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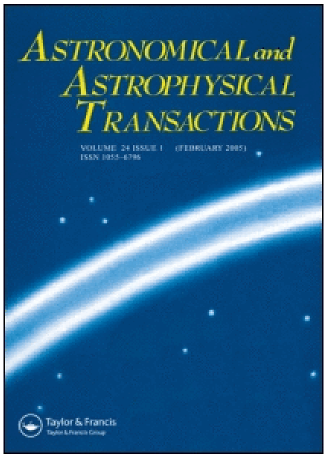

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Time series analysis of the phase curve characteristics of the semiregular star RS Cygni

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TIME SERIES ANALYSIS OF THE PHASE CURVE CHARACTERISTICS OF THE SEMIREGULAR STAR RS CYGNI

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RS Cyg is studied by using 3986 observations from the AFOEV database. The second-order trigonometric fit to the mean light curve corresponds to the period $P = 426^{\mathrm{d}}.64 \pm 0^{\mathrm{d}}.03$, which differs from the value $417^{\mathrm{d}}.39$ listed in GCVS IV. Long-term variations of P in a range 420^{d}–432^{d} are detected. The maxima are often split with a mean separation of $(0.314 \pm 0.034)P$. The application of various complementary fitting algorithms (using trigonometric polynomials, running parabolae and running sine approximations) are discussed with special attention to the criteria of *determination* of the free parameters based on the statistical properties of the smoothing function. Some methodological aspects of the time series analysis of the groups of data with a systematic difference between them are discussed.

KEY WORDS Semi-regular stars, data reduction, RS Cyg

1 INTRODUCTION

RS Cyg is one among several well-studied stars which have been monitored for decades by amateur astronomers. According to GCVS IV (Kholopov *et al.*, 1985) the star is of the SRa type. Its brightness varies from $6^{\mathrm{m}}.5$ to $9^{\mathrm{m}}.5$ (V), the ephemeris for the minimum is the following: min HJD $= 2438300 + 417.39 \cdot E$. The asymmetry is not listed as "the shape of the light curve strongly varies. Double maxima are sometimes observed (min II at the phase $0^{\mathrm{P}}.5$)" (GCVS IV).

We have used visual observations from the AFOEV database (Schweitzer, 1993) to study the temporal behaviour of the brightness as the initial characteristic, and of the parameters of the individual cycles as characteristics of the pulsations.

For the time series analysis we have applied various methods: the periodogram analysis based on the least squares fits by trigonometric polynomials of first and higher orders (Andronov, 1994); the approximation of the individual cycles by running parabolae fit (Andronov, 1990, 1997) and running sine fit (Chinarova *et al.*, 1994).

2 OBSERVATIONS AND REDUCTION TO A "STANDARD" SYSTEM

From all AFOEV data we have deleted uncertain values and estimates "fainter than". Alltogether 3986 data points remained (1921–1996, JD 2423012–50261), 2806 of which had no remarks. To study possible systematic differences between the groups of observations obtained by various amateurs the following procedure was used:

(1) The "standard" photometric system was chosen. For this system the "unmarked" data were used which were obtained by various observers from visual estimates using the same magnitudes of the comparison stars. The "marked" data correspond to other photometric systems – photographic, broad-band red–infrared CCD systems, etc. They were separated into other data files for comparison with nearby "standard" observations.

(2) The running parabolae fit was computed for the "unmarked" data for a grid of values of the filter half-width Δt. The optimal value of the filter half-width $\Delta t = 158^{\mathrm{d}}$ was determined (see section 6 for more details on the criterion). The smoothed values m_s of the "standard" curve were computed at the *times* of "marked" observations.

(3) The characteristic curve "smoothed" value vs. "original" brightness m_0 was plotted and analysed for each data group. If the individual observational system is close to a "standard" one, the diagram is close to the line $m_s = m_0$ showing obvious observational errors. The observations in other distinct photometric systems may not be used for any monotonic (e.g. polynomial) fit as the track of the star during pulsations at the two-colour diagram may resemble a loop. Only if this loop is close to a line (one has to test this hypothesis separately!) may one use the procedure discussed here. Another possibility is that two observers use different photometric systems (e.g. visual and photovisual) but different sets of magnitudes of comparison stars. Because of the difference of the colour indices of the comparison stars and the variable there may be deformations of the magnitude scale.

For the star RS Cyg discussed in this work no corrections were made for the majority of data groups. However, the data marked with "E" (observer "RIP" – J. Rupero-Osorio) were excluded as they showed no resemblance to a "standard" curve. The 11 data items marked with "B" (observer "DPA" – A. Diepvens) were reduced to the "standard system" by using the best parabolic fit. The light curve by this author is very similar to the "standard" one. However, the m_s–m_0 diagram distinctly deviates from a straight line. The statistically significant degree of the polynomial was determined by using Fischer's criterion (e.g. Korn and Korn, 1961). The systematic difference between the polynomials of the first and second orders reached $0\overset{\mathrm{m}}{.}5$ at the levels of minimum and maximum.

The light curve is shown in Figure 1. The range is $6\overset{\mathrm{m}}{.}7$–$10\overset{\mathrm{m}}{.}0$ and is close to $6\overset{\mathrm{m}}{.}5$–$9\overset{\mathrm{m}}{.}5$ listed in GCVS IV. The mean value is $\overline{m} = 7\overset{\mathrm{m}}{.}955$, the r.m.s. deviation

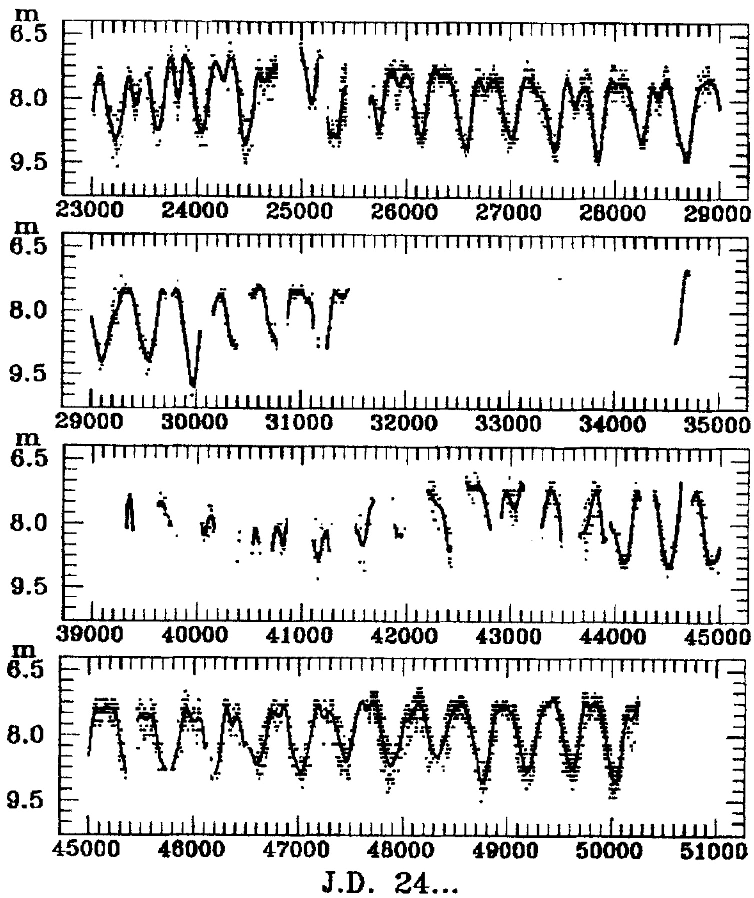

Figure 1 Light curve of RS Cyg from the AFOEV database. Sequence of points: running parabolae fit with $\Delta t = 80^d$.

from the mean is $\sigma_O = 0\overset{m}{.}607$. The most numerous observations (more than 100) were obtained by U. Bieth (433), G. Loreta (372), E. Schweitzer (322), P. Vedrenne (276), F. Mandre (267), G. Krisch (248), M. Hiraga (164), U. Chaumont (144), A. Nemeth–Bubin (130). We thank the amateur astronomers for their intensive studies.

3 PERIODOGRAM ANALYSIS

The periodogram analysis was carried out by using the program FOUR-1 (Andronov, 1994). The test function is $S(f) = 1 - \sigma^2_{O-C}/\sigma^2_O$, where σ^2_O and σ^2_{O-C} are the variances of the observed values and of the deviations $(O - C)$ of the data from the best sine fit of the given trial period $P = 1/f$.

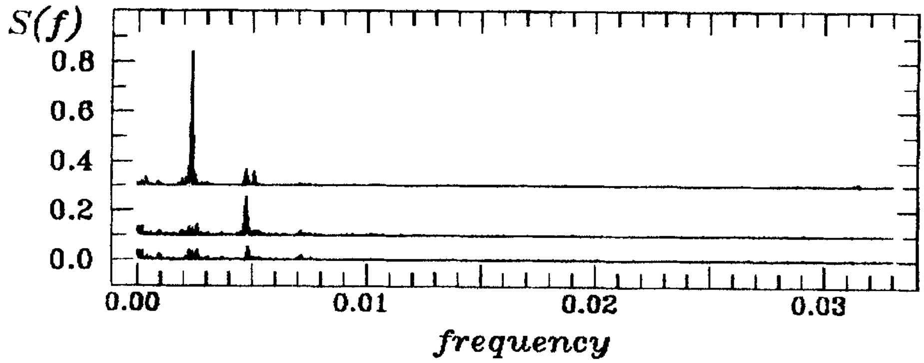

Figure 2 Periodograms of the original (top, shift 0.3) observations and their residuals $(O - C)$ from the trigonometric polynomials of the first- (middle) and second- (bottom) order fits.

The periodogram (Figure 2 top) shows a most prominent peak at frequency $f = 0.0023436 \pm 0.0000002$ cycles/day (c/d) corresponding to a period $P = 1/f = 426\overset{d}{.}69 \pm 0\overset{d}{.}04$, an initial epoch for the maximum $T_{\rm max} = 2437874.6 \pm 0\overset{d}{.}9$, an amplitude $0\overset{m}{.}649 \pm 0\overset{m}{.}009$ and r.m.s. deviation from the fit $\sigma_{(O-C)} = 0\overset{m}{.}41$. The maximal value $S(f) = 0.542$ exceeds the value expected for the white noise by a factor of 1080. Much lower peaks occurred at $P = 213\overset{d}{.}24 \pm 0\overset{d}{.}04$ ($r = 0\overset{m}{.}197$, $S(f) = 0.053$), $P = 211\overset{d}{.}14 \pm 0\overset{m}{.}4$ ($r = 0\overset{m}{.}227$, $S(f) = 0.070$) and $P = 196\overset{d}{.}7 \pm 0\overset{d}{.}4$ ($r = 0\overset{m}{.}210$, $S(f) = 0.058$). The first one is exactly half of the main period and corresponds to a first harmonic, whereas the second period seems to be an independent one.

The periodogram was also computed for the residuals $(O - C)$ from the one-wave sine fit (Figure 2 middle). It shows the highest peak at a first harmonic of the $426^{\rm d}$ wave and at $P = 211\overset{d}{.}2 \pm 0\overset{d}{.}03$ ($r = 0\overset{m}{.}227 \pm 0\overset{m}{.}008$, $S(f) = 0.155$), in good agreement with the results obtained for the original observations. However, the peak at $197^{\rm d}$ practically disappeared; thus it was spurious.

The third periodogram was computed for the residuals of the observations from the third-order trigonometric polynomial fit (see next section). The highest peak occurred at the frequency $P = 208\overset{d}{.}54 \pm 0\overset{d}{.}04$ ($r = 0\overset{m}{.}128 \pm 0\overset{m}{.}008$, $S(f) = 0.054$). Thus one may conclude that this and the $211^{\rm d}$ peak are aliases of the first harmonic and are not realistic.

For all three periodograms no prominent peaks are seen at periods shorter than $196^{\rm d}$, including the second harmonic.

4 TRIGONOMETRIC POLYNOMIAL FITS

As one may see from the previous section, the contribution of the first harmonic is significant. Moreover, the complicated shape of the individual light curves argues for multiharmonic fits rather than for one-wave approximations. We have used

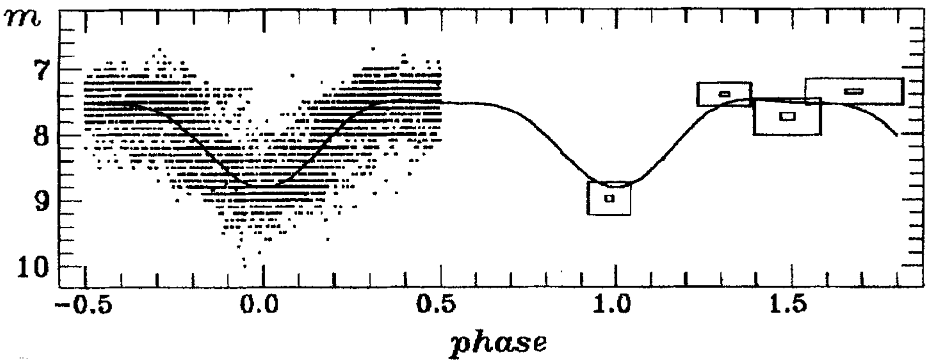

Figure 3 Points: phase light curve of RS Cyg according to elements ( ); *solid line*: its approximation by a third-degree trigonometric polynomial fit; *Boxes*: the border of the region $|m-\overline{m}| < 1\sigma_m$ and $|\phi - \overline{\phi}| < 1\sigma_\phi$, where $\overline{m}$ and $\overline{\phi}$ are weighted mean values of the brightness and phase of the given extremum and σ_m and σ_ϕ are corresponding r.m.s. deviations (outer box) and the standard error of the mean (inner box).

the program FOUR-M (Andronov, 1994). The initial value of the frequency was corrected by using the method of differential corrections. To determine the statistically significant degree of the trigonometric polynomial m we have used Firscher's test. For the whole data set the value $m = 2$ was obtained, corresponding to the extremely low "false alarm probability" $Pr = 10^{-111}$, i.e. the probability of obtaining the amplitude of a given height for pure white noise. For $m = 3$ the value $Pr = 10^{-1.6} = 2.5\%$ is much higher and we have accepted the null hypothesis for $m = 3$.

The coefficients of the trigonometric polynomial fits for the light curves of the δ Cep-type stars were studied by Kukarkin and Parenago (1937) as functions of the photometric period. Recently the interest in comparison of the observations with theoretical models of pulsations has grown as one may see, for example, from the papers by Kovacs *et al.* (1986) and Antonello (1994).

The best third-degree trigonometric polynomial fit for RS Cyg corresponds to the elements

$$\begin{array}{rl} \text{Min HJD} = 2438091.5 & + \; 426^{\rm d}\!.64 \cdot E \\ \pm 9 & \qquad \pm 3 \end{array} \tag{1}$$

The initial epoch for the maximum HJD $2438258.3 \pm 3^{\rm d}\!.2$ ($m = 7^{\rm m}\!.48 \pm 0^{\rm m}\!.01$) is much more poorly defined. The brightness at the primary minimum is $m = 8^{\rm m}\!.82 \pm 0^{\rm m}\!.01$, the asymmetry is $M - m = 0.391 \pm 0.008$ and the full amplitude is $1^{\rm m}\!.34 \pm 0^{\rm m}\!.01$.

The amplitudes of the main wave and its first harmonic are $r_1 = 0^{\rm m}\!.649 \pm 0^{\rm m}\!.009$ and $r_2 = 0^{\rm m}\!.197 \pm 0^{\rm m}\!.009$ in exellent agreement with results obtained from one–wave fits. The corresponding phases of maxima of these contributions with respect to the maximum of their sum are $\phi_1 = 0.103 \pm 0.002$ and $\phi_2 = -0.271 \pm 0.007$.

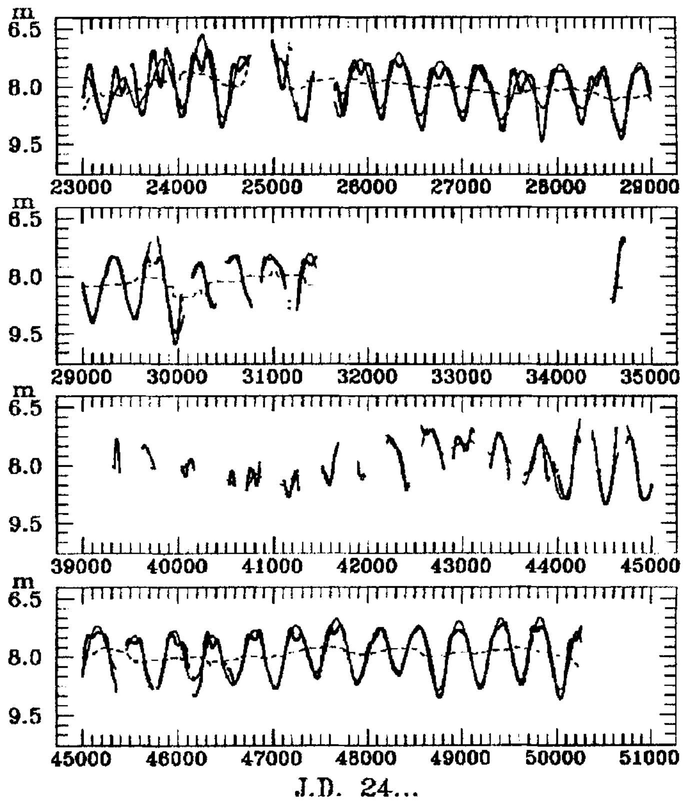

Figure 4 Approximations of the individual cycles by running parabolae (*points*) with $\Delta t = 80^{\rm d}$ and running sines (*solid line*) with $\Delta t = P/2 = 213.^{\rm d}3$.

Additional parameters were introduced by Chinarova *et al.* (1997) and Marsakova *et al.* (1997). These are the maximum slopes dm/dt of the ascending and descending branches and their reversals dt/dm (i.e. the characteristic time of brightness change by $1^{\rm m}$). For RS Cyg, $dt/dm = 73.^{\rm d} \pm 1.^{\rm d}5$ and $77.^{\rm d}2 \pm 1.^{\rm d}7$ respectively. The ratios of the slopes dm/dt to those achieved for the sine function of the same period and amplitude are 1.39 ± 0.03 and 1.31 ± 0.003, respectively. These values significantly differ from unity, but are close to each other. This argues not for asymmetry of ascending and descending branches, but for their relatively short durations as compared with the sine fit.

The phase light curve is shown in Figure 3. The maximum of the mean curve is not split as one may see at many individual cycles. For $m = 3$ the secondary minimum and maximum are seen at the phases 0.12 and 0.22, but the depth of the secondary minimum is $0.^{\rm m}04$ and does not exceed 2σ. Thus formally the maximum may be formally described as a "flat" one instead of a "double". For phasing, the epoch of the minimum should be used, which is much better determined.

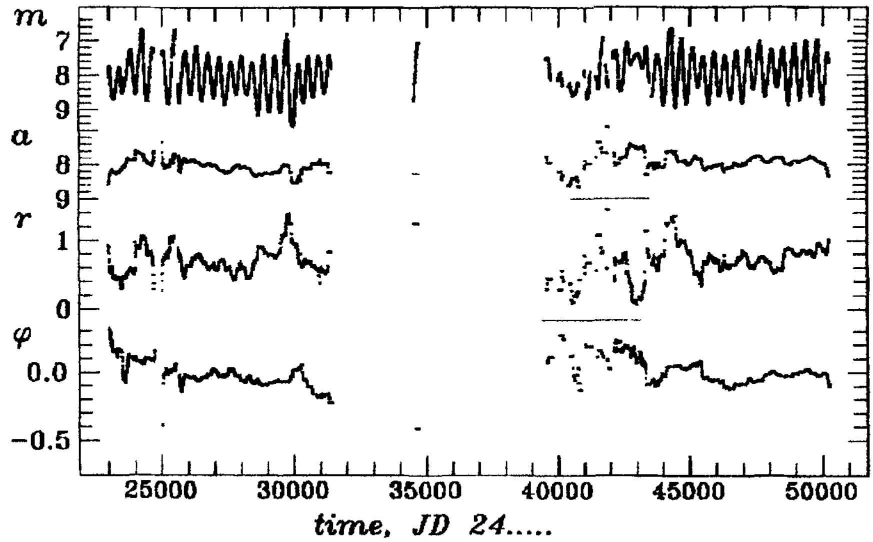

Figure 5 The dependence on time of the parameters of the running sine fit.

5 RUNNING SINE FITS

To study cycle-to-cycle variations of the light curve we have used the method of running sines (RS) proposed by Chinarova *et al.* (1994). As in other running approximations, the part of the data located in the interval from $t_0 - \Delta t$ to $t_0 + \Delta t$ is used for the approximation, where t_0 is the moment of time for which the smoothed value should be computed and Δt is the filter half-width. We have used the sine fit in the form $m(t) = a + r\cos(2\pi(f(t - T_0) - \phi))$, where the phase ϕ corresponds to a maximum if using the initial epoch T_0 and the period $P = 1/f$.

As in all other methods, the RS fit is very sensitive to gaps in the observations. Thus the fit for a trial time was accepted if the number of data points in the smaller interval from $t_0 - 0.5\Delta t$ to $t_0 + 0.5\Delta t$ exceeds 3; there are observations before and after t_0; the smoothed value at t_0 is in the interval from $(1+\alpha)x_{\rm min} - \alpha x_{\rm max}$ to $(1+\alpha)x_{\rm max} - \alpha x_{\rm min}$ (we have used $\alpha = 0.05$); the error estimate of the smoothed value does not exceed σ_O. If any of these conditions is not fulfilled, the data are not saved in a file. This allows us to reject the majority of bad values automatically.

The computed light curve is shown in Figure 4 in comparison with a running parabolae (RP) fit (see next section). One may note that the RS fit shows no rapid variations like split maxima and thus corresponds to the mean characteristics of variations with the main period only. The filter half-width is $\Delta t = P/2$, thus the time interval equal to the complete period is used for each t_0. The smoothing curve exhibits properties of the variations: cycle-to-cycle and long-term changes of the mean brightness a and amplitude r. In Figure 5 we show separately the variations of all RS parameters with time: m (range from $6^{\rm m}\!.7$ to $9^{\rm m}\!.5$), a ($6^{\rm m}\!.9$–$8^{\rm m}\!.7$), r ($0^{\rm m}\!.08$–

$1^{\mathrm{m}}.44$) and ϕ. Some bad points remained after automatic filtration; they are clearly seen in Figure 4. The changes are well pronounced. They are chaotic showing no evidence for secondary periodicities.

The most surprising is the apparent trend of the phase with time which is seen both in the first and second parts of the observations separated by a relatively large gap. This argues for another value of the period than that obtained from the trigonometric polynomial fit. For JD2423013–31452 the value of the period corresponding to the phase trend is $421^{\mathrm{d}}.5$. No peak in the periodogram is associated with this value. The second large peak corresponds to $P = 418^{\mathrm{d}}.33 \pm 0^{\mathrm{d}}.04$, closer to the GCVS value $417^{\mathrm{d}}.39$. Its height is only 87% of the largest one. This value is an alias corresponding to 64 cycles between the first and last detected minimum instead of 63 for $P = 426^{\mathrm{d}}.64$. The corresponding number of the statistically significant degree of the trigonometric polynomial is $m = 3$. However, the phase curve shows larger scatter than that in Figure 3.

The 421^{d} period is between the values 418 and 426, thus the phase difference must be 63.5 cycles which also may not be accepted. The trend of the phases for the $418^{\mathrm{d}}.33$ period has an opposite sign. Thus we interpret the trend as a result of long-term period changes rather than of another constant period.

6 RUNNING PARABOLAE FITS

The RP fit was proposed by Andronov (1990). More detailed study of the statistical and spectral properties of the test function may be found in Andronov (1997). It has only one free parameter – the filter half–width Δt, the optimal value of which was determined by computing the "scalegram"–dependence on Δt of the following parameters: σ_1, σ_2, estimates of the unit weight errors of the observations computed according to equations (19) and (22) of Andronov (1997); σ_3, the r.m.s. deviation of the observations from the RS fit; the proportionality coefficient R between the unit weight error and the error estimate of the smoothing function $\sigma[m(t)]$. As two values σ_1 and σ_2 are used, there are two estimates of $\sigma[m(t)]$: $\sigma_4 = R\sigma_1$ and $\sigma_5 = R\sigma_2$. The parameter σ_C/σ_5 is the "signal/noise" ratio. As in all other fits, one has to find a compromise between the level of systematic deviations of the fit from the shape of the signal and the statistical weight of the smoothing value.

This "scalegram" is shown in Figure 6. The value corresponding to the maximum of the "signal/noise" ratio is $\Delta t = 158^{\mathrm{d}}.5$. It is smaller than $0.54P$ which is expected for the one-wave signal with noise because of the asinusoidal shape of the light curve. The corresponding fit also shows the split maxima, but the amplitude of these variations is significantly reduced because of so large a value of Δt. Another criterion is the minimization of σ_2. This parameter is equal to $0^{\mathrm{m}}.27$ for $\Delta t = 40^{\mathrm{d}}$ and increases by 1% for $\Delta t = 80^{\mathrm{d}}$ and by 12% for $t = 158^{\mathrm{d}}.5$. We have chosen $\Delta t = 80^{\mathrm{d}}$ corresponding to a better approximation of the complicated shape of the light curve.

The smoothed values were used only for the times t_0 for which the fit fulfils the conditions similar to that for the running sines. The preliminary times of

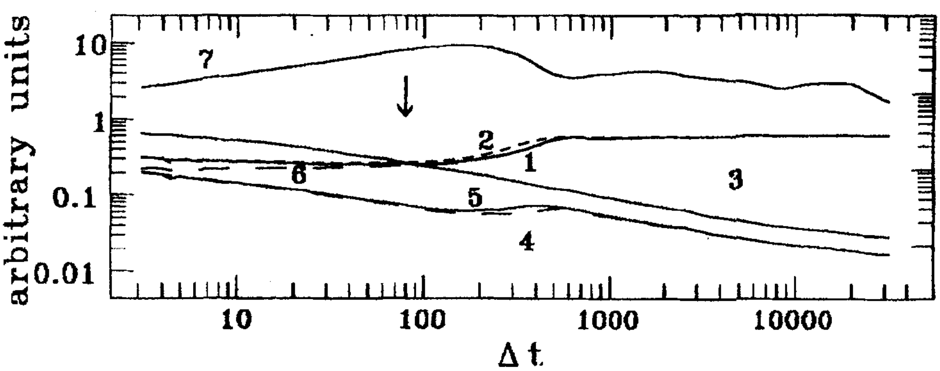

Figure 6 The dependence on the filter half-width Δt of the parameters of the running parabolae fit.: *1*, σ_1; *2*, σ_2; *3*, R; *4*, σ_4; *5*, σ_5; *6*, σ_3; *7*, "signal/noise". The vertical arrow marks the adopted value $\Delta t = 80^{\rm d}$.

the extrema derived from the grid of the smoothed values were then corrected by using the method of differential corrections as desribed by Andronov (1997). The characteristics of the extrema are listed in Table 1 and are shown in Figure 7.

The phase of the primary minimum exhibits trends corresponding to period values $422^{\rm d}_{\cdot}4$ ($n = 18$ minima at JD 2423225–30820), $419^{\rm d}_{\cdot}7$ ($n = 10$, JD 2441157–46600) and $431^{\rm d}_{\cdot}9$ ($n = 8$, JD 2447023–50038). These values were computed by using the algorithm PERMIN (Andronov, 1991). These results are consistent with

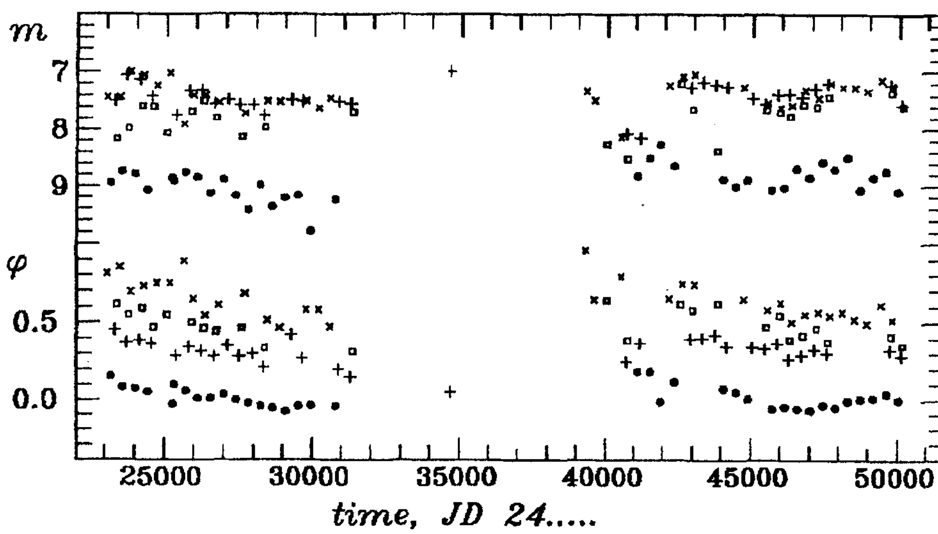

Figure 7 The dependence on time of the brightness and phase of the extrema determined by using the running parabolae fit with $\Delta t = 80^{\rm d}$: *filled circles and open boxes* – primary and secondary minimum, *direct and inclined crosses* – primary and secondary maximum.

Table 1. Characteristics of the extrema: moments of time (JD 24...), brightness and corresponding error estimates

t_e, d	$\sigma[t_e]$	m_e	$\sigma[m_e]$	t_e, d	$\sigma[t_e]$	m_e	$\sigma[m_e]$	t_e, d	$\sigma[t_e]$	m_e	$\sigma[m_e]$
		min 1		26280	7	7.31	0.02	46005	37	7.69	0.11
23225	17	8.94	0.11	26694	10	7.55	0.04	46365	11	7.77	0.07
23621	30	8.74	0.10	27149	20	7.47	0.05	46804	45	7.56	0.08
24044	12	8.78	0.07	27546	7	7.56	0.04	47250	25	7.60	0.14
24461	17	9.06	0.11	27979	28	7.56	0.05	47640	10	7.42	0.08
25280	11	8.85	0.07	28370	4	7.74	0.04	49789	9	7.35	0.04
25334	16	8.90	0.12	29313	24	7.49	0.06	50190	18	7.60	0.06
25743	6	8.76	0.08	29673	6	7.50	0.06			max 2	
26149	9	8.84	0.05	30923	20	7.52	0.04	23078	13	7.43	0.10
26577	7	9.12	0.05	31328	12	7.55	0.06	23523	11	7.44	0.27
27013	10	8.88	0.05	34700	19	6.99	0.10	23882	5	7.00	0.08
27426	7	9.16	0.05	40757	11	8.08	0.18	24324	26	7.06	0.09
27843	4	9.42	0.09	41235	15	8.16	0.20	24758	20	7.24	0.05
28261	8	8.98	0.06	42955	13	7.26	0.06	25184	27	7.03	0.18
28682	5	9.36	0.03	43383	9	7.17	0.04	25672	18	7.91	0.25
29099	5	9.21	0.04	43819	7	7.21	0.07	25995	22	7.40	0.05
29542	9	9.16	0.07	44213	10	7.24	0.07	26378	12	7.40	0.04
29970	6	9.78	0.06	45065	10	7.44	0.07	26832	20	7.53	0.05
30820	2	9.24	0.16	45489	42	7.56	0.10	27716	26	7.72	0.05
41157	9	8.82	0.08	45928	16	7.38	0.05	28497	5	7.50	0.03
41583	39	8.50	0.24	46311	6	7.37	0.07	28904	41	7.51	0.05
41927	37	8.27	0.16	46748	8	7.43	0.07	29806	17	7.50	0.09
42408	19	8.64	0.10	47190	18	7.28	0.12	30233	20	7.63	0.05
44096	15	8.87	0.03	47607	9	7.20	0.05	30613	12	7.45	0.03
44513	10	8.99	0.04	49751	9	7.22	0.05	39354	7	7.32	0.28
44922	29	8.89	0.04	50159	37	7.56	0.07	39647	38	7.49	0.08
45747	8	9.06	0.22			min 2		40562	4	8.14	0.07
46178	12	9.01	0.35	23420	6	8.17	0.06	42209	39	7.23	0.13
46601	7	8.69	0.08	23819	12	7.99	0.08	42676	18	7.08	0.05
47023	19	8.84	0.05	24263	7	7.59	0.14	43101	7	7.04	0.05
47464	9	8.57	0.06	24639	10	7.60	0.05	44768	9	7.26	0.05
47884	16	8.70	0.07	25099	7	8.07	0.10	45593	12	7.53	0.08
48329	11	8.49	0.11	25932	7	7.69	0.05	46038	19	7.61	0.12
48761	21	9.06	0.06	26343	7	7.50	0.03	46413	18	7.57	0.09
49190	13	8.84	0.05	26761	11	7.80	0.05	46860	22	7.31	0.12
49629	9	8.73	0.04	27624	10	8.14	0.04	47293	22	7.44	0.10
50039	12	9.08	0.06	28423	7	7.97	0.03	47710	8	7.19	0.03
		max 1		31399	15	7.70	0.05	48147	23	7.26	0.05
23351	8	7.49	0.08	40071	27	8.28	0.19	48553	17	7.27	0.03
23746	5	7.06	0.07	40817	13	8.52	0.18	48969	110	7.32	0.03
24177	14	7.14	0.11	42621	47	7.20	0.05	49447	7	7.13	0.04
24595	7	7.42	0.05	43032	10	7.64	0.06	49832	15	7.22	0.04
25414	8	7.75	0.07	43903	18	8.38	0.26				
25866	7	7.33	0.03	45547	45	7.66	0.07				

that obtained by the RS fit neglecting the splitting of the maximum. The phases of the secondary minimum and both maxima are variable with respect both to the mean and individual primary minimum.

The brightness at all extrema show variability from cycle to cycle and at longer time scales. The periodogram computed by using the program FOUR-0 (Andronov,

1994) shows a highest peak at the period of 24200 ± 1400 ($r = 0^{m}.32 \pm 0^{m}.07$). Its significance is rather low, as the "false alarm" probability is $\approx 7\%$. The second lower peak at $\approx 7700^{d}$ is closer to the duration of the 5600^{d} wave seen after JD 2441000. Thus these variations are not of a periodic nature and these values are only estimates of the time scale.

The mean values, r.m.s. deviations from the mean and the range of changes of the characteristics of extrema are listed in the following table:

Extr.	n	ϕ	$\sigma[\phi]$	ϕ_{min}	ϕ_{max}	$\overline{m}$	$\sigma[m]$	m_{min}	m_{max}
min 1	36	−0.018	0.061	−0.078	−0.009	8.98	0.25	8.27	9.78
max 1	32	0.311	0.076	0.051	0.451	7.41	0.18	6.99	8.16
min 2	24	0.490	0.094	0.313	0.640	7.74	0.28	7.20	8.52
max 2	34	0.678	0.138	0.466	0.960	7.35	0.19	7.00	8.14

7 CONCLUSION

The shape of the light curve undergoes cycle-to-cycle changes. The maximum is often split into two humps. The most stable in phase is the primary minimum. The phase shows variable trends corresponding to the period changes in the range 420–432^{d}. No periodic variations of the characteristics of the individual cycles was found, or any other periodic component of the brightness variations besides the main period. Besides the cycle-to-cycle changes the irregular variations of the characteristics are present at a ≈ 15 yr time scale. The amplitude of the brightness variations at the primary minimum is $1^{m}.51$; at the primary maximum $1^{m}.17$.

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