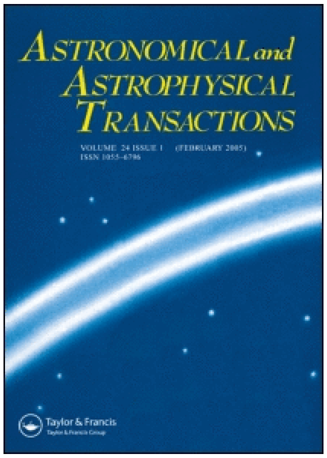

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Over-reflection and instability of shock waves in an inhomogeneous medium

M. A. Eremin [a]; I. G. Kovalenko [a]; D. V. Lukin [a]
[a] Department of Physics, Volgograd State University, Volgograd, Russia

Online Publication Date: 01 August 1999
To cite this Article: Eremin, M. A., Kovalenko, I. G. and Lukin, D. V. (1999) 'Over-reflection and instability of shock waves in an inhomogeneous medium', Astronomical & Astrophysical Transactions, 18:1, 109 - 116
To link to this article: DOI: 10.1080/10556799908203042
URL: http://dx.doi.org/10.1080/10556799908203042

Astronomical and Astrophysical Transactions, 1999,
Vol. 18, pp. 109–116
Reprints available directly from the publisher
Photocopying permitted by license only

OVER-REFLECTION AND INSTABILITY OF SHOCK WAVES IN AN INHOMOGENEOUS MEDIUM

M. A. EREMIN, I. G. KOVALENKO, and D. V. LUKIN

Department of Physics, Volgograd State University 2ya Prodol'naya, 20, Volgograd 400062, Russia

(Received December 15, 1996)

We present a resonant description of hydrodynamic instabilities of shock waves in inhomogeneous media. This approach (1) allows us to clarify the physical mechanisms of instabilities; (2) provides a natural classification of all hydrodynamic instabilities; (3) allows us to simplify the search and prediction of instabilities, reducing the analysis to studying the coefficients of transformation (reflection) of perturbations at the shock front. We apply the developed formalism to the analysis of the resonance characteristics of a model of an accelerating shock wave in an exponential atmosphere and a model of a galactic shock wave. Analysis shows that instability in these models is caused by the effect of spontaneous emission of waves by the shock front while the true resonant effects are insignificant. Finally, we predict that the standing shock wave in an accretion flow on to a point mass must be unstable for the same reasons.

KEY WORDS Accretion, interstellar matter, hydrodynamic instabilities, shock waves

1 INTRODUCTION

Various factors promote the instability of shock waves in the interstellar and the intergalactic media: these are magnetic fields, radiative losses of energy, self-gravitation, etc. Meanwhile instability may arise for exclusively hydrodynamic reasons. This is the case, for example, for the instability of shocks accelerating in a medium with a sharp density gradient (Gurevich and Rumyantsev, 1969; Chevalier, 1990; Lio and Chevalier, 1994), instability of the decelerating spherical Sedov–Taylor blast wave (Ryu and Vishniac, 1987, 1991; Goodman, 1990), or instability of the galactic shock wave standing within the gravitational well of the spiral arm (Kovalenko, 1997; Kovalenko and Lukin, 1997).

We believe that all these instabilities of shock waves in an inhomogeneous medium can be explained from a unified point of view in terms of the resonant approach. A brief description of this approach is given in the present paper (a full version will be published in *Astronomy and Astrophysics*).

The main idea of the resonant mechanism of the instability of a shock wave can be formulated as follows. The postshock flow is, generally speaking, inhomogeneous. Suppose this inhomogeneity is a layer of finite thickness L, not necessarily constant in time. This layer is bounded ahead by the shock front and either continuously passes behind into the homogeneous flow or is bounded behind by some surface, say, another shock front, a rigid boundary, a contact discontinuity, a sonic surface or a critical layer. This layer of inhomogeneity can be considered as a typical waveguide. Indeed, let a sound wave hit the shock front from the postshock side. The oscillating shock front responds by generating outgoing sound, vortical and entropic perturbations. The outgoing waves pass through the layer of inhomogeneity and leave the waveguide along with the flow (provided, of course, that the rear boundary is not rigid or does not make contact with the vacuum). Passing through the inhomogeneity and interacting with the rear boundary, the outgoing waves generate secondary sound waves. One of them, the so-called fast sound wave, moves downstream and leaves the waveguide, whereas the second one, the slow sound wave, moves upstream and is incident on the shock front, after which the process of reflection occurs repeatedly. Thus the oscillations within the waveguide support themselves without excitation from outside.

A researcher traditionally finds the overall structure of the wavefunction of perturbations along with the eigenvalue ω_* through solving the self-consistent Sturm–Liouville problem (this reduces to the integration of a system of differential equations) with the appropriate boundary conditions and never thinks about the resonant properties of the wave process. We instead divide the process of solving the eigenvalue problem into two subproblems. We consider separately the process of reflection and transformation of the sound wave on the shock front and the process of reflection and transformation of the sound wave on the postshock inhomogeneity.

To perform this we expand any small perturbation locally in modes moving upstream and downstream. Let these modes be δq_i, $i = 1, \ldots, 4$ (the fifth mode drops out due to the translational symmetry of the problem along the shock surface). Let the mode δq_1 be the acoustic mode moving upstream while the other three modes travel downstream. These four modes coincide with the eigenfunctions in the case of homogeneous flow and thus become separated, but couple in the inhomogeneous case, so that the eigenfunctions of the inhomogeneous flow consist of linear combinations of modes δq_i.

The wave δq_1 incident on the shock front generates an outgoing fast acoustic mode δq_2, a vortical mode δq_3 and an entropic mode δq_4. Then we can introduce the complex coefficients of the transformation of waves $T_{i1}^{f} = \delta q_i/\delta q_1$, $i = 2, 3, 4$, at the shock front, where the ratio is taken at the point just behind the shock front.

The modes δq_2, δq_3, δq_4, passing through the layer of inhomogeneity, generate the mode δq_1, which reaches the shock front. We can introduce additionally the coefficients of transformation in the postshock flow $T_{1i}^{ps} = \delta q_1/\delta q_i$, $i = 2, 3, 4$, where, again, the ratio must be calculated at the point just behind the shock front.

Oscillations within the waveguide arise if the condition

$$T_{12}^{ps}T_{21}^{f} + T_{13}^{ps}T_{31}^{f} + T_{14}^{ps}T_{41}^{f} = 1 \tag{1}$$

is fulfilled. The relationship (1) is in fact a dispersion equation allowing us to determine the frequency of oscillations ω_*. (We search for perturbations in the form $\exp(-i\omega t)$ in the steady state case or in the form $\exp(-i\omega \log t)$ in time-dependent self-similar flow.) If ω_* has a positive imaginary part, oscillations, and hence the shock front, are unstable.

We classify all shock instabilities according to the behaviour of the coefficients of the transformation.

(1) The first class are instabilities generated by the shock front itself. In this case the coefficients T^f go to infinity at some complex frequency ω^f_∞ with $\mathrm{Im}(\omega^f_\infty) > 0$. This corresponds to an anomalously high response of the front to the incident perturbations, or, in other words, the shock front emits unstable waves without excitation from outside.

If the postshock flow is homogeneous (or we can negect the secondary reflections in the postshock flow) the coefficients T^{ps} vanish so that ω^f_∞ is a root of equation (1). If we allow for the postshock inhomogeneity, the root of equation (1) ω_* shifts away from ω^f_∞ but still lies in its vicinity and has a positive imaginary part. As a rule, the root ω_* lies in this case in the area of over-reflection, that is, the coefficients $|T^f|$ exceed unity and are large compared to $|T^{ps}|$. We discuss some examples of this kind of instability in the next section.

(2) In the second class, alternatively, the coefficients $|T^{ps}|$ are infinite at some complex frequency ω^{ps}_∞ with positive image part, so that the source of instability is now located downstream. This case is particularly found in the Sedov–Taylor hollow blast waves expanding in a power-law radially stratified stellar atmosphere with a density decreasing with radius. In this case postshock material is concentrated into a thin shell inside of which is a vacuum. Goodman (1990) and Ryu and Vishniac (1991) showed that growing oscillations of the blast wave are forced by the convective instability of the flow developing at the inner edge of the shell.

(3) A true resonant instability arises if the roots $\omega^{f,ps}_\infty$ of both triple coefficients T^f and T^{ps} do not have positive image components, but the root ω_* does. In the first two classes one of the two boundaries in the flow is not a mandatory ingredient of instability (instability exists though T^f or T^{ps} vanishes). The interaction between both boundaries becomes crucial in the third class.

Except for the possibility of classifying shock instabilities and thus of revealing their physical mechanisms, the introduction of the coefficients T^f and T^{ps} provides another significant advantage.

We note that the coefficients T^{ps} carry information about the global structure of the postshock flow and hence the problem of finding them is equivalent to the procedure of solving the Sturm–Liouville problem. On the contrary, the coefficients T^f reflect the local process of transformation of waves on the shock front and can be found from algebraic equations derived from the perturbed shock boundary conditions. Analysing the frequency characteristics of the functions T^f we are able to determine the conditions of spontaneous emission of the shock front. If the root ω^f_∞ has a positive imaginary part, or at least there exists an over-reflection on the shock front at some $\mathrm{Im}\,(\omega) > 0$, we may expect the existence of an instability of class (1). Thus we obtain a powerful method of predicting this kind of instability without solving the Sturm–Liouville problem. This approach becomes especially advantageous when the correct statement of the rear boundary conditions or the numerical integration of the Sturm–Liouville problem present difficulties.

2 THE MODELS OF SHOCK WAVES

We selected three models of shock waves in inhomogeneous media for demonstration of an application of the approach developed here.

2.1 Shock Wave in an Exponential Atmosphere

A simple model of an accelerating shock wave is a one-dimensional model of a plane-parallel shock in an atmosphere with exponentially decreasing density. This model is used, e.g., for the analysis of qualitative characteristics of the early stages of a supernova explosion, or the break-through of a layer of interstellar gas by a shock wave in the galactic fountain.

The dynamics of a shock wave is self-similar. The similarity solution was developed by Raizer (1964). Gurevich and Rumyantsev (1969) predicted instability of a shock wave against long-wavelength perturbations. Chevalier (1990) found increments of instability for arbitrary wavelengths k along the shock surface. The dispersion curve of Chevalier is shown in Figure 1 by the dashed line. Instability occurs if $kL < 13$, where L is the scale height of the atmosphere.

The dependence of $\mathrm{Im}\,(\omega^f_\infty)$ is plotted by the solid line in Figure 1. Though one finds a certain quantitative difference between the two curves, their qualitative behaviour is similar. Analysis shows that the curve of Chevalier lies in the area of over-reflection, which means that the instability of a shock wave in an exponential atmosphere is caused by the local instability of the shock front, whereas the resonant effects are insignificant.

2.2 Galactic Shock Wave in a Gravitational Well

A reasonable approximation of a galactic shock wave is a model of steady–state plane–parallel flow with a shock jump in a localized inhomogeneity of gravitational potential $\Psi(x)$. Let the gravitational potential have a well shape with depth $\Psi_0(\Psi_0 < 0)$ and width L. The gas enters the well from $x = -\infty$ with a supersonic velocity $u_{0\infty} > 0$. The shock front is settled at some point X_s inside the well. The postshock flow is everywhere subsonic. The structure of the flow is described by the Bernoulli integral, the law of mass flux conservation and the condition of isentropy of the flow (Kovalenko and Levy, 1992).

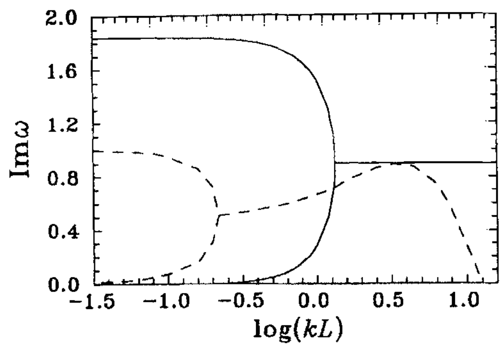

Figure 1 Imaginary parts of the dimensionless frequency spectra of perturbations of a shock wave in an exponential medium (Chevalier, 1990, dashed line) and the frequency of spontaneous emission of the shock front (solid line). The adiabatic index is $\gamma = 4/3$.

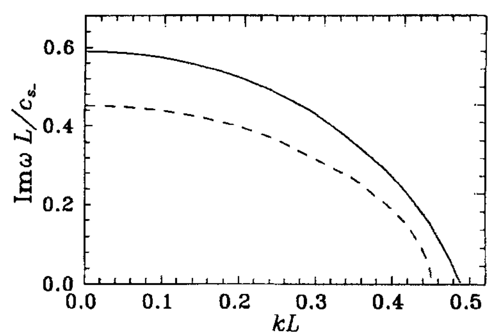

Figure 2 Imaginary dimensionless frequency spectra of perturbations of a steady-state shock wave in a gravitational potential $\Psi(x) = \Psi_0 \cos^2(x/2\pi L)$. Here $\Psi_0/c_{s-}^2 = -2$, $X_s = L/\pi$, $2\pi\Psi'(X_s)/c_{s-}^2 = 0.9$, $M_- = 2.85$, $\gamma = 5/3$. The results of exact calculations of Kovalenko and Lukin (1997) are plotted by the dashed line; the solid line represents the frequency of spontaneous emission.

In the case $k = 0$ the root of the equation $|T^f| = \infty$ can be expressed analytically:

$$\omega_\infty^f = \frac{i}{c_{s+}}\Psi'(X_s)\frac{(\gamma-1)(2\gamma M_-^2 - \gamma + 1)[M_+ + 1/(\gamma-1)]}{M_+(2\gamma M_-^2 - \gamma + 1) + 2M_-^2 + \gamma - 1}. \tag{2}$$

Here the subscripts "−" and "+" refer to the states of the gas just before and just behind the shock front, respectively, the prime stands for the derivative with respect to x, γ is the ratio of specific heats, and M is the Mach number.

One can see from equation (2) that the shock front emits unstable waves if $\Psi' > 0$, that is, on the rear side of the well relative to the flow. The flow is stable on the front side, where $\Psi' < 0$. Figure 2 compares the dependences on kL of the root ω_* (it takes purely imaginary values), obtained by solving the Sturm–Liouville problem (Kovalenko and Lukin, 1997), with the frequency of spontaneous emission ω_∞^f (it is purely imaginary as well). The coefficients of the transformations

Table 1. Absolute values of the coefficients of transformation for the model of a galactic shock wave

kL	ω_*	$\|T^f_{21}\|$	$\|T^f_{31}\|$	$\|T^f_{41}\|$	$\|T^{ps}_{12}\|$	$\|T^{ps}_{13}\|$	$\|T^{ps}_{14}\|$
0.0	0.452 i	0.72	0.00	3.76	0.30	0.00	0.21
0.2	0.398 i	1.94	5.31	4.96	0.72	0.26	0.20
0.4	0.188 i	16.2	22.1	13.1	0.97	0.66	0.16

calculated for three different values of ω_* are presented in Table 1. We see that the root ω_* again lies within the range of over-reflection.

2.3 Spherical Accretion with a Shock on to a Point Gravitational Object

Another modification of the problem of stability of a shock wave in a gravitational well, now in spherical geometry, is the problem of the stability of accretion of matter on to a point Newtonian mass.

According to the classical Bondi solution (1952) an accretion from rest at infinity occurs either in a subcritical regime, in which the inflow is subsonic everywhere, or in a critical regime with the passage through a sonic point to supersonic flow. In the latter case a steady–state spherical shock front may appear at the point $R_{\rm sh}$ inside the sonic sphere; the corresponding postshock flow is then subsonic up to the origin.

Referring to the results of the previous subsection one can suggest that the shock front is stable, since it stays at the front (relative to the inflow) side of the well. However the effects of non-planar curvature begin playing their role now. For the radial oscillations $l = 0$ we find the root

$$\omega^f_\infty = \frac{i}{c_{s+}}\left[\frac{2v_{r-}v_{r+}}{R_{\rm sh}} - \Psi'(R_{\rm sh})\right]\frac{(\gamma-1)(2\gamma M_-^2-\gamma+1)[|M_+|+1/(\gamma-1)]}{|M_+|(2\gamma M_-^2-\gamma+1)+2M_-^2+\gamma-1}. \quad (3)$$

Here $v_{r-,+}$ is the unperturbed radial velocity and $\Psi(r) = -Gm/r$ is the potential of the point mass m.

It can be readily shown that the factor $2v_{r-}v_{r+}/R_s - \Psi'(R_{\rm sh})$ is always positive, therefore the shock front must emit unstable waves in the range $0 < l < l_{\rm cr}$, where l is the orbital wavenumber. Figure 3 portrays the dependence of ω^f_∞ on l.

The stability of the shock-free critical Bondi solution was shown by Garlick (1979). The stability of the solution with shock was not studied. We suppose that it was not done because the flow has a singularity at the origin and "there are no natural inner boundary conditions" as was emphasized by many authors beginning with Bondi.

Meanwhile we can state the correct Sturm–Liouville problem if we allow for the supercritical inflow for which the postshock flow may pass through the sonic point. Multiple passage through sonic points is allowed in non-adiabatic inflows as well (Chang and Ostriker, 1985). Then the eigenvalue problem reduces to a finite range

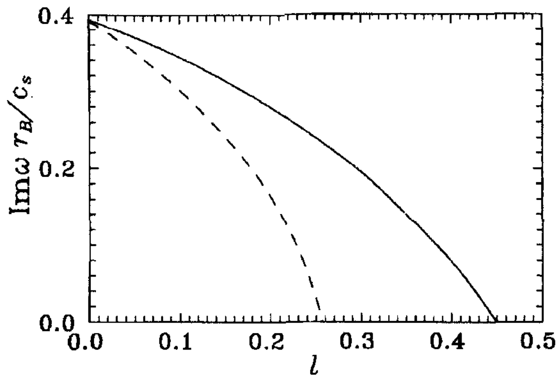

Figure 3 Dimensionless increments of instability of a shock wave for a model of adiabatic supercritical accretion. The exact eigenvalue (Eremin, 1997) is shown by the dashed line; the frequency of spontaneous emission is shown by the solid line. Here $M_- = -2$, $\gamma = 4/3$; $r_B = (\gamma-1)Gm/c_{s\infty}^2$ is Bondi's radius.

of radii between $R_{\rm sh}$ and the sonic radius R_s. The calculations of Eremin (1997) confirm the existence of an instability (dashed line in Figure 3) in the supercritical regime. Since the instability is driven by the shock front itself, the role of the inner boundary conditions must be insignificant so we predict instability in case of critical inflow as well.

3 CONCLUSION

The analysis of the stability of shock waves in a homogeneous medium (D'jakov, 1954; Kontorovich, 1957; Erpenbeck, 1962, etc) can be carried out analytically since it is reduced to an algebraic procedure of the analysis of reflection (transformation) coefficients at the shock front. In the present paper we generalize this approach to the inhomogeneous case. This allows us, first, to formulate the eigenvalue problem in terms of the resonant approach which gives an insight into the physics of the instability and in particular allows us to identify the role of the shock front as a source or as a damper of instability. Second, it provides a convenient method of search and prediction of possible instabilities caused by the local instability of the shock front, by avoiding solving the differential Sturm–Liouville problem. We hope that the three examples presented in the paper demonstrate the real advantages of this method.

References

Bondi, H. (1952) *Mon. Not. Roy. Astron. Soc.* **112**, 195.
Chang, K. M. and Ostriker, J. P. (1985) *Astrophys. J.* **288**, 428.
Chevalier, R. A. (1990) *Astrophys. J.* **359**, 463.
D'jakov, S. P. (1954) *Zhurn. Experim. Teor. Fiz.* **27**, 283.

Eremin, M. A. (1998) *Astron. Astrophys. Trans.*, in press.
Erpenbeck, J. J. (1962) *Phys. Fluids* **5**, 1181.
Garlick, A. R. (1979) *Astron. Astrophys.* **73**, 171.
Goodman, J. (1990) *Astrophys. J.* **358**, 214.
Gurevich, L. E. and Rumyantsev, A. A. (1969) *Astron. Zh.* **46**, 1158.
Kontorovich, V. M. (1957) *Zhurn. Experim. Teor. Fiz.* **33**, 1525.
Kovalenko, I. G. (1998) *Astron. Astrophys. Trans.*, in press.
Kovalenko, I. G. and Levy, V. V. (1992) *Astron. Astrophys.* **264**, 406.
Kovalenko, I. G. and Lukin, D. V. (1998) *Astron. Astrophys.*, in press.
Lio, D. and Chevalier, R. A. (1994) *Astrophys. J.* **435**, 815.
Raizer, Yu. P. (1964) *Zh. Prikl. Mat. Tekh. Fiz.*, No. 4, 49.
Ryu, D. and Vishniac, E. T. (1987) *Astrophys. J.* **313**, 820.
Ryu, D. and Vishniac, E. T. (1991) *Astrophys. J.* **368**, 411.