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SIMULATION OF STOCHASTIC DYNAMICS IN A FOUR-BODY PROBLEM (COMPUTER EXPERIMENT)

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The results of computer experiments on modelling stochastic dynamics in a planetary four-body problem are discussed. A scissors effect in this problem was obtained. This result was first observed by the authors as a solution for a stochastic planetary four-body problem.

KEY WORDS **Bernoulli's scheme, stochastic variables, finite-dimensional probability space**

1 INTRODUCTION

Computer experiments studying body ejection from the sphere of action of the giant planets Jupiter and Saturn were conducted on the basis of a stochastic ejection model in a spatial four-body problem (Kulikova *et al.*, 1993). The basis for the computer experiments is the Monte Carlo algorithms developed according to Bernoulli's scheme. This scheme has been developed by one of the authors (Myshev, 1993) to obtain mapping of solutions for a dynamic N-body system in the finite-dimensional probability space. Computer experiments were aimed at obtaining the probability characteristics of a celestial-mechanical planetary four-body system when the bodies are ejected at distances of the giant planet's satellite orbits. First, to obtain a probability description of the structure of a set of trajectories of bodies ejected into heliocentric (elliptic and hyperbolic-parabolic) as well as planetocentric Keplerian orbits. Secondly, to obtain estimates of probable body ejection into the above classes of orbits for different initial conditions of ejected bodies and body configuration at the moment of ejection. Thirdly, to obtain numerical estimates of variations in Keplerian orbital elements, into which the bodies are ejected.

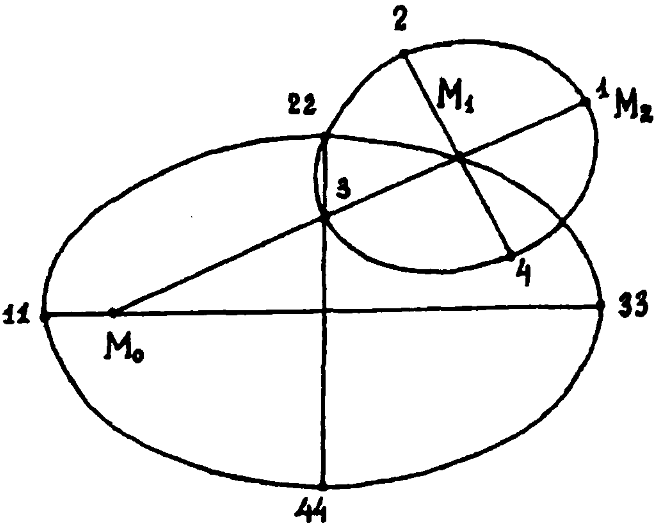

Figure 1 The geometric problem.

2 RESULTS OF COMPUTER EXPERIMENTS

Figure 1 shows the geometry of body positions: the Sun (M_0), a planet (M_1) and a satellite (M_2) for the problem considered. The ejected body M_3 is not shown here because the initial ejection conditions are stochastic variables.

In sudying ejection of M_3 from the sphere of Jupiter's action, the orbit of Io (M_2) was taken as the initial distance. The initial velocity values of M_3 ejection were taken within $v_0 \in (7.8\text{–}15)$ km s^{-1} and $v_0 \in (15\text{–}30)$ km s^{-1}. In this case, the probability of direct as well as oblique ejections was considered. Model calculations were performed with Jupiter at perihelion and the satellite in positions 1 and 3 (see Figure 1).

Figures 2–4 present the probability density of the a, e, i elements of heliocentric elliptic orbits of M_3 obtained from computer experiments. Curves 11 and 12 refer to M_3 ejection at $v_0 \in (7.8\text{–}15)$ km s^{-1} and curves 21 and 22 to $v_0 \in (15\text{–}30)$ km s^{-1}. Curves 12 and 22 refer to position 3 of M_2, the orbit at the moment of M_3 ejection, and curves 11 and 21 to position 1. The probability densities for semi-major axes (Figure 2) show that in the case considered the most probable orbits are those with $a \in (3\text{–}5)$ au. These orbits possess greater eccentricity with $e \in (0.7\text{–}1)$ (Figure 3). For inclination angles, the probability densities given in Figure 4, show that the probability maximum is in a region of direct motions $i_{\max}(0°\text{–}30°)$.

Model calculations also indicate that M_3 ejection from the sphere of Jupiter's action into a heliocentric elliptical orbit with $a < 2.5$ au is unlikely. The probability of M_3 ejection into a long-period orbit with $a > 35$ au in our case is within the accuracy of Monte Carlo methods. The probability densities obtained for elements

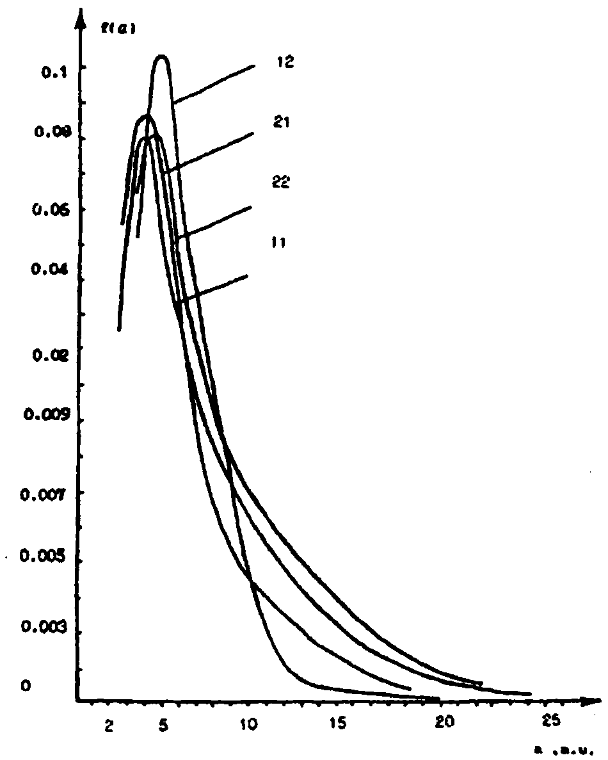

Figure 2 The probability density of semi-major axes.

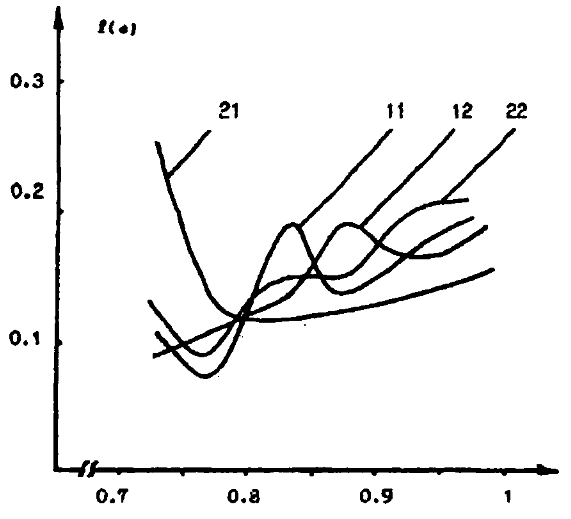

Figure 3 The probability density of eccentricity.

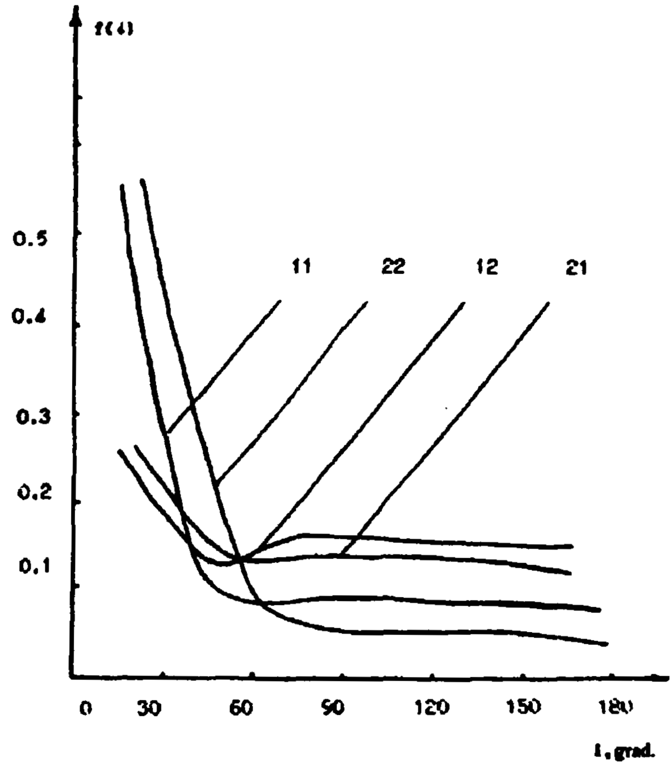

Figure 4 The probability density of inclination angles (ejection from Jupiter's sphere of action).

a, e, i and presented in Figures 2–4, qualitatively characterize the structure of a set of orbits as a result of the stochastic dynamics of M_3.

Ejections from Saturn's sphere of action were studied Computer experiments have been performed for a lot of configurations of M_0, M_1 and M_2 and various initial conditions for M_3. The orbits of Saturn's satellites (Titan and Rhea) were chosen as variations in the initial distances of M_3. Model calculations were carried out for all configurations of M_0, M_1, M_2 presented in Figure 1. The initial ejection velocities were taken to be within the following limits: (1) $v_0 \in (3.7–4.7)$ km s^{-1} and $v_0 \in (4.7–6.7)$ km s^{-1} considering the probability of direct and oblique ejections; (2) $v_0 \in (7–9)$ km s^{-1} considering the probability of direct ejections.

Figures 5 and 6 give the probability densities for elements a, e, i of heliocentric elliptic orbits of M_3 at $v_0 \in (4.7–6.7)$ km s^{-1} with M_3 in position 11 and M_2 in positions 1, 2, 3 and 4 (see Figure 1).

The probability densities of semi-major axes show that in a region of short-period motions the maximum of probability densities a is within $a_{\max} \in (6–9)$ au for all configurations of M_0, M_1 and M_2. For each configuration the range a of

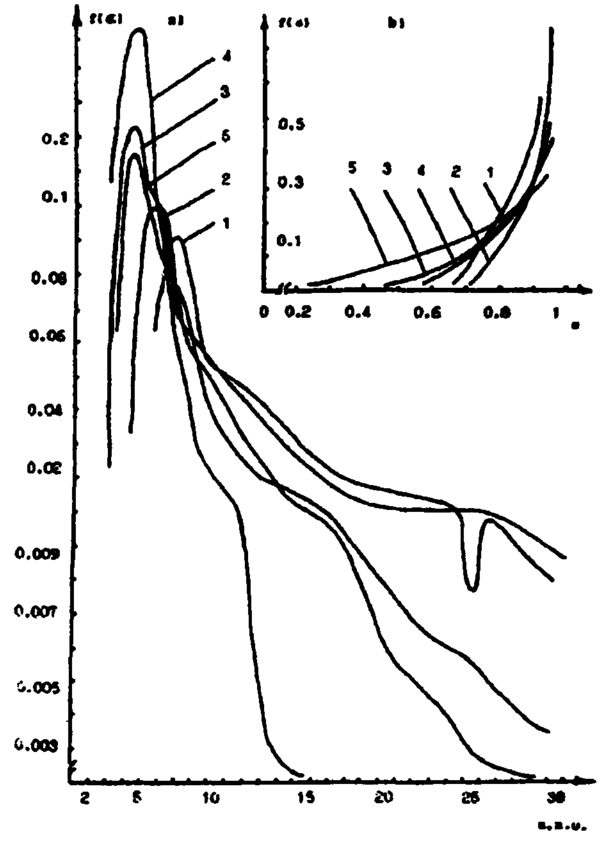

Figure 5 The probability density of: (*a*) semi-major axes; (*b*) eccentricity. Curves 1, 2, 3, 4 refer to the model of M_3 ejection with the account of probability of direct and oblique ejections for corresponding M_2 positions in the orbit; curve 5 refer to the model of M_3 ejection taking into account the probability of only direct ejections for orbital position 3 of M_2.

value variations is different (Figure 5*a*, curves 1, 2, 3 and 4). For the configurations where M_2 is in position 3, most probably M_3 is ejected into a long-period orbit in which the secondary maximum of probability density a is within $a_{max} \in (61-62)$ au. Curve 5 in all figures refers to the configuration of M_0, M_1 and M_2 where M_2 is in position 3 and M_3 ejections are direct. The probability densities of orbital eccentricities given in Figure 5*b* show that these orbits possess greater eccentricity. In direct ejections (Figure 5*b*, curve 5) the probability of ejection into orbits with smaller eccentricity is higher.

The probability densities a and e show that the aphelia of M_3 orbits will be most probably concentrated near the orbit of M_3 with a small shift to Uranus's orbit. This region of the most probable concentration of aphelia will have a layered structure.

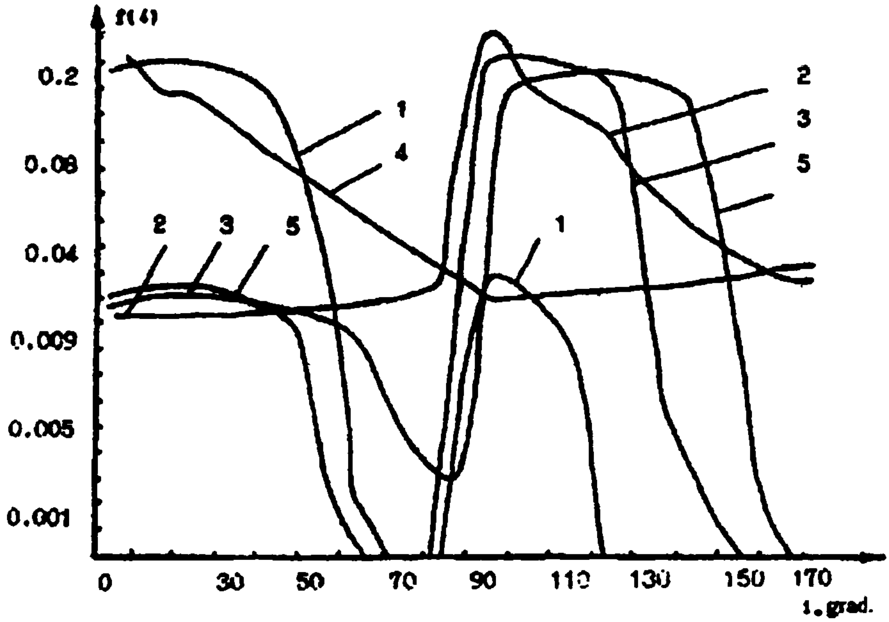

Figure 6 The probability density of inclination angles (ejection from Saturn's sphere of action).

The probability densities presented in Figure 6 (curves 1, 2, 3 and 4) for inclination angles of heliocentric elliptic orbits of M_3 show that they have maxima both in the region of direct motions and the region of reverse motions. The positions of these maxima are stipulated by configurations of M_0, M_1 and M_2 at the moment of M_3 ejection.

For planetocentric orbits of M_3 the following characteristics of their elements are observed. The range of a variations is rather extended and is beyond the sphere of action of M_1 with a_{max} near a planetocentric distance of $R_2 = 1.5 \times 10\ \mathrm{km}^6$ for M_2. The probability density e ranges within $e \in (0\text{–}1)$ and has a monotonic increase at $e \to 1$. For direct and oblique ejections of M_3 orbits with direct motion are most probable.

M_3 is ejected into planetocentric as well as heliocentric orbits: elliptical and hyperbolic-parabolic. Figure 7 presents the probability densities which show how the probability of M_3 ejection into the above orbits depends on configurations of M_0, M_1 and M_2 at the moment of M_3 ejection at a given v_0 (in our case $v_0 \in (4.7\text{–}6.7)\ \mathrm{km\ s^{-1}}$). As seen from the diagrams presented in this figure, there is a sciscors effect in the problem considered, i.e. curves 2 and 3 look like scissors capturing of M_3 in its elliptic or hyperbolic-parabolic heliocentric orbits. This effect mainfested in the problem considered for all configurations of M_0, M_1 and M_2 and initial conditions of M_3 ejection. It should be noted that a scissors effect in such problems was not found earlier and this result was first observed by the authors in a solution for a stochastic planetary four-body problem.

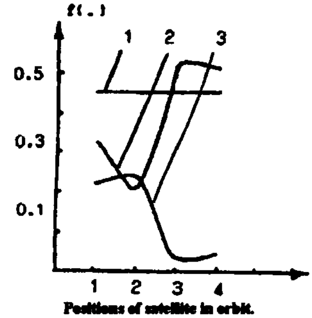

Figure 7 Curve 1, planetocentric orbits; curve 2, heliocentric elliptical orbits; curve 3, heliocentric hyperbolic-parabolic orbits.

So, the presented (Figures 2–6) probability densities of elements a, e, i for M_3 orbits give a probability description of the structure of M_3 orbit classes. Being applied to similar problems these results are original.

3 CONCLUSIONS

The results of computer experiments on modelling stochastic dynamics in a planetary four-body problem allow us to come to some conclusions. First, a set of heliocentric elliptic orbits of M_3 is not limited by the region between the giant planet' orbits and their neighbourhoods and involves different classes of orbits of known and unknown small bodies. Secondly, for planetocentric orbits of M_3 the region of probable ejection is not limited by the M_2 orbit and involves a more distant region of the system M_1. Thirdly, the structure of a set of M_3 orbits both in a class of the system M_0 and system M_1 orbits is stipulated by many factors such as geometric configurations of bodies M_0, M_1 and M_2 at the moment of M_3 ejection and initial conditions: ejection pattern and initial velocities of M_3.

In conclusion, the results of computer experiments on modelling stochastic dynamics show that stochastic methods of computer simulation in the approaching century of supercomputers have the most potential. Studies perfomed by the authors, on the basis of principles of stochastic modelling show that they can be used for a wide class of problems: planetary dynamics, stellar dynamics, cometary and meteor astronomy, data processing (Kulikova *et al.*, 1993; Myshev, 1993; Kulikova and Myshev, 1995) and others.

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