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POST-NEWTONIAN APPROXIMATION AND COLLAPSED STARS

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In this work we confirm the conclusions of papers Aquilano, Castagnino and Lara (1994); Aquilano, Castagnino and Neira (1994). We use of the post-Newtonian approximation to show that the oscillations of shells could be very important in the physics of astronomical objects with collapsed stars.

KEY WORDS Collapsed stars, relativistic effects, the shell oscillations

1 INTRODUCTION

In X-ray and gamma-ray bursters, the rapid variations of luminosity are generally interpreted as being due to their rotation, but these variations can alternatively be explained as radial oscillations of compact objects (Abramowicz, 1989). Energy fluctuations in stellar binary systems endowed with thermomechanical oscillations can be used to describe this astrophysical event.

X-ray bursters radiate an extraordinarily large amount of energy, mainly X-rays, reaching their maximum luminosity in milliseconds (Lewin and Clark, 1980; Shore, Livio and van den Heuvel, 1994). It is usually assumed that these explosions are related to neutron stars, white dwarfs or black holes, that belong to binary systems and receive matter from their normal companion, forming an accretion disk around them; the matter is later deposited over the star's surface (if it is a neutron star or white dwarf). This matter is mainly hydrogen whose temperature rises to a critical value, causing thermonuclear fusion which produces helium and, later on, carbon. When carbon is formed X-rays are emitted, heating the rest of the matter and producing an increase of the stellar luminosity.
We shall consider a spherical atmosphere, neglecting the possible cylindrical shape of the accretion disk (but this is permissible using the post-Newtonian method) and the influence of the normal companion which we consider to be very far away from the neutron star, and only the radiation component in the stress-tensor of the right-hand side of the Einstein equation, neglecting the matter component.

This work is a continuation of previous papers Aquilano, Castagnino and Lara (1994); Aquilano, Castagnino and Neira (1994), but with post-Newtonian approximation (Aquilano, Castagnino and Neira, 1994, Weinberg, 1972), and can be considered as a concluding remark to show that there are objects in the universe where our formalism may eventually yield the detection of a relativistic effect.

2 THE EQUATIONS OF MOTION

We can see that our post-Newtonian approximation method yields the equation of motion (see Aquilano, Castagnino and Neira, 1994),

\[
\frac{dv}{dt} = -\frac{\bar{m}}{R^2} - 2\bar{m}^2/R^3 + (3\bar{m}/R^2)v^2 + \left(\frac{d\tau}{dt}\right)^3(-\bar{m}/2R^2 - L/m_o) \tag{1}
\]

\[R(A - B) = m_0 \tag{2}\]

where \(m_0\) may be interpreted as the gravitational mass of the shell (see Aquilano, Castagnino and Neira, 1994), \(\bar{m}\) is the mass of the central body (the neutron star), \(R\) is the radius of the shell, \(L\) is the shell luminosity, and

\[A = (R^2 - 2\bar{m}/R + 1)^{1/2}, \quad B = (R^2 - 2m_0/R + 1)^{1/2} \tag{3}\]

where the primes represent the time derivatives.

We now introduce two relations, a law of shell emission (see Aquilano, Castagnino and Lara, 1994),

\[L = 4\pi\sigma R^2 T^4 \tag{4}\]

where \(\sigma\) is the Stefan–Bolzmann constant; and the other relation is

\[
\frac{d\tau}{dt} = \left[\frac{(1 - 2\bar{m}/R)^2}{(1 - 2\bar{m}/R + R^2)}\right]^{1/2} \tag{5}
\]

which, in our particular case we can expand up its order \(v^4\) (Aquilano, Castagnino and Neira, 1994):

\[
\frac{d\tau}{dt} = (1 - 2\bar{m}/R - R^2 - 2\bar{m}R^2/R)^{1/2}. \tag{6}
\]

The shell has a constant proper mass and emits as much radiation as that absorbed from the central body. In other words, the radiation emitted to the exterior by this system is not produced at the expense of the matter present in the shell, but originates in the central body.

The Newtonian approximation of this problem will be reached when we have low velocities, \(R' \ll 1\). Thus in this case, we have \(A = B \approx 1\) and \(m_0 \approx m\) (see Castagnino and Umerez, 1983), therefore the gravitational mass approaches the proper mass, which is consistent with the assumption that this approximation is a Newtonian one.
Figure 1 Phase diagram: velocity vs. radius for a neutron star of three solar masses and different initial velocities. We can see the oscillations around the stable singular point.

3 COMPARISON WITH ASTRONOMICAL DATA

Therefore, the shell equation of motion will be

\[
\frac{dv}{dt} = -\frac{m}{R^2} - 2\dot{m}^2/R^3 + (3m/R^2)R^2 + (1 - 1/R - R^2/R)^{3/2} \\
\times (-m/2R^2 - 4\pi\sigma R^4T^4/m)
\]  

(7)

where we use the following astrophysical data for a central body (neutron star): \( \dot{m} = 3M_\odot \) \((M_\odot = \text{solar mass})\) according to Oppenheimer and Volkoff (see Oppenheimer and Volkoff, 1939), \( R_0 = 0.0c \) to 0.3c \((c = \text{light velocity})\), and \( m = 0.002, 0.02 \) and \( 0.2M_\odot \), with a temperature of \( 10^7 \) K, and the adiabatic cooling is neglected because the model is very simple, and we study the first parts of the curves.

We obtain stable oscillations for several values of the parameters (see Figure 1) and the most interesting result is that the oscillations have near of millisecond periods (see Figure 2). In all cases, the oscillations are a very simple mathematical calculus (a "toy model").
Figure 2 Luminosity vs. time, showing the luminosity pulsations.

4 CONCLUSION

In past work Castagnino and Umérez, 1983; Hamity and Gleiser, 1978; Hamity and Spinosa, 1984, it was found that the general relativistic effects will be very difficult to detect, because the central mass density is smaller, but now in bursters with neutron stars we have shown that it could be possible to detect a post-Newtonian effect using our model.

In this paper we obtain oscillations near to half a millisecond of time, a very important conclusion for this astrophysical event (see Benvenuto, Harvath and Vucetich, 1991).

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References