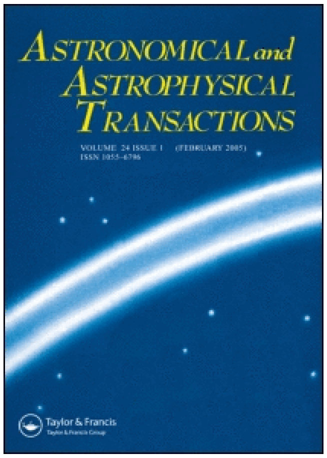

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BOSE INSTABILITY IN KERR BLACK HOLES

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Bose instability in rotating (Kerr) black holes (BHs) consists of an exponential increase in time of small perturbations of Bose mass fields, corresponding to superradiative, quasibound levels. The minimal time of dumping of the angular momentum on the 2P envelope is much less than the time of dumping of the angular momentum by superradiation for primordial BHs when the mass of the particles $m \ll \frac{M_{\rm Pl}^2}{M}$. Very fast dumping of the angular momentum occurs when $0.46 \geq \frac{mM}{M_{\rm Pl}^2} \geq 0.203$ (for π), $0.353(\eta)$, $0.065(D^0)$. Electrically charged particles cannot develop Bose instability due to the ionization of bound levels by electromagnetic radiation emitted by the BH itself. The neutral particles produce γ-bursts of energies 67.5, 274.5, 932 MeV correspondingly. The duration of bursts is 1.26×10^{-17} s(π^0), 2.99×10^{-18} (η), 8.55×10^{-19} s (D^0). The radiated energies are 1.20×10^{35} erg, 8.67×10^{34} erg, 8.55×10^{33} erg, corresponding to powers of the order of magnitude 10^{52} erg s^{-1}. Other consequences for BHs evaporation are discussed.

KEY WORDS Kerr black holes, scalar bosons, superradiation, bound levels, ionization, γ-bursts

1 INTRODUCTION

Bose instability of rotating black holes (Kerr BHs) is related to an exponential increase in time of small perturbations of a test mass field, corresponding to superradiative quasi-bound states with energies

$$E \leq \min\{\mu c^2, \hbar m \Omega_H\}, \tag{1}$$

where μ and E are, respectively, the rest mass and energy of the particles, m is the projection of the momentum on the BH's axis, M and $J = Mac$ are mass and angular momentum of the BH and the angular velocity of the BH is written as

$$\Omega_H = \frac{ac^3}{2GMr_+}, \tag{2}$$

where

$$r_+ = GM/c^2 + (G^2M^2/c^4 - a^2)^{1/2}.$$

On the level of a second quantized quantum field theory (QFT) this corresponds to the occurrence of spontaneous and induced particle creation processes on quasi-bound superradiative levels. Only spontaneous generation of fermions may occur due to the Fermi exclusion principle, but bosons may accumulate on such levels by induction (or stimulation). On the level of Klein–Gordon, Dirac and other similar QF equations this corresponds to the fact that $s = \frac{1}{2}, \frac{3}{2}, \ldots$, mass equations only support damping (Ternov *et al.*, 1980), while the $s = 0, 1, 2, \ldots$, mass equation may change the sign of the imaginary part of the energy (Damour *et al.*, 1976)

$$E = E^{(0)} - i\gamma, \tag{3}$$

$$E^{(0)} \equiv Re E < \mu c^2,$$

$$\gamma \equiv Im E = \begin{cases} > 0, & \text{for } s = \frac{1}{2}, \frac{3}{2}, \ldots; \\ > 0, & \text{for } s = 0, 1, \ldots \text{ and } E^{(0)} > \hbar m \Omega_H; \\ \leq 0, & \text{for } s = 0, 1, \ldots \text{ and } E^{(0)} < \hbar m \Omega_H. \end{cases}$$

In other words, bosons support self-stimulated generation (and consequence–accumulation) on quasi-bound superradiative states (1) in which the wave function increases as $\Phi \sim e^{\lambda t}$, where $\lambda = -\gamma$, for $E^0 \leq \hbar m \Omega_H$ and the number of particles and the energy density increase as

$$N = \frac{1}{2} i \int \{\Phi^*(\partial^0 \Phi) - \Phi(\partial^0 \Phi)^*\} \sqrt{-g} d^3x \sim e^{2\lambda t} C, \tag{4}$$

$$\varepsilon = \int k^{\nu}_{(t)} T^0_{\nu} \sqrt{-g} d^3x = \int T^0_0 \sqrt{-g} d^3x \sim e^{2\lambda t}, \tag{5}$$

where $k^{\nu}_{(t)} = \delta^{\nu}_t$ is time like the Killing vector of Kerr metrics. One could show by an alternative method that the probability of the transition of a system (BH+bosons) from an initial state $|N_{\vec{k}}, 0 >$ with $N_{\vec{k}}$ bosons with quantum number $\vec{k}$ and 0 antibosons into a final state $|N_{\vec{k}} + 1, 1 >$ with $N_{\vec{k}} + 1$ bosons and 1 antiboson will be proportional to the square of the matrix element

$$| < 0, n_k | T^0_0 | n_k + 1, 1 > |^2 = |c|^2 (N_k + 1). \tag{6}$$

When $N_{\vec{k}} = 0$, this is just the probability of a spontaneous generation of a pair boson-antiboson from which one particle localizes on the quasi-bound state and the other inside the BH. Otherwise equation (6) gives the probability of self-stimulated generation of pairs.

So, we have considered the following equations for the number of particles on the superradiative levels, mass and angular momentum variations of a BH (Gaina, 1989)

$$\frac{dN_{\rm nlm}}{dt} = \lambda_{\rm nlm}(N_{\rm nlm} + 1), \tag{7}$$

$$\frac{d(Mc^2)}{dt} = -\sum_{\rm nlm} \lambda_{\rm nlm} E^{(0)}_{\rm nlm}(N_{\rm nlm} + 1), \tag{8}$$

$$\frac{dJ}{dt} = -\sum_{\mathrm{nlm}} \lambda_{\mathrm{nlm}} \hbar m (N_{\mathrm{nlm}} + 1). \tag{9}$$

Equation (7) gives the number of particles on the quasi-level with quantum numbers $n \equiv 1 + l + n_r, l, m$ ($n_r = 0, 1, 2, \ldots,\ l = 0, 1, 2, \ldots$) for scalar particles. Generally, equations (7)–(9) are non-linear, admitting solutions only in special cases.

2 BOSE INSTABILITY IN KERR BH's

Let us limit our examination to spinless particles only, as the solutions for the vector particles and other boson mass particles are still unknown in Kerr backgrounds.

As has been shown (Gaina, 1989) only the case

$$\mu M \leq M_{\mathrm{Pl}}^2 \equiv \frac{\hbar c}{G} \tag{10}$$

is of interest, if one excludes the case of very large $\mu \to M_{\mathrm{Pl}}$. The probabilities of particle generation were calculated (Ternov *et al.*, 1978; Detweiler, 1980; Gaina and Kochorbe, 1987). The main contribution to the change of mass and angular momentum of the BH comes from the generation and accumulation of particles on the 2P level. The dynamic equations for the number of particles and angular momentum are (below we use the system of units $c = \hbar = G = 1$)

$$\frac{dN_{\mathrm{np}}}{dt} = \lambda_{\mathrm{np}}(N_{\mathrm{np}} + 1), \tag{11}$$

$$\frac{dJ}{dt} \simeq -\frac{dN_{2\mathrm{p}}}{dt} \tag{12}$$

while the mass change is negligible.

Using the law of conservation of total angular momentum $J = J_0 - \sum_n N_{\mathrm{np}}$ in the system BH+bosons we obtain the law of variation of the number of particles and angular momentum of the BH in explicit form

$$N_{\mathrm{np}} = J_0' \frac{1 - \exp[-(J_0' + 1)\mu^9 M^6 t/48]}{1 + J_0' \exp[-(J_0' + 1)\mu^9 M^6 t/48]}, \tag{13}$$

$$J = J_0 - N_{2\mathrm{p}}, \tag{14}$$

where

$$J_0' = J_0 - J_{\mathrm{st}}, \tag{15}$$

J_{st} being the BH angular momentum at which superradiance at the given level stops. The exact value of J_{st} is

$$J_{\mathrm{st}} = \frac{4E^{(0)} M^3 m}{m^2 + 4[E^{(0)}]^2 M^2}, \tag{16}$$

while for the case $\mu M \ll 1$ one obtains

$$J_{\rm st} \simeq 4\mu M^3 \ll M^2. \tag{17}$$

Note, that the time of dumping of the angular momentum of the BH into the levels is

$$\tau_J \approx 48 t_{\rm Pl} \left(\frac{M_{\rm Pl}}{\mu}\right)^3 \left(\frac{M_{\rm Pl}^2}{\mu M}\right)^6 \frac{\ln(J_0 - J_{\rm st} + 1)}{(J_0 - J_{\rm st})} \tag{18}$$

for $\mu M \ll M_{\rm Pl}^2$ which approximately equals

$$\tau_J \approx 96 t_{\rm Pl} \left(\frac{M_{\rm Pl}}{\mu}\right) \left(\frac{M_{\rm Pl}^2}{\mu M}\right)^8 \ln \frac{M}{M_{\rm Pl}}. \tag{19}$$

The mass of the envelope of bosons in the 2P state is $\Delta M = M_0 - M_{\rm st} \approx \mu N_{2p} \approx \mu J_0' \approx \mu(J_0 - J_{\rm st}) \approx \mu M a_0$. It will be much less than the mass of the BH itself if $\mu M \ll M_{\rm Pl}^2$.

A discussion of other details of the dumping of the angular momentum of the BH, caused by Bose instability is given elsewhere (Gaina, 1989). The time of loss of the angular momentum of a rotating BH by superradiation is (Zel'dovitch, 1971, 1972)

$$\tau_{\rm superrad} \sim 8\pi e^{\xi} \left(\frac{M}{M_{\rm Pl}}\right)^3 t_{\rm Pl}, \tag{20}$$

where ξ is of order unity. Then, the ratio

$$\frac{\tau_{\rm superrad}}{\tau_J} = \frac{\pi}{12} e^{\xi} \left(\frac{M}{M_{\rm Pl}}\right)^2 \left(\frac{M\mu}{M_{\rm Pl}^2}\right)^9 \ln^{-1} \frac{M}{M_{\rm Pl}} \tag{21}$$

may be much greater than unity if $M \gg M_{\rm Pl}$. We do not now consider the cases $\mu \leq M_{\rm Pl}$, $M \geq M_{\rm Pl}$ and $\mu M \sim M_{\rm Pl}^2$. For the last case we can give some estimations based on analytical approaches developed, while an exact treatment should be given numerically.

The probability of pair production for the case $\mu M \gg M_{\rm Pl}^2$ was calculated by Zouros and Eardley (1979) for scalar bosons and improved by Gaina (1989). The corresponding time of relaxation (dumping) of angular momentum for an extremely rapidly rotating BH with unfilled levels is less than the age of the Universe for BHs with masses $\mu M \leq (23 \div 26) M_{\rm Pl}^2$. The characteristic range of variation of the specific angular momentum of the BH is $0.6 \leq a/M < 1$.

One should emphasize that thermal effects will be small if the temperature of BHs $kT_{\rm BH} = \sqrt{1 - a^2/M^2}/4\pi r_t \ll E^{(0)} \approx \mu c^2$. From this it is easy to obtain the criterion for macroscopic tunnelling:

$$\sqrt{1 - (ac^2/GM)^2} \ll 4\pi\mu M/M_{\rm Pl}^2. \tag{22}$$

It can be satisfied easily for rapidly rotating ($ac^2 \to GM$) or macroscopic ($\mu M > M_{\rm Pl}^2$) primordial or stellar BHs, that is, these cases generation of particles is caused by the ergosphere of the BH and not by Hawking tidal generation.

3 THE ENERGETIC SPECTRUM OF QUASI-BOUND LEVELS

There are very different energy spectra in the long wavelength ($\mu M \ll M_{\rm Pl}^2$, $or r_+ \ll \lambda_c^{/}$) and short wavelength ($\mu M \gg M_{\rm Pl}^2$, $or r_+ \gg \lambda_c^{/}$) limits. In the first case we have a full hydrogen-like spectrum for $a = 0$

$$\frac{E_n^0}{\mu} = 1 - \frac{\mu^2 M^2}{2n^2}. \tag{23}$$

S quasi-bound levels appear for $\mu M \leq 0.25 M_{\rm Pl}^2$, P quasi-bound levels appear for $\mu M \leq 0.46 M_{\rm Pl}^2$, D quasi-bound levels appear for $\mu M \leq 0.74 M_{\rm Pl}^2$ and so on, nl quasi-levels appear for $\mu M \leq \frac{\sqrt{3}}{6} M_{\rm Pl}^2$ if $l \gg 1$ (quasi-classical limit) as shown by Gaina and Chizhov (1980). Such a criterion is still unknown for the Kerr metrics. The extremely rotating Kerr BH was examined by Gaina and Zaslavskii (1992). It was shown that the marginally stable corotating orbit is damped for spinless particles. However, it is known (Gaina and Ternov, 1988) that the spectrum (24) is a good approximation also for Kerr metrics if

$$\mu M \ll l + \frac{1}{2}. \tag{24}$$

So, one could expect that the criterion for the existence of 2P and 3D quasi-bound levels for Kerr BHs is roughly the same as for Schwarzshild BHs.

4 ELEMENTARY PARTICLES AND THE MASS RANGES FOR BOSE INSTABILITY

Assuming $\mu M \approx 0.45$ one derives from equation (18) the minimal dumping time of the angular momentum for an extremely rotating primordial BH:

$$\tau_{J(\min)} = 5.7 \times 10^4 \mu^{-1} \ln\left(\frac{M}{M_{\rm Pl}}\right). \tag{25}$$

By taking into account the lifetime of most mesons $\tau_L < 10^{-8}$ one finds that boson instability cannot develop for BHs with masses $M > 5M_{\rm Pl}^2/\mu_\pi = 3.8 \times 10^{16}$ g. If we assume, however, that boson instability occurs for neutrino pairs (assuming the neutrino to have mass) with a characteristic time determined by equation (26), we obtain an upper limit on the mass of a BH subjected to boson instability: $M < 2.4 \times 10^{23}$ g. For known bosons the masses ranges for BHs subjected to Bose instability are given in Table 1.

From this table it is easy to see that the bosonic instability cannot develop for η. On the other hand one should cover η decay in the gravitational field of a BH as well as for other particles which could meet ours expectations for accumulations of η mesons. One must take into account also that estimations for the upper limit mass of the BH subjected to vector instability were made on the basis of a scalar equation and the actual value of the τ_J may be less for $W^\pm$ and Z^0.

Table 1. Mass ranges for the known bosons

Particle	Lifetime $\tau_L(s)$	Low limit mass(g)	Upper limit mass(g)	Mass of the particles (Mev)
$\pi^{\pm}$	2.6×10^{-8}	5.8×10^{13}	8.5×10^{14}	140
π^0	0.8×10^{-16}	7.0×10^{14}	8.85×10^{14}	135
η	2.4×10^{-19}	3.0×10^{14}	2.18×10^{14}	549
$K^0, \bar{K}^0$	10^{-9}	2.1×10^{13}	2.4×10^{14}	498
D^0	5×10^{-13}	1.6×10^{13}	6.42×10^{13}	1864
$D^{\pm}$	10^{-12}	1.5×10^{13}	6.4×10^{13}	1869
$F^{\pm}$	2×10^{-13}	9.4×10^{12}	5.9×10^{13}	2020
$W^{\pm}$	3×10^{-25}	4.8×10^{12}	1.2×10^{12}	83×10^3
Z^0	-//-	4.2×10^{12}	1.08×10^{12}	93×10^3

5 STOPPING THE INSTABILITY FOR ELECTRICALLY CHARGED PARTICLES: ELECTROMAGNETIC TRANSITIONS AND PHOTOIONIZATION

Of course, we do not taken into account the annihilation of $\pi^{\pm}$, $K^{\pm}$, $D^{\pm}$ and $F^{\pm}$ during their generation and accumulation near BH. Particles of opposite charges may annihilate rapidly during their generation. But one must take into account the influence of a strong gravitational field near the horizon of a BH.

Let us assume that electrically charged particles and their antiparticles could accumulate on quasi-levels, for instance, on a 2P quasi-bound level in the field of a highly rotating BH ($a \rightarrow M$) of mass $M \leq 0.45 M_{\rm Pl}^2/\mu$, i. e. near the upper limit of Bose instability. In this case there are three processes which can stop the instability: (1) electromagnetic transitions $2p \rightarrow 1s$, or other transitions on non-superradiative levels; (2) annihilation of particles into two fotons ($\pi^+ + \pi^- \rightarrow 2\gamma$, $K^+ + K^- \rightarrow 2\gamma$ and so on); (3) photoionization of bound levels by the electromagnetic radiation emitted by the BH itself.

The equation which governs the number of particles on the 2P level for a doublet of charged particles is

$$\frac{dN_{2P}}{dt} = \lambda_{2p}(N_{2p} + 1) - W_{2p\rightarrow 1s} N_{2p} - \frac{1}{2} \times W_{\rm ann} N_{2p} - W_{\rm ion} N_{2p}^2. \qquad (26)$$

It is not difficult to calculate the probability of normal dipole transitions $2p \rightarrow 1s$ in the field of such a BH assuming that a hydrogen-like spectrum of bound states is achieved. One could note that there are also anomalous transitions with $\Delta m = 0$ which may have the same order of magnitude in the field of an extremely rotating BH (Gaina, 1992), but we do not examine such transitions here. The probability $W_{2p\rightarrow 1s}$ will be:

$$W_{2p\rightarrow 1s} = \left(\frac{2}{3}\right)^8 \alpha\mu(\mu M)^4. \qquad (27)$$

Transitions (28) impose further restraints on the masses of BHs supposed to Bose instability:

$$M \geq^4 \sqrt{\frac{2^{10}\alpha}{3^7}} \frac{M_{\rm Pl}^2}{\mu} = 0.24 \frac{M_{\rm Pl}^2}{\mu} = 0.24 \frac{M_{\rm Pl}^2}{\mu}, \tag{28}$$

that is, the actual low limit mass for instability will be greater for $\pi^{\pm}$, $D^{\pm}$, $F^{\pm}$, $K^{\pm}$.

The total cross-section of annihilation of particles of opposite charges on the quasi-levels may also very easily be estimated in the non-relativistic limit:

$$\sigma_{\rm ann} = \frac{\alpha^2}{2\mu E^{(0)}} \approx \frac{\pi\alpha^2}{2\mu^2}.$$

Akhiezer and Berestetzki (1981) give an exact formula for the cross-section of ionization of 2P atomic levels. In the case of N 2P electrons one has:

$$\sigma_{2p} = N_{2p} \frac{2^{10}\pi^2\alpha}{9\mu I_{2p}} \left(\frac{I_{2p}}{\hbar\omega}\right)^5 \left(3 + 8\frac{I_{2p}}{\hbar\omega}\right) \frac{e^{-4\eta \arctan \frac{\eta}{2}}}{1 - \exp(-2\pi\eta)}, \tag{29}$$

where

$$\eta = \frac{Z_{\alpha c}}{\sqrt{1 - \frac{\mu^2}{\varepsilon^2}}}$$

where ε is the energy of the photoelectrons.

In the limit $\hbar\omega \gg I_{2p}$ one has approximately:

$$\sigma_{2p} \simeq \frac{2^8\alpha\pi}{3\mu I_{2p}} \left(\frac{I_{2p}}{\omega}\right)^{9/2} N_{2p}.$$

Adapting this formula for a BH we obtain the cross-section of photoionization of one particle on the 2P level ($Z\alpha \to \mu M/M_{\rm Pl}^2$) :

$$\sigma_{2p} = \frac{\pi\alpha(\lambda_c')^2}{12\sqrt{2}} \left(\frac{\mu}{\omega}\right)^{9/2} (\mu M)^7 N_{2p}. \tag{30}$$

An important feature of the cross-section (31) is the dependence of the rate of ionization on the number of particles on the level.

Let us calculate the probability of ionization of one electrically charged 2P scalar particle by electromagnetic radiation emitted by the BH itself as a result of superradiance. The probability will be:

$$W_{2p}^{(\rm ion)} = \frac{1}{S} \int \frac{dn_p}{dt\, dw} \sigma_{2p}\, dw, \tag{31}$$

where

$$\frac{dn_p}{dt\, dw} = \frac{1}{2\pi}\langle n \rangle = \frac{\Gamma_{1\omega 1mp}}{2\pi} = \frac{4}{9}\frac{8\pi M r_+}{2\pi^2} M^2(\omega - \Omega_H)\omega^3 \tag{32}$$

is the number of p-photons emitted by the BH. In the limit $a \to M$ one has:

$$\frac{dn_p}{dt\, dw} = \frac{8M^3\omega^3}{9\pi}. \tag{33}$$

After the integration of equation (32) with $S = a_0^2 = \frac{1}{\mu^4 M^2}$ one has:

$$W_{2p}^{(\mathrm{ion})} = \frac{8\alpha}{81\pi}\mu(\mu M)^{11} \tag{34}$$

For N_{2p} particles localized on the 2P level this formula must be multiplied by N_{2p}. Thus, photoionization is slow compared with dipole transitions for small occupation numbers, but may suppress the last ones for great N_{2p}. It is easy to estimate the number of particles on the 2P level after photoionization ignoring the dipole transitions and annihilation of pairs. Equation (29) will have the form:

$$\frac{dN_{2p}}{dt} = \lambda_{2p}(N_{2p}+1) - W_{2p}^{(\mathrm{ion})} N_{2p}^2. \tag{35}$$

Assuming $N_{2p} \gg 1$ one obtains for the equilibrium number of particles:

$$N_{2p} = \frac{\lambda_{2p}}{W_{2p}^{(\mathrm{ion})}} = \frac{81\pi}{96\alpha}(\mu M)^{-3}, \tag{36}$$

that is, for a BH with incipient instability ($\mu M \approx 0,45$) one derives

$$N_{2p} = 3985.$$

So, photoionization stops Bose instability efficiently for electrically charged mesons $\pi^{\pm}$, $K^{\pm}$, $D^{\pm}$, $F^{\pm}$.

6 CONCLUSIONS

Self-stimulated generation and accumulation of bosons in the fields of highly rotating BHs (Bose instability) is an efficient mechanism of lowering angular momentum for primordial BHs. Electrically charged particles cannot accumulate near the BH due to electromagnetic transitions, annihilations and photoionization. However, π^0, D^0, K^0 can rapidly accumulate near BHs and produce γ bursts. η could also produce γ bursts. The powers of bursts will be

$$\left(\frac{dE}{dt}\right)_{\mathrm{int}} \simeq \frac{1}{288}\frac{c^5}{G}\left(\frac{\mu M}{M_{\mathrm{Pl}}^2}\right)^{10}\frac{1}{\ln\frac{M}{M_{\mathrm{Pl}}}},$$

that is, $\simeq 9.5\times 10^{51}$ erg s^{-1} (for π^0), 2.9×10^{52} erg s^{-1} (for η), 10^{52} erg s^{-1} (for D^0), if one assumes the BH to be near the threshold of instability (i. e. $\mu M \approx 0.45 M_{\mathrm{Pl}}^2$). This corresponds to effective masses of radiated energies of the order of magnitude 100 Mt. The energies of γ-photons radiated will be 67.5 MeV (for π^0), 274.5 MeV (for η), 932 MeV (for D^0).

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