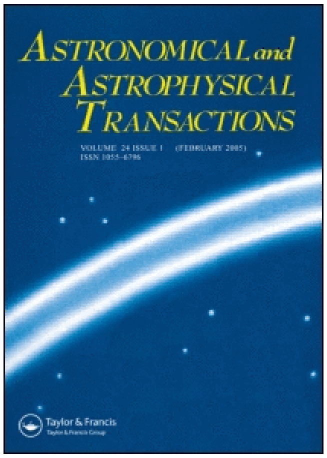

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ON THE DETERMINATION OF STELLAR CHARACTERISTICS IN DIMENSIONLESS PARAMETERS

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Main stellar parameters are analysed. All the parameters are normalized to the corresponding present-day solar values. The measure of excess (A) of gravitational energy GMm/R, preventing gas particles of the stellar atmosphere from irreversible escape, over thermal gas energy $\frac{3}{2}kT_e$ is introduced. It is shown that for all the stars of any given type, the ratio $Mm/(RT_eA)$ is constant and equal to 3.1×10^{13} kg^2 m^{-1} K^{-1}. The dimensionless quantity $B^0 = \frac{M/M_\odot}{(R/R_\odot)\cdot(T_e/T_\odot)}$ defining the main stellar parameters, is derived. The conditions for correlation between luminosity $L/L_\odot$ and B^0 are found: $\log(L/L_\odot) > 0$, $\log B^0 < 0$; $\log(L/L_\odot) < 0$, $\log B^0 > 0$. These conditions are supported by theoretical and observational works on the determination of the parameters M, R, T_e, L for 30 types of stars. Based on the parameters of the stars studied, plots of $\log(L/L_\odot) - \log(T_e/T_\odot)$ and $\log(L/L_\odot) - \log B^0$ are constructed. These diagrams allow us to estimate for a given star its $T_e/T_\odot$, B^0, $M/M_\odot$ and many other parameters from $\log(L/L_\odot)$.

KEY WORDS Stars, stellar parameters, stellar evolution

In stars (in particular, stellar atmospheres), every particle is constantly subjected to action of the gravity force and ejection forces (thermal gas pressure, radiation, degeneration forces, turbulence, stellar rotation, etc.). In stellar atmospheres, under the action of the combined ejection forces, individual particles may acquire an escape radial velocity of the order of the second cosmic velocity V_∞. This velocity is determined by the mean density of the star $\langle \rho_l \rangle$ per unit length of its radius:

$$V_\infty = \sqrt{2GM/R} = \sqrt{2G\langle \rho_l \rangle}. \tag{1}$$

For the Sun $\langle \rho_{l\odot} \cong 2.86 \times 10^{21} \rangle$ kg m^{-1}, $V_{\infty\odot} \cong 617.7$ km s^{-1}. Here G is the gravitational constant, M is the stellar mass and R is the stellar radius. Particles of the stellar atmosphere, which have such an initial kinetic energy, commit work

$$\frac{mV_\infty^2}{2} = \frac{GMm}{R}, \tag{2}$$

where m is the particle mass. For a solar-atmosphere proton, this work is 3.2×10^{-16} J.

As noted above, one of the forces that eject particles from stellar atmospheres is thermal gas pressure. The kinetic energy of translational thermal motion of particles of a stellar atmosphere is determined by the effective temperature T_e (Allen, 1973; De Jager, 1980; Bisnovatyj-Kogan, 1989; Kaplan, 1977; Chandrasekhar, 1939):

$$\frac{3}{2}kT_e = \frac{mV_T^2}{2}. \tag{3}$$

Here V_T is the root-mean-square velocity of particles of the stellar atmosphere for T_e, and k is the Boltzmann constant. Below it will be shown that the thermal gas pressure per unit particle of the stellar atmosphere is by a factor of many smaller than the gravity force.

Let the gravity force acting upon particles of the stellar atmosphere be opposed simultaneously by several ejecting forces. Also let the gravitational energy per particle exceed the kinetic energy of translational thermal motion by a factor A. From (2) and (3), it follows that for particles of stellar atmospheres the measure of excess of gravitational energy over thermal energy is determined by the ratio of the mean linear density of stellar matter to the effective temperature:

$$A = \frac{2GMm}{3kRT_e} = \frac{2G\langle\rho_l\rangle m}{3kT_e}. \tag{4}$$

For the Sun, $T_{e\odot} = 5784$ K, $A_\odot = 2663.5$.

From (4), it follows that for a star of any type the combination

$$\frac{Mm}{RT_eA} = \frac{3}{2}kG \tag{5}$$

is a constant equal to 3.1×10^{-13} kg^2 m^{-1} K^{-1}.

In what follows we normalize all the main parameters to corresponding present-day parameters of the Sun: $M/M_\odot$, $R/R_\odot$, $T_e/T_\odot$, $L/L_\odot$.

The luminosity L and the combination M/RT_e (henceforth denoted with B^0) become

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \left(\frac{T_e}{T_\odot}\right)^4, \tag{6}$$

$$B^0 = \frac{M/M_\odot}{(R/R_\odot)\cdot(T_e/T_\odot)} = \frac{\langle\rho_l\rangle}{\langle\rho_{l\odot}\rangle}\cdot\frac{T_\odot}{T_e}. \tag{7}$$

Thus, to describe the stellar characteristics we can use two dimensionless parameters: $L/L_\odot$ and B^0.

From (7), it follows that B^0 is determined by the ratio of the mean linear density of the stellar matter (M/R) to the effective temperature in solar units. With (1), (3) and (4), we may present the combination B^0 as

$$B^0 = \frac{A}{A_\odot} = \left(\frac{V_\infty}{V_{\infty\odot}}\cdot\frac{V_{T\odot}}{V_T}\right)^2. \tag{8}$$

Table 1. **Parameters of some types of stars**

Type	$\frac{M}{M_\odot}$	$\frac{R}{R_\odot}$	$\frac{T_e}{T_\odot}$	$logB^0$	$log\frac{L}{L_\odot}$	A	$\frac{M/M_\odot}{R/R_\odot}$
MS O4	72.0	15.78	7.943	–0.242	6.0	1526.5	4.36
O5	39.81	17.78	6.355	–0.45	5.7	945.7	2.24
O7	30.0	8.758	6.745	–0.294	5.2	1353.3	3.42
O9	24.0	8.404	6.138	–0.332	5.0	1238.8	2.86
B0	17.0	7.32	5.496	–0.375	4.7	1390.6	2.32
B2	9.95	5.45	3.963	–0.353	3.86	1182.8	1.76
B3	6.80	4.20	4.197	–0.197	3.74	1691.6	1.62
B4	5.84	3.90	2.904	–0.288	3.03	1374.6	1.49
Al	2.2	1.8	1.6215	–0.123	1.35	1514	1.222
A5	2.09	1.736	1.585	–0.124	1.3	2000	1.204
F2	1.02	1.23	1.207	–0.1814	0.114	1750	0.83
G2	1	1	1	0	0	2663.5	1
G5	0.933	0.933	0.982	0.009	–0.1	2712	1
K5	0.89	0.87	0.773	0.122	–0.569	3524	1.02
M2	0.389	0.501	0.602	0.114	–1.5	3458	0.775
WR	20.0	5.95	24.0	–0.51	5.7	825.8	3.36
"	16.0	2.366	17.3	–0.407	5.7	1041.6	6.76
"	12.0	1.058	17.3	–0.184	5.0	1744.9	11.34
"	10.0	1.677	13.74	–0.35	5.0	1190.8	5.97
SG B5	25.12	31.62	2.82	–0.55	4.8	751.2	0.8
A5	12.59	50.12	1.41	–0.75	4.0	474.2	0.25
G5	12.59	125.89	0.794	–0.90	3.8	343.7	0.10
K5	15.85	398.11	0.562	–0.15	4.2	189.1	0.04
β Cep	25.80	13.95	4.246	–0.360	4.80	1164.2	1.85
"	21.94	11.6	4.055	–0.331	4.56	1244.1	1.89
"	18.15	9.21	3.872	–0.293	4.28	1356	1.97
"	15.01	7.32	3.698	–0.296	4.0	1478.5	2.05
δ Cep	9.59	140.7	0.883	–1.113	4.06	205.13	0.07
"	7.15	70.3	0.956	–0.97	3.60	285.0	0.102
"	6.21	41.9	1.08	–0.863	3.30	365.0	0.148
WVir	10.0	158.5	0.53	–0.924	3.30	317.0	0.053
"	6.3	79.43	0.594	–0.876	2.90	354.3	0.08
"	3.98	25.11	0.791	–0.70	2.40	532.8	0.158
PNN	0.55	0.17	15.56	–0.682	3.233	554.1	3.24
"	0.54	0.19	12.02	–0.623	2.869	634.0	2.84
"	0.53	0.18	8.81	–0.476	2.30	889.8	2.94
Giants	5.0	25.12	0.658	–0.518	2.07	807.2	0.2
"	3.98	15.85	0.778	–0.491	1.96	860.5	0.25
"	3.16	10.0	0.865	–0.437	1.716	975.0	0.316
"	2.50	6.31	0.968	–0.388	1.54	1092.2	0.396
RR Lyr	0.6	5.3	1.094	–0.998	1.60	277.0	0.113
"	0.56	5.0	1.10	–0.992	1.567	271.7	0.112
"	0.55	4.08	1.16	–0.913	1.47	311.7	0.136
HB	1.0	4.38	1.37	–0.78	1.87	442.2	0.228
"	0.8	2.588	1.73	–0.75	1.75	479.5	0.310
"	0.6	1.50	2.18	–0.73	1.70	490.2	0.40
δ Sct	2.0	3.3	1.33	–0.341	1.53	1214.8	0.60
"	1.7	1.9	1.37	–0.184	1.10	1726.3	0.89
"	1.6	2.6	1.24	–0.300	1.20	1718.3	0.80
SD	1.26	1.2	1.14	–0.036	0.386	2453	1.05
"	0.63	0.724	0.74	0.07	–0.804	3133	0.87

Table 1. Continued

Type	$\frac{M}{M_\odot}$	$\frac{R}{R_\odot}$	$\frac{T_e}{T_\odot}$	$\log B^0$	$\log\frac{L}{L_\odot}$	A	$\frac{M/M_\odot}{R/R_\odot}$
"	0.15	0.20	0.525	0.165	−2.524	3889	0.79
WD	1.0	0.014	0.42	1.470	−2.166	78588	71.4
"	0.7	0.012	1.80	1.516	−2.84	87380	58.3
"	0.4	0.009	4.54	1.59	−3.83	103895	44.0

Note. MS, main sequence; WR, Wolf–Rayet; SG, supergiants; PNN, planetary nebula nuclei; HB, horizontal branch; SD, subdwarfs; WD, white dwarfs.

The quantity B^0 is a function of three independent variables: $M/M_\odot$, $R/R_\odot$, $T_e/T_\odot$. The luminosity $L/L_\odot$ is a function of two independent variables. Therefore, B^0 may be informative for several stellar characteristics. Relationships (7) and (8) confirm this. Below we will present information about the correlation of central temperatures $T_{\rm cen}$, effective temperatures T_e and temperatures of stellar coronae $T_{\rm cor}$ (Kaplan, 1977; Mihalas, 1978).

From (6) and (7), we find that for stars of various types the relationship between $L/L_\odot$ and B^0 is

$$\frac{M}{M_\odot} = \frac{B^0\sqrt{L/L_\odot}}{T_e/T_\odot} \tag{9}$$

and

$$\log\frac{L}{L_\odot} > 0, \quad \log B^0 < 0; \quad \log\frac{L}{L_\odot} < 0, \quad \log B^0 > 0. \tag{10}$$

We tested conditions (10) using the data from more than a hundred observational and theoretical works published before 1996 and devoted to the determination of M, R, T_e and L. Some of them are listed in the references (Bernabeu, 1992; Boyarchuk and Lyubimkov, 1982; Cherepashchuk, 1989; Cox, 1980; Glushneva, 1983, 1985, 1987, 1989a,b, 1990; Haniff *et al.*, 1995; Howarth and Prinja, 1989; Imshennik, 1975; Kudritzki, 1988; Lyubimkov and Savanov, 1983; Merezhin, 1994; Pel, 1978; Pottasch, 1984; Rachkovskaya, 1988; Surdin and Lamzin, 1992; Svechnikov, 1986; Tutukov, 1986).

Examples of the parameters (mostly averaged) of the previously studied stars of various types are given Table 1. From these data, we see that for a given stellar type, the intervals of values of $M/M_\odot$, $R/R_\odot$, $T_e/T_\odot$, B^0, $L/L_\odot$ are strictly limited, and (what is important) are numerically simple. The main point is that we find no stars that do not satisfy conditions (10).

However, among about one thousand stars studied previously, we found 15 stars (Table 2) whose parameters do not fit conditions (10). These are Trumpler's stars (Chandrasekhar, 1939), some nuclei of planetary nebulae (Sharova, 1992) and ε Aur (Martynov, 1981). However, a careful examination in each case showed that their parameters were (for different reasons) inaccurate. In some cases, there were simply typographic misprints. Hence it follows that observational and theoretical data on the parameters $M/M_\odot$, $R/R_\odot$, $T_e/T_\odot$, $L/L_\odot$ of the stars studied, which satisfy

Table 2. Stars with uncertain parameters

$\frac{M}{M_\odot}$	$\frac{R}{R_\odot}$	$\frac{T_e}{T_\odot}$	$\log B^0$	$\log \frac{L}{L_\odot}$	$\frac{M/M_\odot}{R/R_\odot}$
		Trumpler's stars			
57.54	4.57	11.03	0.057	5.49	12.6
97.72	7.24	5.53	0.387	4.69	13.5
151.36	6.61	8.37	0.438	5.33	22.9
295.12	19.05	6.20	0.398	5.73	15.5
223.87	15.14	5.53	0.426	5.33	14.8
398.11	16.60	4.39	0.737	5.01	24.0
77.62	9.12	5.53	0.187	4.89	8.5
		Nuclei of planetary nebulae			
0.52	0.03	16.94	0.0	1.87	17.3
0.55	0.005	93.36	0.07	3.28	110
0.53	0.014	57.40	0.36	2.25	71.4
0.53	0.007	45.29	0.22	2.32	75.7
0.52	0.03	15.56	0.05	1.72	17.3
0.52	0.02	19.90	0.12	1.50	26
0.52	0.04	12.10	0.03	1.54	13
0.52	0.01	17.63	0.47	1.0	52
		ε Aur			
23	1.4	2.59	0.80	1.95	16.43

Note. Data on Trumpler's stars are taken from Chandrasekhar (1939), on nuclei of PN from Sharova (1992), on ε Aur from Martynov (1981).

condition (10) within the acceptable errors, have been determined correctly and deserve confidence.

Some of the table data are plotted in the diagrams $\log(L/L_\odot) - \log(T_e/T_\odot)$ (Figure 1) and $\log(L/L_\odot) - \log B^0$ (Figure 2). On these plots, the Sun is at the origin of the coordinates. From the plots, we see that clustering of stellar types and evolutionary tracks are conserved as in the Hertzsprung–Russel diagram. There is invariance with respect to transformation of coordinates. Thus, the diagrams do not contradict the accepted notions of astrophysics. The diagrams have a fiducial significance also because the tabular data plotted have withstood test (10).

Finding on these diagrams the places of the stars studied on the basis of their values of $\log(L/L_\odot)$, $\log(T_e/T_\odot)$ and $\log B^0$, we see, first, that all the stars on the $\log(L/L_\odot) - \log B^0$ diagram are divided into two groups. In the first quadrant, there are stars with $\log(L/L_\odot) > 0$, $\log B^0 < 0$; in the third those with $\log(L/L_\odot) < 0$, $\log B^0 > 0$. Second, we see that each type of star occupies its own, strictly limited domain, a narrow strip or, sometimes, just a line (main-sequence stars, cepheids, W Vir-type stars, horizontal-branch stars, subdwarfs, white dwarfs, etc.). The scatter of points is due not only to the actual stellar parameters, but also to instrumental effects, dates and methods of observations, etc.

By now, a large amount of observational data on M, R, T_e and L for various types of stars has been accumulated. These data on stellar parameters and the corresponding diagrams can be represented in the form of a computer database.

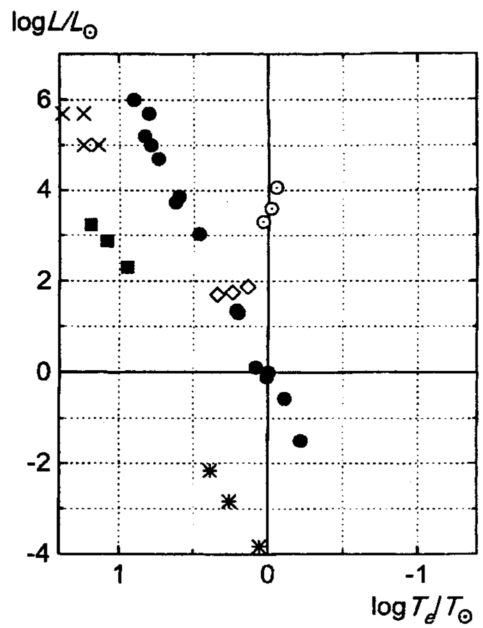

Figure 1 The diagram $\log(L/L_\odot) - \log T_e/T_\odot$. Filled circles – main-sequence stars, crosses – Wolf–Rayet stars, open circles – cepheids, filled squares – nuclei of planetary nebulae, diamonds – horizontal-branch stars, asterisks – white dwarfs.

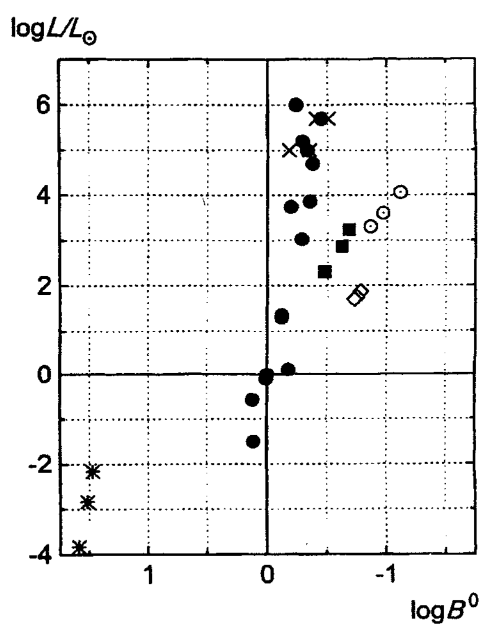

Figure 2 The diagram $\log(L/L_\odot) - \log B^0$. Notation is the same as in Figure 1.

When studying (or verifying) the parameters of a star of a given type, after obtaining $\log(L/L_\odot)$ for it, we find from the diagrams its $\log(T_e/T_\odot)$ and $\log B^0$. Then from (11) we find $M/M_\odot$. After that (Kaplan, 1977; Mihalas, 1978), we estimate the central temperature $T_{\rm cen}$ and corona temperature $T_{\rm cor}$:

$$\frac{T_{\rm cen}}{T_{\rm cen\odot}} \cong \frac{M/M_\odot}{R/R_\odot}, \tag{11}$$

$$T_{\rm cor} \cong \frac{GMm_p}{48R} \tag{12}$$

and other parameters. Thus, the diagrams presented allow us to make spectrophotometric standardization of many types of stars, to verify the determined parameters M, R, T_e, L of stars, to reveal errors in stellar parameters, and to numerically estimate various stellar characteristics.

We summarize the results of the work as follows.

(1) In this work, all main stellar parameters are normalized to the present-day solar parameters, thus numerically simplifying the values of all main stellar characteristics.

(2) From fundamental notions of physics and astronomy, the function M/RT_e, defining many stellar parameters, is derived.

(3) We introduce a measure of excess A of the gravitational energy, confining particles of stellar atmospheres from irreversible escape, over the thermal energy of expulsion.

(4) It is found that for stars of any type combination $Mm/(RT_eA)$ is a constant, equal to 3.1×10^{-13} kg^2 m^{-1} K^{-1}.

(5) We introduce a parameter B^0 which quantitatively characterizes the possibility of preventing particles of stellar atmospheres from irreversible escape.

(6) The conditions to be satisfied by all stars are obtained: $\log(L/L_\odot) > 0$, $\log B^0 < 0$; $\log(L/L_\odot) < 0$, $\log B^0 > 0$. Results of more than a hundred theoretical and observational works, carried out before 1996 and determining the parameters M, R, T_e, L confirm that there are no stars that do not satisfy conditions (10).

(7) We present diagrams that allow us, based on the value of $\log(L/L_\odot)$ of the star studied, to find its $T_e/T_\odot$, B^0 and $M/M_\odot$. This result allows us to make spectrophotometric standardization of many types of stars, to verify the determined parameters M, R, T_e, L of stars, to reveal errors in stellar parameters, and to estimate temperatures of stellar coronae and interiors.

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