Astronomical and Astrophysical Transactions, 1998,
Vol. 15, pp. 271–275
Reprints available directly from the publisher
Photocopying permitted by license only

CHAOTIC DYNAMICS IN THE N-BODY PROBLEM (STOCHASTIC APPROACH: MODELS AND METHODS)

A. V. MYSHEV

Institute of Nuclear and Power Engineering, 249020, Obninsk, Kaluga region, Russia

(Received May 8, 1996)

The author is developing a stochastic approach to formalize the mathematical models and to study the models describing the behaviour of dynamic systems in celestial mechanics and stellar dynamics. The essence of the approach is that the mathematical models take a stochastic form and the behaviour of this set is described by a probability space. The mapping of solutions for dynamic systems is constructed in this space.

KEY WORDS Stochastic dynamic, probability space, probability measure, stationary state, metric space

1 INTRODUCTION

Any system in celestial mechanics, even rather strictly determined in the sense of mathematical formalism as well as in studying its mathematical models, exhibits a chaotic dynamics. The present-day theories of motion in the framework of the N-body problem, as a rule, are developed in conservative dynamic systems. It is impossible to consider a great number of gravitational and nongravitational factors in such systems. Both the conservative and dissipative systems display a chaotic dynamics. The dynamics of such a type in dissipative systems has not been studied in the framework of the N-body problem either in celestial mechanics or stellar dynamics. The temporal behaviour of such systems may be either determinate or stochastic. Hence, both the determinate and stochastic chaos manifests itself in them. The determinate chaos is mainly under study at present. In many cases an ergodic theory is used. This theory is acceptable only for a restricted class of problems being true for stationary or "semistationary" processes. The nature of such processes is unknown in a strict sense. In the celestial mechanics and stellar dynamics with uncertain parameters and a great variety of factors in a system, the dynamic systems exhibit a stochastic dynamics very clearly. To investigate this dynamics, the development of new formalization approaches to the problem and of methods for studying the mathematical models of such problems is necessary. The author is developing the stochastic approach to formalize mathematical models and to study the models describing the behaviour of dynamic systems in celestial mechanics and stellar dynamics. The essence of the approach is that the mathematical models take a stochastic form and the behaviour of this set is described by a probability space. The mapping of solutions for dynamic systems is constructed in this space.

2 PROBLEM STATEMENT (MODELS)

The chaotic dynamics in the N-body problem, as a rule, is stipulated by objective and subjective uncertainties. Among objective uncertainties is a complex wave character of the gravitational and nongravitational disturbances which are included in the mathematical model in a determinate manner. Subjective uncertainties (conventionally at least) can be stipulated by the following reasons: a) gravitational and nongravitational disturbances are stochastically described; b) uncertain initial conditions of a dynamic system; c) a stochastic character of variations of some (may be all) variables in a dynamic system; d) a linear character of the operators approximating the input mathematical model and etc. This type of uncertainties is stochastically described in the mathematical model.

When studying the chaotic dynamics in the N-body problem with objective and subjective uncertainty factors present, the mathematical models in this problem can be constructed in the following ways.

In the first way of problem formalization, the right sides of the differential equations are determinate functionals of determinate and stochastic variables and random parameters. Then the equations and initial conditions may be written as

$$\begin{cases} dX/dt = F_d(t, X_d, X_{st}, Y) \\ X_{d|t=t_0} = P_1(t_0, X_d^0) \\ X_{st|t=t_0} = P_2(t_0, X_{st}^0) \\ Y = P_3(t, Y) \end{cases} \tag{1}$$

where F_d are determined force vectors; X_d is a vector of determined variables; X_{st} is a vector of stochastic variables; Y is a vector of random parameters.

In the second way of formalization, the right sides of the differential euqations include both the determined and stochastic functionals, and the problem takes the form

$$\begin{cases} dX/dt = F_d(t, X_d, X_{st}, Y) + F_{st}(t, X'_d, X'_{st}, Y'); \\ X_{d|t=t_0} = P_1(t_0, X_d^0); \quad X'_{d|t=t_0} = P'_2(t_0, X'^0_d); \\ X_{st|t=t_0} = P_3(t_0, X_{st}^0); \quad X'_{st|t=t_0} = P'_4(t_0, X'^0_{st}); \\ Y = P_5(t, Y); \quad Y' = P_6(t, Y'); \end{cases} \tag{2}$$

where F_{st} is a vector of the stochastic functionals, other symbols are similar to the first way.

It is clear that the first way of problem formalization is a special case of the second one.

The third way of formalization consists in the fact that Eqs. (1) and (2) can be supplemented by partial differential equations of the Vlasov or Kolmogorov–Fokker–Planck type. In this case such equations in a vector form can be presented as follows:

$$\partial P/dt + F\partial P/dX + P\partial F/dX = 0, \tag{3}$$

where X is a vector of stochastic variables; F is a vector of functionals in the right side of Eqs. (1) and (2); P is a vector of probability measures.

The development of another mathematical models of the dynamic systems specified by a particular problem, in a strictly mathematical sense, is a combination of the above ways.

3 METHODS FOR STUDYING MATHEMATICAL MODELS (BERNOULLI'S SCHEMES)

Two methods for studying the chaotic dynamics of different dynamic systems are used now: first, the method of moments; second, the problems of type (1) and (2) are supplemented by Eq. (3). The application of the theory of moments is limited by a second-order theory. The theory of solving equations of type (3) is applied to linear equations and requires stationary conditions and a Markov behaviour of the process under study to be satisfied. The applied and theoretical studies of chaotic dynamics show that these methods are suitable for a narrow class of problems.

The suggested methods (Bernoulli's schemes) of computer modeling nonlinear correctly stated problems (1) and (2) are always applicable; they have no restrictions and do not require the conditions of a stationary state, ergodicity and the Markov property to be satisfied. It is the primary merit of the methods suggested and the universality of their application. Let us briefly characterize the methods and the scheme of their development.

The primary idea of the suggested methods is that the behaviour of the sets (1) and (2) is described by a probability space $(B,\ F,\ P)$, where B is a space of expected states of a dynamic system in the extended phase space ($b \in B$ is a trajectory in B); F is an event algebra in B; P is a probability measure in the event class F. Since the computer simulation allows one to study dynamic systems with a limited number of variables and parameters, the finite-dimensional spaces $(B,\ F,\ P)$ are considered, i.e. B is finite-dimensionally divided into a subset. P includes all available information about the set (1), (2) at given initial conditions. The solution of this set is constructed as its finite-dimensional mapping in $(B,\ F,\ P)$. Theoretical and practical studies Kulikova, Myshev, Pivnenko (1993) and Myshev (1993) show that it is not always possible to estimate P in the initial space $(B,\ F,\ P)$ and, hence, to construct the mapping of the solution for the problem under study. In these cases it is suggested to map $(B,\ F,\ P)$ in another metric probability space $(Y,\ G,\ \bar{P})$ where the solution of the set (1), (2) is possible. Such a map is constructed from the following prerequisites: first, the initial space (B, F, P) may be of arbitrary character (and not necessarily metric) and is the range for estimating variables and parameters of the set (1), (2); second, the set (1), (2) is solved in the metric space $(Y, G, \bar{P})$ which is the range of parameter variations in the set (1), (2) with a simple functional dependence; third, the choice of $(Y, G, \bar{P})$ is stipulated by the fact that here the probability metric and probability relations between variables and parameters of the set (1), (2) are constructed much easier. One of the ways to construct such maps for systems in celestial mechanics is described by Kulikova, Myshev, Pivnenko (1993).

As shown by Myshev (1993, 1995), in this case the solution is presented as an expectation and a dispersion of the most probable solutions described by the probability space $(Y, G, \bar{P})$. The spectrum-periodogram of the probability measure P and the expectation of solutions as well as their functional correlation dependence is an indicator of the set (1), (2) transition to chaos and outcome of it. A measure of trajectory compactness in the set (1), (2) is the mathematical expectation of the functional $q(E)$ defined in the probability space $(Q, \hat{Q}, \hat{P})$ where Q is the range of functional $q(E)$ variations, E is the parameter vector defined in Y. In many problems of celestial mechanics, both applied and theoretical, the D – criterion is taken as $q(E)$.

When mapping the initial (B, F, P) to the metric $(Y, G, \bar{P})$ space is constructed and the functional dependencies between variables and parameters of these spaces are determined, it is necessary to estimate $\bar{P}$ in $(Y, G, \bar{P})$. The algorithm for this estimation is treated at length by Kulikova, Myshev, Pivnenko (1993). Let us consider the main steps in estimating $\bar{P}$ for computer simulation.

First, the space Y is divided into subsets $A_i(Y = U_i A_i)$ so as to take into account the following errors: a) an error in the difference operator approximating the problem (1), (2); b) approximation errors accumulated in a computer; c) a shift error and statistical Monte–Carlo error. These conditions (called agreement conditions) are calculated before the computer simulation and are the input parameters in numerical experiments.

Second, a computer experiment with studying the set (1), (2) dynamics on a given interval is continued until the agreement conditions are fulfilled.

Third, when a computer experiment is completed, P is estimated, solutions of the set (1), (2) are mapped and its dynamics is analyzed.

As the agreement conditions are reached during the final time interval, the subsequent problem of solving the set (1), (2) can be replaced by forecasting $\bar{P}$ in the space $(Y, G, \bar{P})$ using the apparatus of predicting stochastic filters.

In conclusion, the described scheme is simply realized and allows one to obtain the accuracy stipulated by computer architecture.

4 CONCLUSIONS

The method suggested for computer simulation of the stochastic dynamics of the systems (1), (2) according to Bernoulli's scheme is more universal and allows one to obtain the solutions inaccessible for other methods. First, it is explained by the fact that it can be used for all nonlinear problems. The only and necessary condition for the application of Bernoulli's scheme is an adaptivity of the law of large numbers and the central limiting theorem to variables and parameters of the set (1), (2).

The studies of the chaotic dynamics in planetary astronomy, meteoric and cometary astronomy performed according to Bernoulli's scheme Kulikova, Myshev, Pivnenko (1993) and Kulikova, Myshev (1995) as well as the processed observational data are indicative of the prospect of this method in the above fields.

References

Kulikova, N. V., Myshev, A. V., and Pivnenko, E. A. (1993) *Cosmogony of Small Bodies*, Cosmosinform, Moscow.

Myshev, A. V. (1993) *Abstract of the Conference "Theoretical, Apllied and Computer Celestial Mechanics"*, St. Peterburg, October 12–14.

Myshev, A. V. (1995) *Abstract of the Conference "Asteroid Danger-95"*, St. Peterburg, May 23-25.

Kulikova, N. V., Myshev, A. V. (1995) *Earth, Moon, and Planets* **68**, 389.