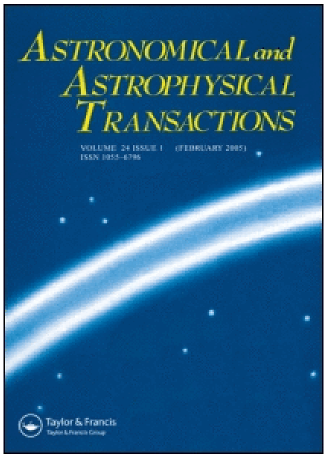

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DISSIPATION INSTABILITIES IN AN ACCRETION DISK

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A model of a geometrically thin gaseous disk in an external gravitational potential is considered. The dynamics of small non-axisymmetric perturbations in the plane of the accretion disk with dissipative effects is investigated. It is shown that the conditions of development and parameters of unstable oscillation modes in the opticaly thick accretion disk strongly depend on the models of viscosity and opacity.

KEY WORDS Accretion disk, acoustic waves, instabilities, turbulent viscosity

1 INTRODUCTION

The possibility of the development of various types of instabilities in thin gaseous disks is a very attractive one for understanding different aspects of the accretion-disk (AD) phenomenon. In order to explain the required large values of dissipative coefficients, the concept of turbulent viscosity has been used, which may be caused by the developed turbulence of a gaseous medium arising from the loss of stability. On the other hand, the non-linear evolution of unstable oscillation modes may be responsible for many non-stationary phenomena in accreting systems.

There are four unstable oscillation modes in the framework of the standard α-model of an accretion disk (Shakura and Sunyaev, 1973). Two of them are acoustic (Wallinder, 1990, 1991; Okuda and Mineshige, 1991; Wu and Yang, 1994; Khoperskov and Khrapov, 1995), the third is viscous and the last one is thermal (Lightman and Eardley, 1974; Shakura and Sunyaev, 1976; Taam and Lin, 1984; Szuszkiewicz, 1990). A distinctive feature of the dynamic viscosity $\eta = \sigma\nu$ in the model of the accretion disks is its dependence on the surfase density σ and on the disk half-thickness h (ν is the kinematic viscosity). The perturbation of the dynamic viscosity $\tilde{\eta}$ is responsible for the formation of all four unstable modes. However, in all the above-cited papers on the dynamics of linear perturbation it is suggested that the viscosity simultaneously varies with changing accretion-disk parameters.

In our present work, we extend our research to the time delay of the viscosity influence on the thermal, viscous, and acoustic instabilities. In the construction of different viscous models AD it is suggested that the viscosity is caused by the developed turbulence $\eta \sim \sigma u_t l_t$, where u_t and l_t are the characteristic velocity of the large-scale turbulent pulsations and its characteristic size, respectively (Landau and Lifshits, 1986). The fundamental energy is contained in the large-scale pulsations, however dissipation of energy is the case in small-scale pulsations. Thus, as the local conditions change in the accretion disk, there are two factors, which involve the change delay of the turbulent viscosity. First, since the turbulence may be caused by the non-linear evolution of unstable oscillation modes, it requires a characteristic time τ_1 to form the developed turbulence. The first approximation may be thought of as τ_1 proportional to the build-up time of instabilities. Secondly, there is a delay τ_2, associated with transfer of energy from large-scale pulsations to small-scale pulsations. Hence, as the accretion-disk parameters (temperature and density) change, the value of the actual viscosity is delayed from the instantaneous value of the dynamic viscosity η_* by a characteristic time $\tau = \tau_1 + \tau_2$. For the standard α-model AD $\eta_* \sim \alpha\sigma\Omega h^2$, where Ω is the Keplerian angular velocity. The first approximation law of the relaxation viscosity η to the value η_* may be written in the following form:

$$\frac{d\eta}{dt} = \frac{\eta_* - \eta}{\tau}. \tag{1}$$

We restrict ourselves to the case of small perturbation with $kr \gg 1$ and $m/r \ll k$ (k is the radial wave-number and m is the azimuthal wave-number), which allows us to use the WKB approximation and seek the solution in the form

$$\tilde{f} = f_1 \exp(-i\omega t + ikr + im\varphi), \tag{2}$$

where ω is the complex frequency of a mode. Equilibrium quantities are denoted by the subscript 0.

2 THE INFLUENCE OF THE DELAY OF VISCOSITY

In the general case $\tau > 0$ we obtain a fifth-order dispersion relation. This equation describes five oscillatory modes. Four of them were considered previously at $\tau = 0$ (Khoperskov and Khrapov, 1995). The inclusion of the delay $\tau > 0$ giving rise to a new oscillatory mode is the second-viscous mode, for which $\mathrm{Re}(\omega) = 0$ and $\mathrm{Im}(\omega) < 0$ at any values of the other parameters. Rewriting equation (1) in terms of the result (2) we obtain in the linear approximation:

$$\frac{\eta_1}{\eta_0} = \frac{1}{1 - i\omega\tau}\frac{\eta_{*1}}{\eta_0} = \frac{1}{1 - i\omega\tau}\left[(1 + \delta_\sigma)\frac{\sigma_1}{\sigma_0} + \delta_h\frac{h_1}{h_0}\right], \tag{3}$$

where $\delta_\sigma = \left(\frac{d\ln\nu_*}{d\ln\sigma}\right)_0, \delta_h = \left(\frac{d\ln\nu_*}{d\ln h}\right)_0$.

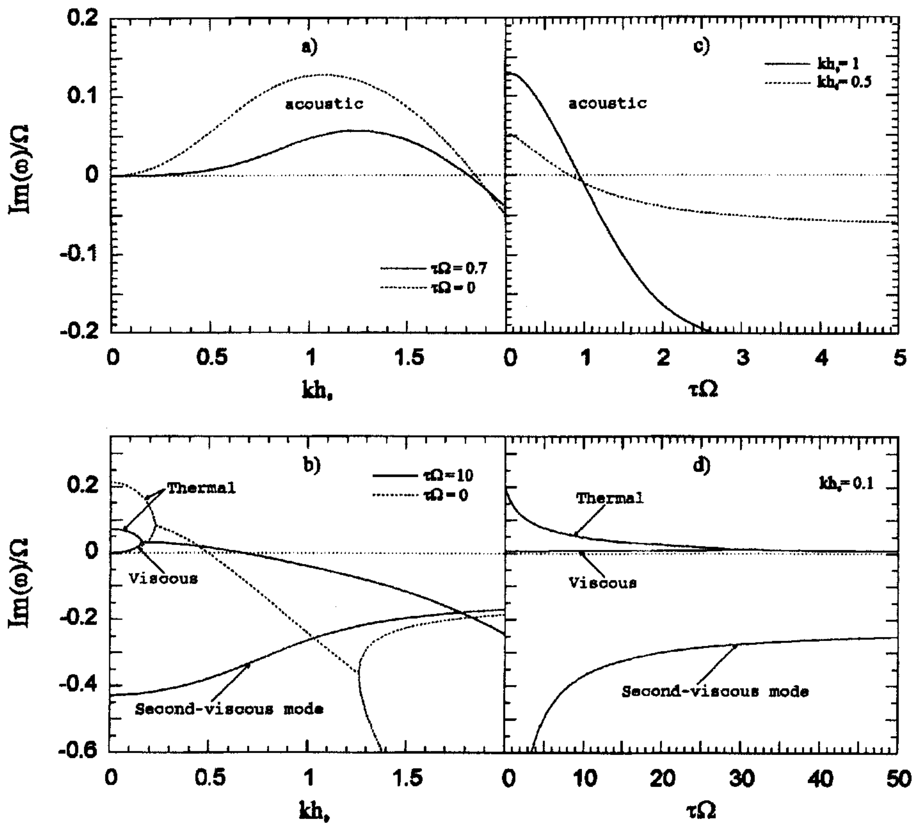

Figure 1 Dependence of the imaginary part of the frequency in terms of the angular velocity $\mathrm{Im}(\omega)/\Omega$ on the dimensionless wave-number kh_0 (a),(b) and on the dimensionless time of delay $\tau\Omega$(c),(d).

For a standard α-model accretion disk in the radiation-pressure-dominated region the increment of all four unstable modes decreases, with the increase of characteristic time of the delay τ, as indicated by Figure 1. Stabilization of the thermal and viscous oscillation modes ($\mathrm{Im}(\omega) \lesssim 0$) occurs at $\tau \gtrsim 100/\Omega$, while the acoustic mode tends to become stable at $\tau \simeq 1/\Omega$.

3 THE INFLUENCE OF OPACITY

The conditions of the development of unstable oscillation modes very strongly depend on the model of opacity. We assume that the opacity $\bar{\kappa}$ is a function of σ and h (in other words, of density and temperature). The linear approximation yields

$$\frac{\bar{\kappa}_1}{\bar{\kappa}_0} = \Delta_\sigma \frac{\sigma_1}{\sigma_0} + \Delta_h \frac{h_1}{h_0}, \quad \left\{ \Delta_\sigma = \left(\frac{d \ln \bar{\kappa}}{d \ln \sigma}\right)_0, \ \Delta_h = \left(\frac{d \ln \bar{\kappa}}{d \ln h}\right)_0 \right\}. \tag{4}$$

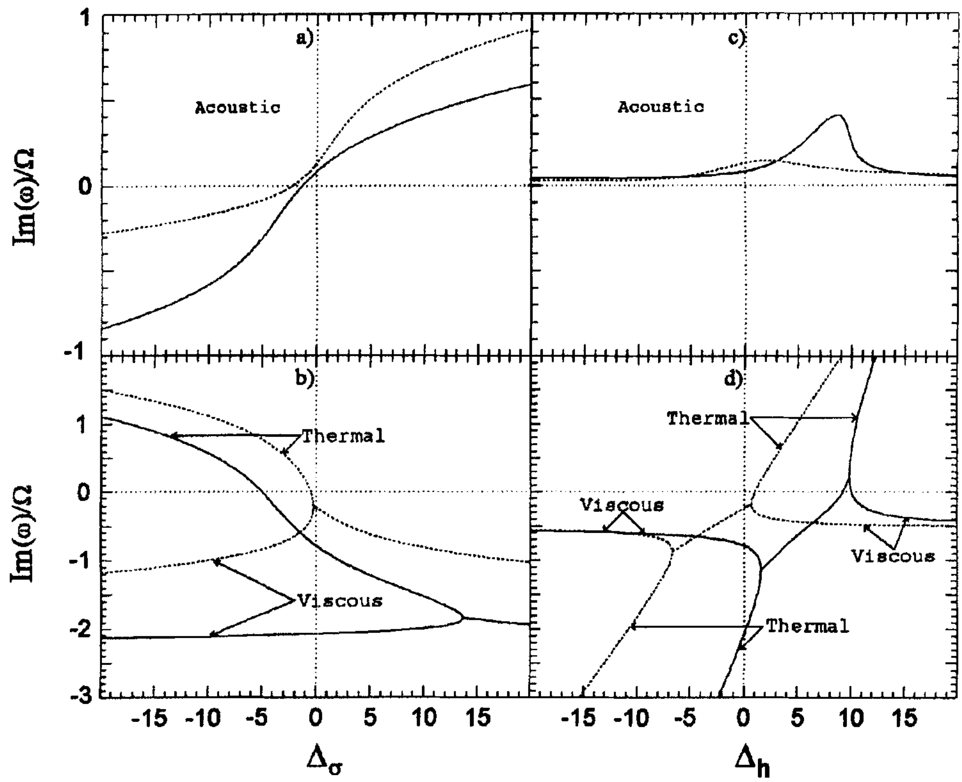

Figure 2 Dependence of the imaginary part of the frequency in terms of the angular velocity $\mathrm{Im}(\omega)/\Omega$ of acoustic, thermal, and viscous oscillation modes on the value of the parameter Δ_σ (a),(b) and on the value of parameter Δ_h (c),(d) at $kh_0 = 1$. The solid and short dashed lines represent $\beta_0 = 0$ and $\beta_0 = 1$, respectively.

In the case of Thomson scattering $\bar{\kappa} = \bar{\kappa}_{es} = 0{,}4\mathrm{sm}^2/g$ ($\Delta_\sigma = 0$, $\Delta_h = 0$), and for the Kramers law $\bar{\kappa} = \bar{\kappa}_{ff} \propto \rho T^{-7/2}$ ($\Delta_\sigma = 1$, $\Delta_h = -8$). For low-temperature protoplanetary disks $\bar{\kappa} \propto T^2$ (Lin, 1981) ($\Delta_\sigma = 0$, $\Delta_h = 4$). In the "hot" limit of the Foulkner model (Foulkner *et al.*, 1983) $\bar{\kappa} \propto \rho T^{-5/2}$ $\left(\Delta_\sigma = \dfrac{2+\beta_0}{2(1+3\beta_0)},\ \Delta_h = -\dfrac{12+\beta_0}{2(1+3\beta_0)}\right)$, but in the "cold" limit $\bar{\kappa} \propto \rho^{1/3}T^{10}$ ($\Delta_\sigma = 1/3$, $\Delta_h = 59/3$). Here $\beta_0 = P_{0\,\mathrm{rad}}/(P_{0\,\mathrm{rad}} + P_{0\,\mathrm{gas}})$, P_{gas} is the gas pressure, P_{rad} is the radiation pressure.

As indicated in Figure 2 the thermal mode of the oscillation becomes unstable even in the case $\beta_0 = 0$ at $\Delta_\sigma < 0$ and $\Delta_h > 0$. Stabilization of the acoustic-mode occurs at a high negative value of Δ_σ. However, in the framework of the standard model AD (Shakura and Sunyaev, 1973) acoustic modes of the oscillations prove to be unstable both in the inner radiation-dominated region ($P_{\mathrm{rad}} \gg P_{\mathrm{gas}}$, $\bar{\kappa} = \bar{\kappa}_{es}$), and in the external gaseous region ($P_{\mathrm{gas}} \gg P_{\mathrm{rad}}$, $\bar{\kappa} = \bar{\kappa}_{ff}$).

4 THE INFLUENCE OF VISCOSITY

As the second (elastic) viscosity μ_0 increases, the increment of the sound waves decreases until the imaginary part vanishes at $\mu_0 = \mu_{0\,\mathrm{crit}}$. The perturbation decays ($\mathrm{Im}(\omega) < 0$) when $\mu_0 > \mu_{0\,\mathrm{crit}}$. As shown by Khoperskov and Khrapov (1995) the quantity $\mu_{0\,\mathrm{crit}}$ weakly depends on β_0, so that $\mu_{0\,\mathrm{crit}} = 2$–$3\nu_0$.

The increments and conditions for instability development very strongly depend on the parameters δ_σ and δ_h. By now there are many models of turbulent viscosity with different values δ_σ, δ_h. For the standard α-model $W_{r\varphi} = -\alpha p$ and $\delta_\sigma = 0$, $\delta_h = 2$. For modifications of α-models $W_{r\varphi} = -\alpha(p_g/p)^{N/2}p$ ($N = \mathrm{const}$) (Lightman and Eardley, 1974; Taam and Lin, 1984; Szuszkiewicz, 1990) and $\delta_\sigma = \frac{N\beta_0}{2(1+3\beta_0)}$, $\delta_h = \frac{4+\beta_0(12-7N)}{2(1+3\beta_0)}$.

The functions $\mathrm{Im}[\omega(\delta_\sigma)]$ and $\mathrm{Im}[\omega(\delta_h)]$ are in qualitative agreement with $\mathrm{Im}[\omega(\Delta_\sigma)]$ and $\mathrm{Im}[\omega(\Delta_h)]$, respectively. In terms of the above-mentioned models of turbulent viscosity, both acoustic modes of the oscillations are unstable at $\mu_0 < \mu_{0\,\mathrm{crit}}$, because for all these models $\delta_\sigma > 0$ and $\delta_h > 0$. The acoustic oscillations stabilize at large negative values of δ_σ.

5 CONCLUSION

A linear analysis has been performed to examine the radial–azimuthal instability of accretion disks. The main results are as follows:

First, the existence of unstable modes is associated completely with the perturbation of dynamic viscosity $\eta = \sigma\nu$ and, consequently, it is determined by the dependence of $\nu(\sigma, h)$. If we set $\tilde{\eta} \equiv 0$, all four oscillatory modes (including the thermal and viscous modes in the radiation-dominated limit) would decay with decrement $\mathrm{Im}(\hat{\omega}) \sim -\nu_0 k^2$.

Second, the azimuthal perturbation wave-number m in terms of the WKB approximation for a short-wave perturbations with $kr \gg 1$ is not affected by the increments of unstable modes and that is responsible for the Doppler shift of the frequency $\hat{\omega} = \omega - m\Omega$.

Third, in the radiation-pressure-dominated accretion disk the thermal and viscous unstable modes stabilize at large values of the characteristic time scale of the delay τ, while the acoustic mode tends to become stable at small values of τ.

Finally, the conditions of the development and parameters of the thermal, viscous and acoustic instabilities in the geometrically thin and optically thick accretion disk strongly depend on the models of viscosity and opacity. In the gas-pressure-dominated accretion disk the thermal mode tends to become unstable with decreasing values of δ_σ, Δ_σ and with increasing values of δ_h, Δ_h. The acoustic mode at any gas-to-radiation pressure ratio tends to become stable with decreasing values of $\delta_\sigma, \Delta_\sigma$. An additional point to emphasize is that the sound waves stabilize at large values of the second (elastic) viscosity.

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