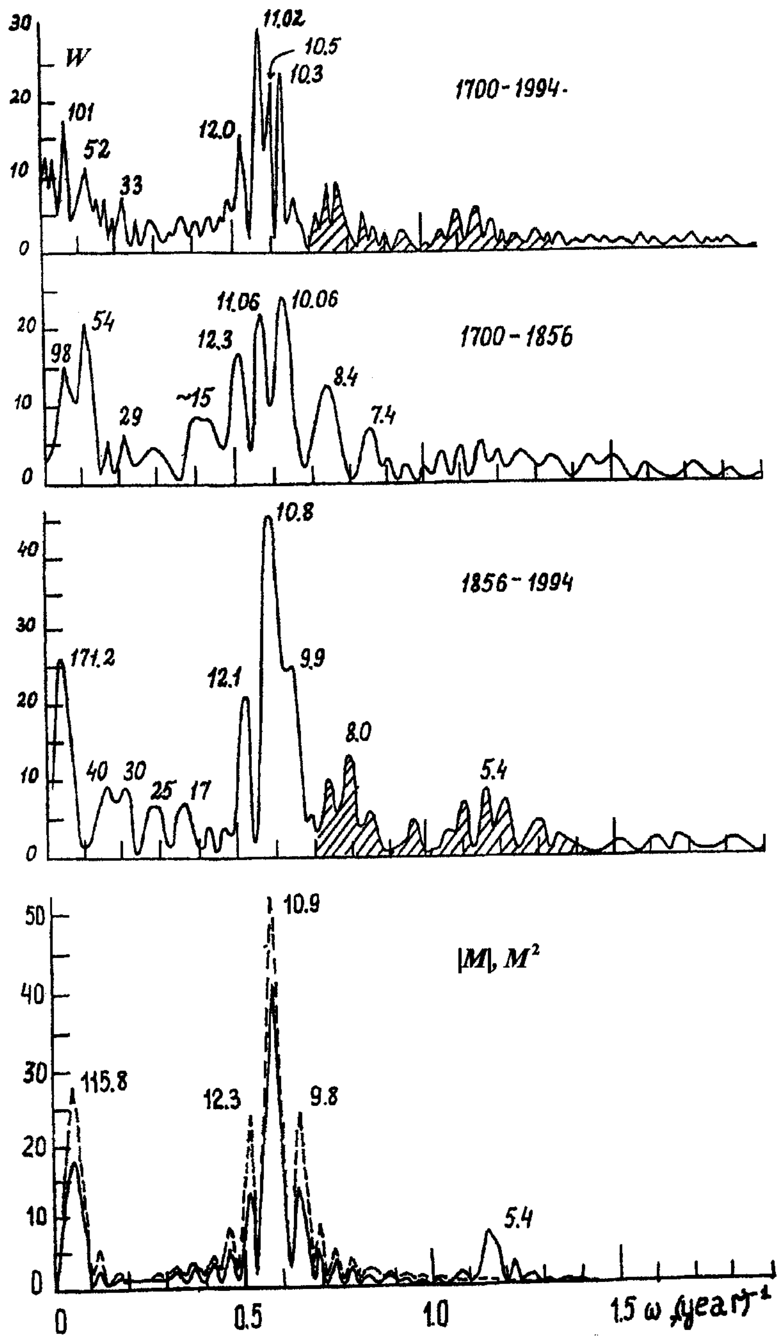

Figure 2 Amplitude spectra of the annual mean Wolf series over various intervals, as well as the linear ($|M|$, solid line) and quadratic (M^2, dashed line) detection of model (1). Numerals at the peaks denote T in years.

4 SPECTRAL ANALYSIS OF THE WOLF NUMBER SERIES

The amplitude spectra of W are illustrated in Figure 2. In the spectrum of the second part of the series, the leading role belongs to the harmonics with $T \simeq 10$–11 years with their amplitude modulation. The spectral amplitude of the basic harmonics is ~ 40 units, $T_\Omega \simeq 100$ years, and $m \simeq 0.3$. This spectrum displays the same complicated structure of the central maximum as the spectrum of $\tilde{W}$. It is also observed in the spectrum of W over the entire interval of carrier harmonics with $T \simeq 11.02$ and $T \simeq 10.5$ years. The hypothesis of the complicated central part of the spectrum in Figure 2 is corroborated if we compare the W spectrum over the second and the whole intervals with the model spectrum (1) after detection by various transformations. It would be useful to recall that the splitting of the W spectrum in the region of the central maximum was considered by Vasilyev (1970), who suggested that the W series be interpreted as an amplitude-modulated process with the main periods of 11 and 10 years. Later this was also noted by Rivin (1989). Still no adequate explanation of this splitting is available in the literature until now.

An important particularity of the W spectra, compared to $\tilde{W}$, is the appearance of significant harmonics of the group, which are often revealed in the lower-frequency part of the spectrum. The spectrum of the W expansion over the entire interval is expected to provide an accurate representation of the lower-frequency part of the spectrum. It shows the most powerful harmonic with $T_\Omega \simeq 100$ years, its sub-harmonics 52, 33 years, as well as longer-period oscillations. Comparison of this part of the W spectrum and the models in Figure 2 shows that the appearance of sub-harmonics is not connected with the expansion over a limited time interval. However the W spectrum over the full interval may have reflected a significant difference between the LF parts of the first and the second halves of the Wolf series.

Another important characteristic of the W spectrum is the presence of two groups of significant harmonics in the high-frequency region ($\Delta T_3 \simeq 7$–8 and $\Delta T_4 \simeq 5$ years) (see the hatching).

Thus the spectrum of the Wolf series seems to be mostly the result of detection by the magnetic cycle.

5 SELECTION OF MODEL TO DESCRIBE THE SPECTRUM COUPLING OF THE WOLF AND WOLF–ANDERSON SERIES

The coupling of spectra of the two Wolf-number series mentioned above must display their relationship in time. In fact, the procedure of changing the sign of the odd cycles is an attempt to restore the original signal from the detected one. For this purpose, it is important to know what type of detection was applied when passing from the Wolf–Anderson to the Wolf series.

Table 1. Relative amplitudes of the main spectral frequencies

Series	Ω	2ω	$2\omega \pm \Omega$	4ω	N (*years*)
(3)	0.19	0.21	0.03	0.04	–
(4)	0.30	0.51	0.15	–	–
$\lvert M \rvert$	0.18	0.40	0.14	0.08	139
M^2	0.28	0.52	0.25	–	139
W	0.27	0.48	0.22	0.10	139

Comparison of models (3) and (4) reveals different relative amplitudes of harmonics at the main frequencies (2ω, Ω, etc.). Let us use this difference as a criterion to estimate which of the harmonics agrees best with what is obtained in the W spectrum.

For this purpose, the amplitudes of the main harmonics of the LF part and the central maximum, normalized to the original oscillation amplitude (carrier frequency), are tabulated in Table 1. They were calculated separately for models (3), (4), $|M|$, M^2, as well as for the second part of the W series.

As seen from the amplitude ratio of the most powerful harmonics the transition from $\tilde{W}$ to W is most adequately described in terms of the quadratic detection model. However the amplitude estimates over a short expansion interval give an error as large as $\sim$ 10–20%. This error, as well as the growth of relative amplitudes $|M|$ as model (1) gets more complicated, make the choice of the detector more difficult.

This conclusion also seems to be true as far as the model by Sonett (1982) is concerned:

$$y = (1 + m\cos \Omega t)(\Delta + \cos \omega t)^2 + \eta(\rho)^2, \tag{5}$$

where we assume by adjustment $T_\Omega = 90$ years, $T_\omega = 22$ years, $m = 0.25$, $\Delta = 0.05$, and $\eta(\rho)$ is Gaussian noise with $\rho \simeq 0.05$ (the letters and brackets have been changed by the author).

An alternative criterion for the distinction between the detectors is that the linear detector has a 4ω harmonic in the spectrum of W. Proceeding from this characteristic alone, one might conclude that the Wolf number was largely due to linear detection of the Wolf–Anderson series, i.e. the relationship between the original and the transformed oscillations could be described to a first approximation by model (3).

However, this approach is far from optimal. Besides the 4ω harmonic, the W spectrum contains the frequency band ΔT_3, which is difficult to interpret. The frequency band ΔT_4 in the W spectrum, observed in Figure 2, may have a different origin than that suggested above, e.g. it may be due to the same reasons as ΔT_3. On the other hand, both suggestions may be true, taking into account that the ΔT_3 band has a more complicated structure than the corresponding band in the $|M|$ spectrum.

Thus, the above hypothesis of a transition from the Wolf–Anderson to the Wolf series by linear detection needs further corroboration. Important evidence can arise from establishing the origin of ΔT_3 and ΔT_4.

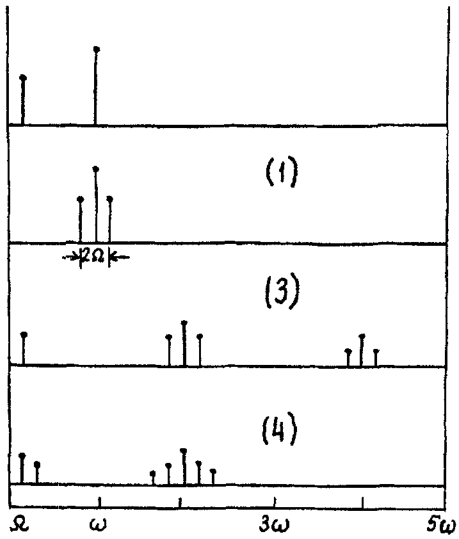

Figure 3 The spectra of the linear (3) and quadratic (4) detection of an amplitude-modulated quasi-harmonic signal (1).

The models of the linear and quadratic detectors have the following common features and differences in the spectral region (Kharkevich, 1957).

Let model (1) be subject to linear detection:

$$|y| = (1 + m \sin \Omega t)|\sin \omega t|. \tag{2}$$

Representing the second term as a Fourier series, we obtain:

$$\begin{aligned} |y| &= \frac{2}{\pi}\{(1 + m \sin \Omega t) - \sum_{k=1}^{\infty} \frac{1}{4k^2 - 1}[\cos 2k\omega t + \frac{m}{2} \sin(2k\omega + \Omega)t \\ &- \frac{m}{2} \sin(2k\omega - \Omega)t]\}. \end{aligned} \tag{3}$$

In the case of quadratic detection (1)

$$\begin{aligned} y^2 &= \frac{1}{2}\{1 + \frac{m^2}{2} + 2m \sin \Omega t - \frac{m^2}{2} \cos 2\Omega t - (1 + \frac{m^2}{2}) \cos 2\omega t \\ &- m \sin(2\omega - \Omega)t + m \sin(2\omega + \Omega)t + m^2 \cos 2(\omega - \Omega)t \\ &+ m^2 \cos 2(\omega + \Omega)t\}. \end{aligned} \tag{4}$$

The schemes of the spectra of model (1) and the results of detection by both detectors over an infinite interval are illustrated in Figure 3. The results of the analysis in Figures 1–3 corroborate the earlier conclusion that transition from $\tilde{W}$ to W is accompanied by detection of the amplitude-modulated magnetic cycle(s), which results in the appearance of 11-year cycle(s) in the W spectrum with their own satellites and modulation frequencies.

Let us see if the lower-frequency part of the W spectra can be used to determine the kind of detector.

In accordance with (3) and (4), the amplitude modulation must be accompanied by the appearance of modulation frequencies in the LF spectrun of the detected signal: Ω in the case of linear detection, and Ω and 2Ω in the case of quadratic detection. Only one modulation period ($T_\Omega \simeq 100$ years) was isolated in the $\tilde{W}$ spectrum by satellites. Therefore if the transition from $\tilde{W}$ to W occurred through linear detection, we should expect only one significant harmonic, T_Ω, in the W spectrum. However, as shown above, the LF part of the spectrum W is more complicated. Its structure contains the frequencies Ω, 2Ω, and 3Ω, decreasing in amplitude, which does not comply with model (4). Rivin (1993–1995) explained the complication of the spectrum as a result of a more complicated frequency composition of the modulating oscillation. However it may partly be due to the model limitations, as well as to spectral differences between the modulating oscillations in the two parts of the series.

6 DISCUSSION OF RESULTS

In the course of the analysis described above we specified the spectral structure of the Wolf–Anderson and Wolf series and tried to describe their coupling in terms of simple detection models. However we failed to reliably establish the kind of coupling, mainly because the present-day studies of solar periodicity do not provide answers to some fundamental questions, such as:

(1) What is the cause of the complicated structure of the central maximum in the $\tilde{W}$ spectrum?

(2) What is the cause of the complicated structure of the lower-frequency part of the W spectrum?

(3) What are the oscillation modes and the sources of ΔT_{1-4}?

Without answering these questions it is hardly possible to conceive a representative analytical model of solar activity or to proceed from cyclic variations of sunspots in the solar photosphere to the physical processes that control the magnetic cycle.

7 CONCLUSIONS

(1) The main harmonics in the spectrum of the Wolf–Anderson series are due to the magnetic cycle. They have a complicated structure. In addition, the spectrum contains two groups of oscillations ($\Delta T_1 \simeq$ 13–14 years and $\Delta T_2 \simeq$ 7–8 years), whose amplitudes make about 20%. Variations with $T \geq 30$ years are not observed in the spectrum. They only produce satellites.

(2) The presence of harmonics $\sim 4\omega$ in the spectrum of W corroborates the possibility of a transition from $\bar{W}$ to W by linear detection. However the transition model still needs additional investigation and correction.

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