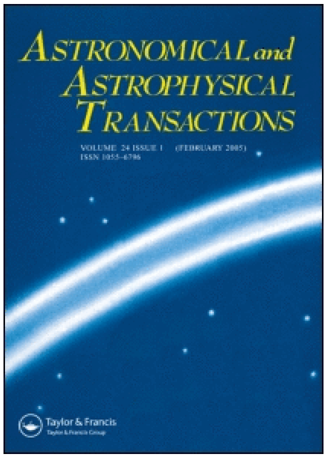

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STOCHASTIC EJECTION FROM THE SUREACE OF SMALL BODIES 1. MODELLING

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The development of a stochastic model of eruptive ejection is used in the framework of a spatially perturbed two-body problem for object 3200 (Phaethon). The computer experiment allowed us to estimate the stochastic measure from a large number of Keplerian orbital elements of fragments ejected from the Phaethon surface and from the range of *D*-criterion variations calculated for these orbits relative to the Phaethon orbit and the average orbit of a possible meteor stream (Geminids?). The ejection rates range within 200–300 m s^{-1}; 1–1.2 km s^{-1}. The ejection was simulated in the perihelion and aphelion of the Phaethon's orbit. The probability functions are obtained for the semimajor axis, eccentricities, and perihelion distances as well as for inclination angles and perihelion argument for a large number of Keplerian orbits of ejected fragments. The analysis of calculated probability functions of the *D*-criterion allowed us to conclude that new formations will be more compact relative to the average orbit of the Geminids and less compact relative to Phaethon's orbit. This formation will also have a rather layered structure in depth and a rather inhomogeneous density in the longitudinal direction. The simulation results obtained correlate well with observational data for the Geminids and in our opinion permit a fuller analysis of the given meteor stream.

KEY WORDS **Stochastic model, probability space, Phaethon, Geminids**

1 INTRODUCTION

One of the trends in solving the problem of the origin of small bodies in the Solar System is associated with Lagrange (Lagrange, 1815) who in the last century proposed a hypothesis of small-body formation by eruptive processes in planets. This idea was later developed by Proctor (Proctor, 1884) and Schulhof (Schulhof, 1891), and at the beginning of this century by Krommelin (Krommelin, 1910).

In the 1930s the hypothesis that interplanetary material is formed partly as a product of volcanic processes in the giant planets was set up again (Vsekhsvyatsky, 1933). When developing this hypothesis (Vsekhsvyatsky, 1953) it was proposed that the satellites of giant planets, on the surface of which high eruptive activity is possible, may also be sources of small bodies in the Solar System and the processes of substance ejection (gas, dust, cometary ice, meteor particles, etc.) should be compared with similar processes in our Galaxy and other galaxies (Vsekhsvyatsky, 1978). Space investigations during the last few decades by ground means and space vehicles have confirmed some aspects of this hypothesis (Keller *et al.*, 1986; Sagdeev *et al.*, 1986; Voyager 2, 1989). Many opponents consider the absence of a strictly proved mechanism providing for substance ejection from the surface of celestial bodies as a weakness of the whole theory of eruption.

However, in the last few decades the eruptive hypothesis has become the subject of study for many scientists. On the one hand, it is connected with eruptive activity in giant planets and comets discovered by space vehicles; on the other hand, with the rapid development of fundamental studies of processes proceeding in the depths and on the surfaces of celestial bodies. In this context, the field of solved problems focused on establishing the nature and the formation of cometary–asteroidal–meteoric matter in space within an eruptive concept has become wider. First, there are astrophysical problems directed at the investigation and study of various mechanisms of substance ejection from the surface of celestial bodies. Second are the problems of celestial mechanics, the solutions of which are associated with determining the space structure of a set of orbits of most probable substance ejection and their evolution.

Theoretical studies in combination with computer experiments and observational data are of particular concern in the analysis of celestial objects. The methods of stochastic formalism of substance eruption at any space point and the formation of new classes of small bodies have been developed using the concept of continuous formation of interplanetary small bodies. Theoretical aspects and the peculiarities of algorithms and methods were considered earlier (Kulikova and Mychev, 1989). This paper analyses the case of possible substance ejection from the surface of 3200 1983 TB object (Phaethon) within a spatially perturbed problem of the bodies.

2 PROBLEM STATEMENT

In conformity to the problem of small-body formation in space as a result of substance ejection from a comet or asteroid surface, the general model may be modified to the model of stochastic substance ejection from the surface of celestial bodies within the space-limited four-body problem considered by Kulikova *et al.* (1993).

The coordinate system $MXYZ$ (Figure 1) is connected with a body (comet, asteroid or another small body), from the surface of which the substance is ejected and which is called as the parent body (PB) in what follows. The MX axis is directed along a radius-vector connecting PB with the body S (the main body), relative to which PB moves in a Keplerian orbit. The positive direction of the MY axis coincides with the direction of movement of PB in the orbit. The MXY plane is the plane of the PB orbit. The MX axis is perpendicular to MXY and vectors **MX**, **MY** and **MZ** form a right triple.

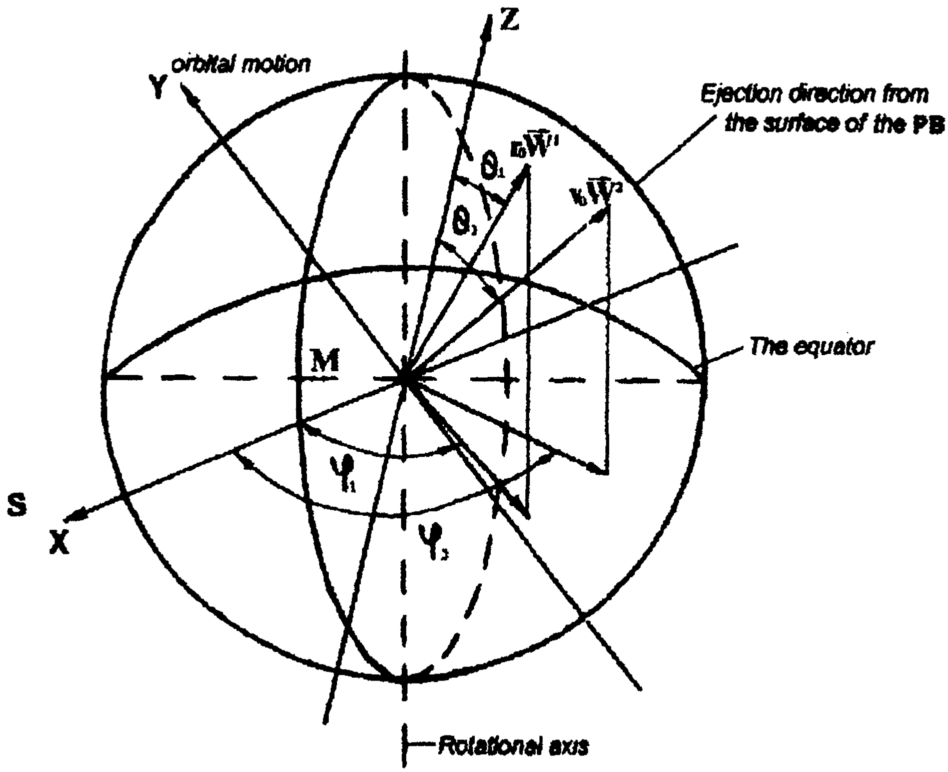

Figure 1 Coordinate systems: NN' is the node line; Ω is the longitude of asceding node of orbit M; ω is the argument of perihelion M; ν is the true anomaly M; r_ω is the perihelion distance of orbit M; i is the angle of inclination of orbit M to the plane X_1SY_1.

By virtue of the fact that the character of the forces under the action of which the fragments are ejected from the PB surface is uncertain, the radius-vector and the fragment velocity vector as well as the ejection velocity values are stochastic variables. We define a stochastic variable for our problem using conventional methods (Kulikova *et al.*, 1993). To define a stochastic (i.e. probabilistic) variable is to give:

(1) the range of all possible values for the radius-vector, (i.e. the set of points on the PB surface or of partit), for the velocity-vector, (i.e. the set of all possible ejection directions), and for fragment ejection velocity values (the range of variation of these values);

(2) a probability distribution in this range, for which the following normalization condition is satisfied:

$$\int P(x)\,dx = 1, \tag{1}$$

where $P(x)$ is the probability that a stochastic variable takes the values between x and $x + dx$.

Denote the components of the radius-vector of the ejected fragments by M_x, M_y and M_z, and the components of the velocity-vector in the coordinate system $MXYZ$ by V_x, V_y and V_z. Then these components may be represented in Cartesian coordinates with stochastic unit vectors W^k $(k = 1,2)$ as:

$$\begin{array}{lll} M_x = W_1^1 r_0, & M_y = W_2^1 r_0, & M_z = W_3^1 r_0 \\ V_x = W_1^2 v_0, & V_y = W_2^2 v_0, & V_z = W_3^2 v_0 \end{array} \tag{2}$$

where r_0 is the distance between an ejected fragment and the centre M at the moment of ejection, v_0 is the initial ejection velocity and

$$\left(W_1^k\right)^2 + \left(W_2^k\right)^2 + \left(W_3^k\right)^2 = 1 \quad (k = 1,2).$$

Between the vector coordinate components W^k $(k = 1,2)$ the following condition is to be satisfied which characterizes the nature of fragment ejection from the PB surface:

$$C_1 < \left(W_1^1 \cdot W_1^2 + W_2^1 \cdot W_2^2 + W_3^1 \cdot W_3^2\right) \le C_2 \tag{3}$$

where $C_1 = \cos\theta_n$, $C_2 = \cos\theta_v$, i.e. θ_n, θ_v are the angles determining the nature of fragment ejection. It is accepted that if $\theta_n = 30°$ and $\theta_v = 90°$, the ejections are "direct"; if $\theta_n = 0°$ and $\theta_v = 30°$ they are "oblique" (Kulikova *et al.*, 1993).

The initial values of fragment ejection velocities v_0 are taken according to the chosen law of probability distribution $v_0 \sim f(v_0)$. The Cartesian coordinates of the unit vectors W^k $(k = 1,2)$ are connected with the direction angles ϕ_j and θ'_j $(j = 1,2)$ by the following relations:

$$\begin{aligned} W_1^j &= \cos\varphi_j \cdot \sqrt{1-\cos^2\theta'_j}, \\ W_2^j &= \sin\varphi_j \cdot \sqrt{1-\cos^2\theta'_j}, \quad W_3^j = \cos\theta'_j. \end{aligned} \tag{4}$$

The values of the functions $sin\,\varphi_j$, $\cos\varphi_j$, $\cos\theta'_j$ specify a "random" direction of the vectors W^k $(k = 1,2)$ and to estimate them various modelling procedures for the probability distribution are used (Neumann, 1951), the choice of which depends both on the statement of the problem and on its computer implementation, and on the operating characteristics of the computer system and the quality of the random number generator.

Such a scheme permits the simple algorithmization of the initial modelled ejection process by a Monte-Carlo method and subsequent studies of this stochastic process in two versions:

(1) ejection is studied at certain points of the PB orbit, i.e. a static version;

(2) ejection is studied for a definite time interval of PB motion along the orbit, i.e. a dynamical version.

For the second version it is necessary to consider the numerical integration of the equation of PB motion within the perturbed-space problem of two bodies:

$$\frac{d^2\mathbf{r}}{dt^2} = -\mu\frac{\mathbf{r}}{r^3} + \dot{\mathbf{r}}', \tag{5}$$

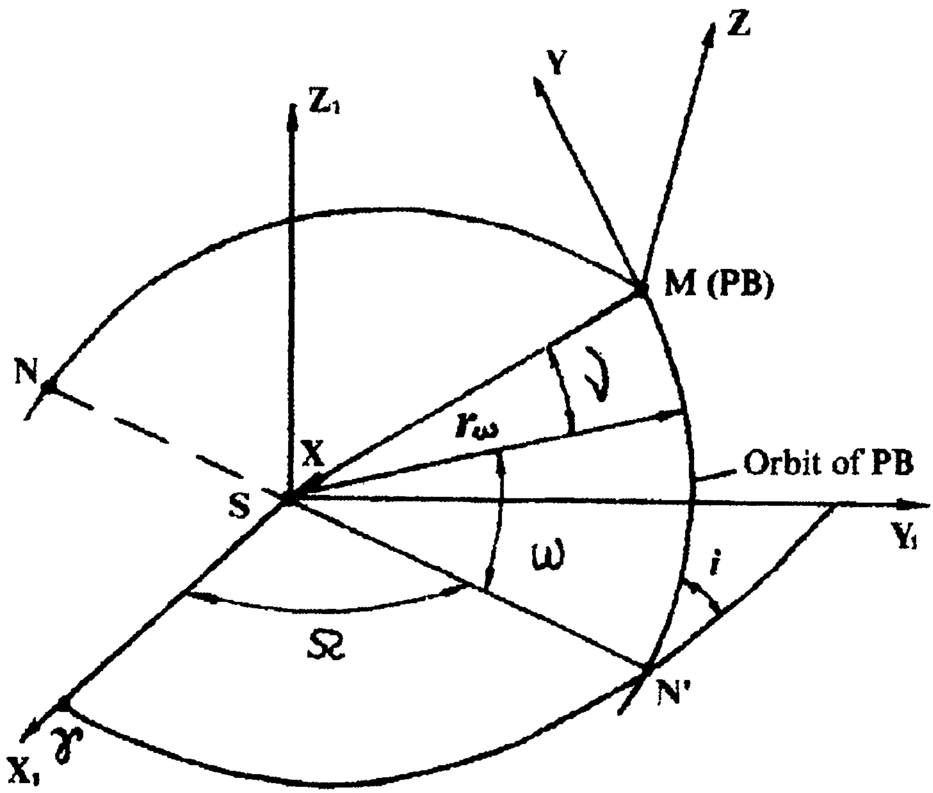

Figure 2 Ejection geometry: the direction angles of the radius-vector and velocity-vector of an ejected fragment.

where $\mu = k^2 \cdot (m_1 + m_2)$, m_1, m_2 are the masses of the main and the parent bodies (PT), k^2 is a Gaussian constant, and $\dot{\mathbf{r}}'$ is the total perturbed acceleration which may include also gravitational and non-gravitational disturbances, the account of which is specified on the basis of the statement of the problem.

For further numerical studies of the model both in the first and second versions, a heliocentric coordinate system $SX_1Y_1Z_1$ (Figure 2) is introduced which is related to the body S; the plane X_1SY_1 is the plane of the ecliptic; the SX_1 axis is directed to the vernal equinox point Υ.

The components of the radius-vector M_x, M_y, M_z and the velocity-vector V_x, V_y, V_z are represented with the help of random numbers in the coordinate system $MXYZ$. These components of the ejected fragments are then reestimated in the coordinate system $SX_1Y_1Z_1$, with allowance for the PB dynamics if the second version of the problem is considered. Conversion from the coordinate system $MXYZ$ to the system $SX_1Y_1Z_1$ is accomplished with the algorithm described by Kulikova *et al.* (1993). Next, the heliocentric Keplerian orbital elements of the ejected fragment are found from classical ratios (Kulikova *et al.*, 1993).

In computer modelling of the process of fragment ejection from the parent-body (PB) surface, we obtain a statistical representation of the set of orbits into which fragment ejection is most probable. Because of limited computer resources, it is difficult to obtain the statistics of fragment orbits sufficient to find a confidence interval for Keplerian orbits of the fragments. Therefore a probability space in which the probability measure is estimated is used to describe this set mathematically. The algorithm for the construction and estimation of the probability measure in the elemental space of the Kêplerian motion is considered explicitly in our monograph. The probability measure includes all available information about the structure of the set of orbits into which the fragments are ejected, and allows us to calculate estimates of the most important characteristics of Keplerian orbits: mathematical expectation and dispersion. To estimate these, the following equations are used

$$\hat{M}\{f_j^i\} = \sum_{i=1}^{n_j} \overline{f_j^i} \cdot P_j(A_j^i), \tag{6}$$

$$\hat{D}\{f_j^i\} = \sum_{i=1}^{n_j} [\overline{f_j^i} - \hat{M}\{f_j^i\}]^2 \cdot P_j(A_j^i), \tag{7}$$

where n_j is the number of intervals which divides the range of the jth element f_j^i – of the Keplerian orbit of an ejected fragment; A_j^i – is the ith interval of the range of variation of the jth element f_j^i; $P_j(A_j^i)$ is the estimate of the probability measure in A_j^i; and f_j^i – is the midpoint of the interval A_j^i.

In the case when it is possible to obtain an analytical form for the probability measure $P_j(A_j^i)$ or its approximation in the range Y_j of the jth element of the Keplerian heliocentric orbit of the ejected fragment, equations (6) and (7) taken the form (Kulikova *et al.*, 1993)

$$\hat{M}\{f_j\} = \int_{Y_j} f_j \cdot P_j(db), \tag{8}$$

$$\hat{D}\{f_j\} = \int_{Y_j} [\hat{M}\{f_j\} - f_j]^2 \cdot P_j(db), \tag{9}$$

where $P_j(db) = P_{f_j}(x)dx$ and $P_{f_j}(x)$ is the probability density of the jth element of the fragment orbit in Y_j.

As a result of fragment ejection from the parent-body (PB) surface, the fragment may form a stream or an association if the set of their orbits meets certain criteria. In meteor astronomy this criterion is orbit similarity. To establish a degree of similarity the so-called $D_{\rm cr}$-criterion is used in our model calculation and the equation for the $D_{\rm cr}$-criterion is

$$D_{\rm cr}^2 = (e_1 - e_2)^2 + (q_1 - q_2)^2 + [2\sin(I/2)]^2 + \{[(e_1 + e_2)/2] \cdot [2\sin(\Pi/2)]\}^2, \tag{10}$$

where

$$[2\sin(I/2)]^2 = \{2\sin[(i_1+i_2)/2]\}^2 + \sin\, i_1 \cdot \sin\, i_2 \cdot \{2\sin[(\Omega_1-\Omega_2)/2]\}^2,$$

$$\Pi = \omega_1 + \omega_2 + 2\arcsin\{\cos[(i_1+i_2)/2]\cdot\sin[(\Omega_1-\Omega_2)/2]\cdot\sec(I/2)\}.$$

In the range of variation of the D_{cr}-criterion value the probability measure is constructed in a similar way to that used for the set of orbital elements of ejected fragments. Then a measure of compactness of orbits forming a stream or an association is the mathematical expectation of D_{cr}-criterion values calculated from (6) and (8), and the dispersion calculated from (7) and (9) will be a measure of scattering of the D_{cr}-criterion value.

3 CALCULATION RESULTS

The numerical experiments were performed in the first version which simulate substance ejection from Phaethon's surface. The initial velocity values v_0 for isotropic fragment ejection ranged within $v_0 \in (200\text{–}300)\ \mathrm{m\,s^{-1}}$ and $v_0 \in (1\text{–}1.2)\ \mathrm{km\,s^{-1}}$. The first value corresponds to substance ejection velocities in sublimation (Sherbaum, 1968), and the second one to ejection from the PB surface as a result of its encounter with the interplanetary medium. The ejection was observed at the perihelion and at the aphelion of Phaethon's orbit for its sunlit side taking account of expected direct ejections. For the exponential distribution of v_0 with probability density function $f(v_0) = (1/b)\exp\{-(v_0/b)\}$, the scale parameter b was taken to be equal to 250 m s^{-1}. This distribution of v_0 was chosen for the following reasons: first, on the basis of physical implications of the hypothesis on the exponential character of substance ejection from the surface of celestial bodies (Vsekhsvyatsky, 1967); second, in order to study the ejection process for various laws of v_0 distribution and to establish the regularities, if any, in the structure of the set of orbits of ejected fragments under these conditions.

Results obtained in modelling the isotropic ejection of fragments from Phaethon's surface at the perihelion and at the aphelion of Phaethon's orbit are analysed in detail in our paper (Kulikova and Mychev, 1995). In short, with substance ejection at the perihelion of Phaethon's orbit at velocities $v_0 \in (200\text{–}300)$ m s^{-1}, the orbits of ejected fragments have the following parameters. The values of the semimajor axis of fragment orbits are most probably concentrated in the area of 0.006 a.u. in width and the perihelia of fragment orbits are rather densely concentrated in the range 0.1395–0.1405 a.u. The range of variation of orbital eccentricities is also extremely small. The variations in the values of the angular elements i, ω, Ω are of a complex oscillatory character. In this case the space of the set of ejected fragment orbits has a layered structure in the formation depth.

At ejection velocities $v_0 \in (1\text{–}1.2)$ km s^{-1} the range of variation of semimajor axis values has a complex configuration and a significant width 1.2720–1.3135 a.u. Simultaneously, the eccentricity values vary within even smaller limits (width of 0.005) than at low ejection velocities and have a simple functional dependence; in

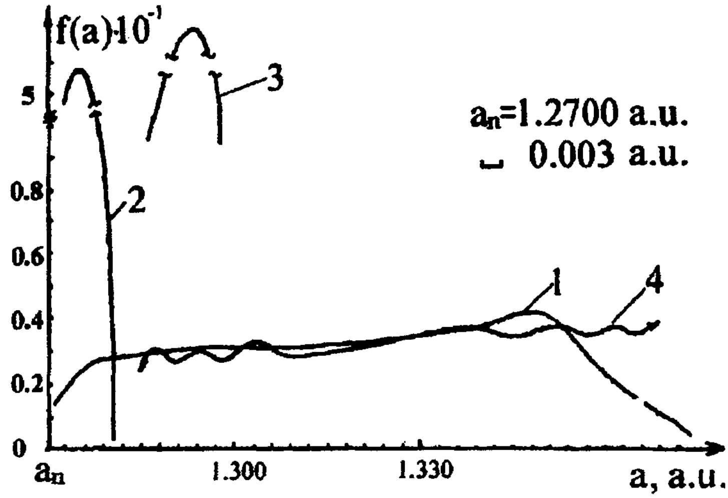

Figure 3 Probability densities of semimajor axis: curves *1,2*, model of exponential ejection: the scale parameter is 250 m s^{-1}; curves *3,4*, model of isotropic ejection: the ejection velocity is 200–300 m s^{-1}; curves *2,3*, ejection at the aphelion of orbit M; curves *1,4*, ejection at the perihelion of orbit M.

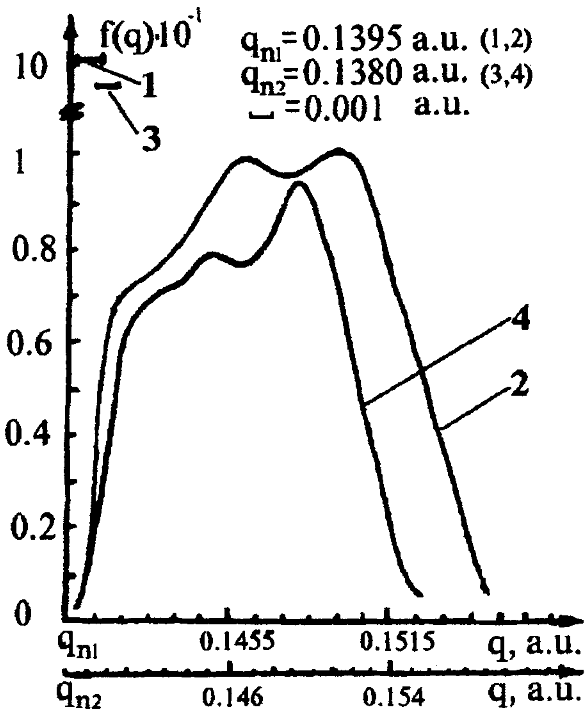

Figure 4 Probability densities of orbital perihelia. Symbols as in Figure 3.

fact the variations in angular element values are well-approximated by a low-degree polynomial.

In modelling fragment ejection at the aphelion of Phaethon the tendencies of variations in orbital element hold true for all calculated velocities. However, the linear dimensions of variation ranges increase for all values and a pronounced oscillatory character of angular element variations is established. In the general case this means that the set of orbits of ejected fragments will have a more or less "layered" structure in the formation depth. So, more or less densely populated (i.e. discharged) regions of different dimensions may form in space.

The functions were calculated for the D-criterion probability density relative to Phaethon's orbit and the Geminids orbit (Kulikova and Mychev, 1995).

Analysis of the calculated data has shown that at low ejection velocities at the aphelion of Phaethon's orbit the set of fragments is less compact relative to the mean orbit of the Geminids than relative to Phaethon's orbit. In this case the local extrema $f(D_{\mathrm{cr}})$ specify the "layered" formation structure. At higher ejection velocities the formations will have a more "blurred" but more extended structure.

In modelling fragment ejection at the perihelion of Phaethon's orbit the values of the mathematical expectations of D_{cr} for Phaethon and the Geminids are comparable and equal to 0.48×10^{-2} and 0.45×10^{-2}, respectively, whereas the despersion of D_{cr} is quite different, i.e. 0.25×10^{-2} and 0.93×10^{-3}. As the value of D_{cr} despersion reflects the extent to which fragment orbits fill the space relative to the object studied, in this case the space filling relative to Phaethon's orbit is more dense than relative to the Geminids orbit. The expected alternation of dense and "discharged" regions of new formations permits the assumption of the presence of a complex layered association of small bodies near Phaethon's orbit as well as near the mean orbit of the Geminids. Hence the object 3200 Phaethon (TB 1983) may be one of the most probable sources of Geminid replenishment.

The exponential ejection of fragments was simulated with the scale parameter for velocities equal to 250 m s^{-1}. It turned out that in fragment ejection at the perihelion of Phaethon's orbit the range of semimajor axis values is much wider than in case of isotropic ejection $a \in (1.2701–1.3751)$ a.u., i.e. its width is 0.1 a.u. and the variations themselves are equiprobable in the whole interval (Figure 3). The perihelia of these fragment orbits are compactly concentrated in the range $q \in (0.139–0.140)$ a.u. (Figure 4). The maximum of the probability density function of eccentricities is shifted to high eccentricity values and differs in form from its analogues for isotropic ejection (Figure 5). For the angular elements i, ω, Ω the probability density function for angles of inclination has two local extrema in the range 21.886–22.007° (Figure 6) and the probability density function for the arguments of perihelion and longitudes of the ascending node vary similarly to such functions in isotropic ejection at velocities $v_0 \in (200–300)$ m s^{-1} (Figures 7, 8).

In modelling the exponential ejection at the aphelion of Phaethon's orbit we have the following characteristics of the Keplerian orbits of ejected fragments. The range of semimajor axis variations is narrower $a \in (1.270–1.279)$ a.u. The form of the probability density function is extremely simple and is well approximated by a low-degree polynomial; it is nearly identical to the relative function for isotropic

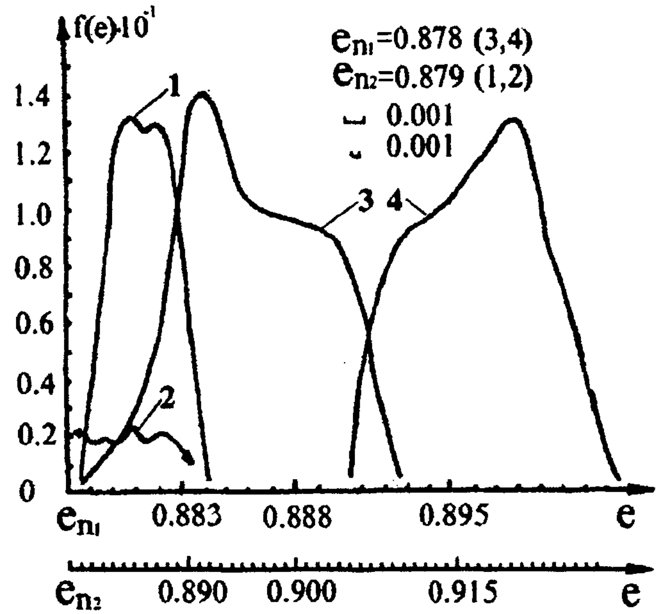

Figure 5 Probability densities of eccentricities. Symbols as in Figure 3.

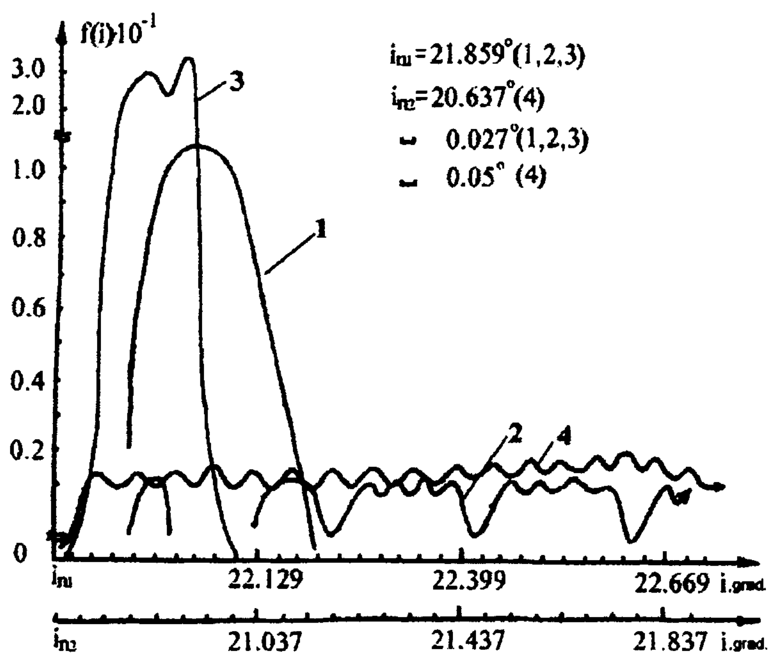

Figure 6 Probability densities of angles of inclination. Symbols as in Figure 3.

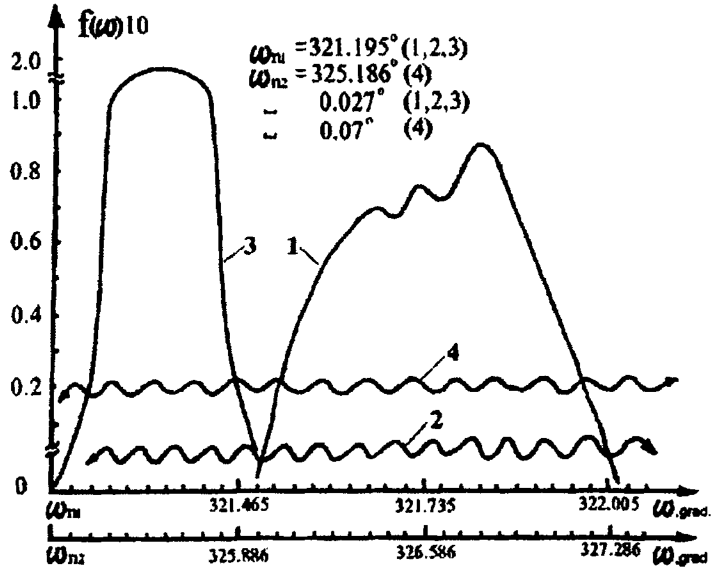

Figure 7 Probability densities of perihelion argument. Symbols as in Figure 3.

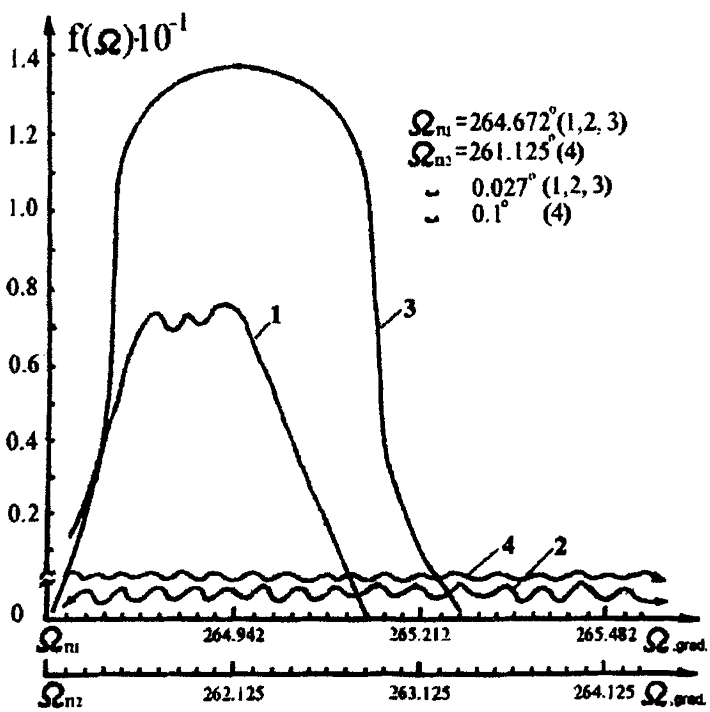

Figure 8 Probability densities of node longitude. Symbols as in Figure 3.

Table 1.

Orbital elements	Ejection model: Isotropic ejection $v_0 \in (200\text{–}300)\ m\ s^{-1}$		Exponential ejection $b = 250\ m\ s^{-1}$ $f(v) = (1/b)\exp\{(b/v)\}$	
	Perihelion			
	M	$D^{1/2}$	M	$D^{1/2}$
a, a.u.	1.3304	0.3242×10^{-1}	1.3313	0.3531×10^{-1}
q, a.u.	0.1385	0.0000	0.1395	0.0000
p, year	1.5309	0.5592×10^{-1}	1.53248	0.6112×10^{-1}
e	0.8946	0.2577×10^{-2}	0.8951	0.2770×10^{-2}
i, (grad.)	22.0752	0.5835×10^{-1}	22.0705	0.3603×10^{-1}
ω, (grad.)	321.7430	0.11243	321.1950	0.2700×10^{-1}
Ω, (grad.)	264.9380	0.01215	264.9390	0.1227×10^{0}
	Aphelion			
	M	$D^{1/2}$	M	$D^{1/2}$
a, a.u.	1.2746	0.2105×10^{-3}	1.2746	0.2177×10^{-2}
q, a.u.	0.1460	0.3518×10^{-2}	0.1461	0.3796×10^{-2}
p, year	1.4355	0.3000×10^{-3}	1.4356	0.3178×10^{-2}
e	0.8850	0.2598×10^{-2}	0.8854	0.2807×10^{-2}
i, (grad.)	22.1000	0.9386×10^{0}	22.0833	0.9746×10^{0}
ω, (grad.)	322.4400	1.3254	322.0560	1.2458
Ω, (grad.)	264.9830	1.9822	264.9900	2.0149

ejection velocities of $v_0 \in (200\text{–}300)$ m s^{-1} within the accuracy of Monte Carlo methods. The probability density functions of perihelia for a given case and for isotropic ejection at low velocities have a similar form with three local extrema and a similar variation range, 0.015–0.016 a.u. The probability density functions of angles of inclination, arguments of perihelion and longitudes of the ascending node have a harmonic form with a large spatial extension. The form of these curves is analogous to similar functions obtained in modelling the isotropic ejection at low velocities at the aphelion of Phaethon's orbit (Figures 6–8).

Table 1 presents the estimates of mathematical expectation and dispersion of the elements a, e, q, i, ω, Ω for the initial fragment ejection velocities. Similar estimates obtained in modelling the isotropic ejection at low initial velocities are given for comparison. It is clear that the values obtained practically coincide within the accuracy of Monte Carlo methods. The greatest discrepancy between estimates is observed for arguments of perihelion (at aphelion – 0.4° and at perihelion – 0.55°); however, in this case they do not exceed 0.1% of the desired value. Therefore, it may be concluded that the space scales of regions of clustering of the orbits of fragments are similar for the two models of ejection within the considered limits of accuracy. The probability functions of the Keplerian orbital elements a, e, q, i, ω, Ω of ejected

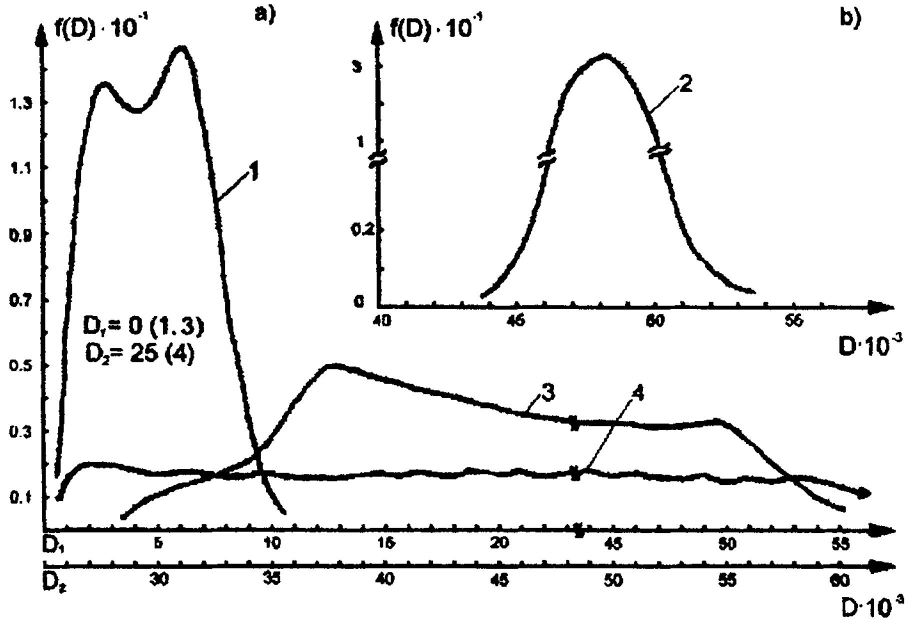

Figure 9 Probability densities for D-criterion values (exponential model of ejection): curves *1,2*, ejection at the perihelion; curve *1*, probability density of D-criterion values relative to Phaethon's orbit; curve *2*, as in curve 1 but relative to the mean Geminids orbit; curve *3*, probability density for D-criterion values relative to Phaethon's orbit in ejection at the aphelion M; curve *4*, relative to the mean Geminids orbit in ejection at the aphelion M.

fragments represent the numerical space parameters of the structure of the set of orbits to which the fragments from Phaethon's surface are most probably ejected.

Figure 9 shows the probability density functions for D_{cr} values calculated both relative to Phaethon's orbit and relative to the observed mean orbit of the Geminids. These functions reflect the degree of compactness of the set of ejection fragment orbits and the effect of such factors as a model of substance ejection, the initial ejection velocity values, and the position of the parent-body in the orbit at the moment of ejection.

It is clear that in fragment ejection at the perihelion of Phaethon's orbit they cluster more compactly around Phaethon's orbit and less compactly relative to the Geminids orbit (Figure 9). The local extrema in the D-criterion probability function shows the "layered" of the set of orbits in space, to which the fragments are most probably ejected. In ejection at the aphelion of Phaethon's orbit the fragments fill the space of Keplerian orbits relatively uniformly.

Table 2 presents estimates of mathematical expectation and dispersion in the D-criterion values for a set of orbits of ejected fragments relative to Phaethon's orbit and the observed mean orbit of the Geminids for two models of ejection at two positions of the parent-body. The table shows that in the case of exponential ejection the fragment orbits are most compactly clustered around Phaethon's orbit

Table 2.

Orbit relative to which the D-creterion value is estimated	*Ejection model*			
	exponential ejection		*isotropic ejection*	
	Perihelion			
	$M[D_{cr.}]$	$D^{1/2}[D_{cr.}]$	$M[D_{cr.}]$	$D^{1/2}[D_{cr.}]$
Phaethon's orbit	0.5192×10^{-2}	0.2519×10^{-2}	0.4815×10^{-2}	0.2555×10^{-2}
Mean orbit of Geminids	0.4811×10^{-2}	0.1188×10^{-2}	0.4550×10^{-2}	0.9296×10^{-3}
	Aphelion			
Phaethon's orbit	0.2147×10^{-1}	0.1022×10^{-1}	0.8409×10^{-2}	0.4097×10^{-2}
Mean orbit of Geminids	0.5165×10^{-1}	0.1744×10^{-1}	0.1285×10^{-1}	0.3576×10^{-2}

in ejection at the perihelion of its orbit. In ejection at the aphelion the fragment orbits are rather uniformly clustered around the parent-body orbit and around the observed mean orbit of the Geminids, but in this case more dense and less dense (discharged) regions are formed in the space of the set of fragment orbits. In the case of the isotropic model of ejection at the perihelion and at the aphelion of Phaethon's orbit the dimensions of the stream relative to Phaethon's orbit are smaller than relative to the Geminids.

So, the Geminids seem to be significantly replenished at the expense of disintegration products of object 3200 Phaethon (TB 1983) only in isotropic ejection of substances at the perihelion of Phaethon's orbit. In other cases such a replenishment is possible but the meteoroid complex of Phaethon itself is more likely.

The results of computer experiments with modelling the process of fragment ejection from Phaethon's surface, in our opinion, convincingly illustrate the possibility of stochastic methods in celestial mechanics. Stochastic modelling of such processes permits (unlike deterministic approaches) the most informative and the fullest description of quantitative and qualitative initial structures of a complex dynamic system of new formations.

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