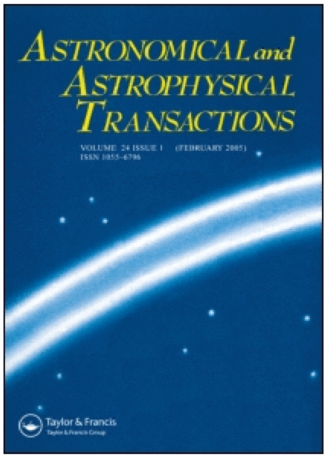

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Dispersion of electromagnetic waves in active moving plasma

E. Lopez [ab]
[a] Central Astronomical Observatory at Pulkovo, St. Petersburg, Russia
[b] Escuela Politecnica Nacional, Quito, Ecuador

Online Publication Date: 01 August 1997
To cite this Article: Lopez, E. (1997) 'Dispersion of electromagnetic waves in active moving plasma', Astronomical & Astrophysical Transactions, 14:1, 79 - 85
To link to this article: DOI: 10.1080/10556799708213574
URL: http://dx.doi.org/10.1080/10556799708213574

Astronomical and Astrophysical Transactions, 1997,
Vol. 14, pp. 79–85
Reprints available directly from the publisher
Photocopying permitted by license only

DISPERSION OF ELECTROMAGNETIC WAVES IN ACTIVE MOVING PLASMA

E. LOPEZ

Central Astronomical Observatory at Pulkovo, 196140, St. Petersburg, Russia, and Escuela Politecnica Nacional, 22650, Quito, Ecuador

(Received November 3, 1995)

Observable radiation contains information on its source as well as on the medium through which it has propagated. Measuring the angular broadening suffered by crossing radiation through any magnetoactive plasma, it is possible to obtain important conclusions about the physical plasma parameters, such as the electron density distribution, magnetic field and macroscopic plasma velocity. In this work we describe the physical picture for propagation of the electromagnetic waves in the magnetoactive plasma, taking into account the macroscopic velocity of the system. In moving magnetoactive plasma, the relations for the dielectric permittivity tensor, the dispersion equations and the refractive indexes are established. These results can be evaluated, for example, by the determination of the angular displacement of the radiation coming to us from far radio sources, after passing through solar corona and solar wind plasmas.

KEY WORDS Refractive index, movement of the system, magnetoactive plasma, invariant method, dielectric tensor of permittivity

1 INTRODUCTION

The refraction scenery of electromagnetic waves in a medium, where a gradient is present in the refractive index, has been studied by Wright and Nelson (1979), de Pater and Ip (1984). These authors considered the spatial variation in the electron density and the influence of magnetic field was neglected. In other works, for example, by Tim Bastian (1995), the electron density fluctuations in the solar corona were used to analyse the problem of scattering of radio waves in the solar corona in the small-angle scattering limit. The inclusion of the magnetic field to describe the anomalous refraction, was carried out in our previous paper (Gnedin and Lopez, 1995). In that work the simultaneous action of gradients in the electron density and magnetic field was used for the determination of the dispersion relations and angular displacement of electromagnetic waves crossing magnetoactive plasma.

The present work is devoted to determining how the macroscopic movement of the dispersive system alters the usual refraction process of electromagnetic waves in magnetoactive plasma. The influence of the global velocity, in the case when isotropic plasma lessens the external magnetic field, was analysed and the expressions for the dispersion equation and the refraction index have been established (Lopez, 1995). In the case of magnetoactive plasma, if using a similar mathematical procedure as in the isotropic case, it is possible to obtain the dispersion equation for the system and determine the plausible angular deviation in crossing electromagmetic radiation. In moving plasma some physical alterations occur due to macroscopic velocity, for instance an isotropic plasma becomes anisotropic and an anisotropic one acquires additional anisotropies of the higher order. The anisotropies of the system are contained in the dielectric permittivity tensor. The dielectric tensor of permittivity plays a big role in the theory of wave propagation. This tensor contains very important information about the optical properties of plasma. The appearance of anisotropies in the medium is described as changes in the components of the permittivity tensor. Generally, these changes are small but important because they bring qualitative modifications to the optical properties of the system and, consequently, to the propagation of the waves.

In this paper, taking into account the global movement of plasma, the general expressions for the dielectric tensor of permittivity are given ($\epsilon_{ij}(w, \mathbf{k})$), the dispersion equation and relations for the refractive indexes in magnetoactive plasma are deduced. The mathematical complications introduced by the employment of a defined coordinate system, are much reduced using the tensor invariant representation. The inverse dielectric permittivity tensor is presented in this invariant representation and in this way the dispersion equation is obtained. This method gives a simple procedure to find the solutions from the dispersion equation and reveals physical characteristics of the system, which are not seen often.

2 THE INVARIANT REPRESENTATION IN A MOVING MEDIUM

The relationship between the elements of the permittivity tensor in the different coordinate systems is not so difficult to deduce from the basic equation of electrodynamics and can be expressed by the following expression:

$$\begin{aligned}\epsilon_{ij} &= \delta_{ij}\left(1-\frac{w'^2}{w^2}\right)+\frac{w'^2}{w^2}\epsilon'_{ij}+\frac{w'^2}{w^2}\frac{v_i k_\mu \epsilon'_{\mu j}}{w'}-\frac{w'^2}{w^2}\frac{v_i k_j}{w'}+\frac{w'}{w}\frac{k_\nu v_j \epsilon'_{i\nu}}{w}-\frac{w'}{w}\frac{k_i v_j}{w} \\ &+ \frac{w'}{w}\frac{v_i v_j}{w' w}(k_\mu \epsilon'_{\mu\nu} k_\nu - k^2), \end{aligned} \tag{1}$$

where $\epsilon_{ij}(w, \mathbf{k})$ is the dielectric permittivity tensor in the laboratory system (xyz), $\epsilon'_{ij}(w', \mathbf{k}')$ is the dielectric permittivity tensor in the rest system $(x'y'z')$, w is the radiation frequency in the system xyz, w' is the radiation frequency in the system $x'y'z'$, $\mathbf{k}$ is the wave vector in the system xyz, $\mathbf{k}'$ is the wave vector in the system $x'y'z'$ and $v(x, y, z)$ is the plasma velocity.

The tensor ϵ'_{ij} for magnetoactive plasma, in the rest frame, should have the common form:

$$\epsilon' = \begin{pmatrix} \epsilon_\perp & ig & 0 \\ -ig & \epsilon_\perp & 0 \\ 0 & 0 & \epsilon_\parallel \end{pmatrix}, \tag{2}$$

where

$$\epsilon_\perp = 1 - \frac{w_p^2}{w^2 - w_b^2} \quad g = \frac{-w_p^2 w_b}{w(w^2 - w_b^2)} \quad \epsilon_\parallel = 1 - \frac{w_p^2}{w^2}$$

where w_p is the plasma frequency and w_b is the gyrofrequency. Putting equation (2) into equation (1), it is possible to find the most general expression for the dielectric tensor of permittivity in moving plasma under the influence of an external magnetic field. Components of this tensor can be written as:

$$\epsilon_{xx} = 1 + \frac{w'^2}{w^2}(\epsilon_\perp - 1) + 2\frac{w'^2}{w^2}\frac{v_x k_x}{w'}(\epsilon_\perp - 1) + \frac{w'^2}{w^2}\frac{v_x^2}{w'^2}(\epsilon_\perp k_\perp^2 + \epsilon_\parallel k_z^2 - k^2), \tag{3}$$

$$\epsilon_{yy} = 1 + \frac{w'^2}{w^2}(\epsilon_\perp - 1) + 2\frac{w'^2}{w^2}\frac{v_y k_y}{w'}(\epsilon_\perp - 1) + \frac{w'^2}{w^2}\frac{v_y^2}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2], \tag{4}$$

$$\epsilon_{zz} = 1 + \frac{w'^2}{w^2}(\epsilon_\parallel - 1) + 2\frac{w'^2}{w^2}\frac{v_z k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'^2}{w^2}\frac{v_z^2}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2], \tag{5}$$

$$\begin{aligned} \epsilon_{xy} &= \frac{w'^2}{w^2}ig + \frac{w'^2}{w^2}\frac{ig}{w'}(v_x k_x + v_y k_y) + \frac{w'^2}{w^2}\frac{(\epsilon_\perp - 1)}{w'}(v_x k_y + v_y k_x) \\ &+ \frac{w'^2}{w^2}\frac{v_x v_y}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2], \end{aligned} \tag{6}$$

$$\begin{aligned} \epsilon_{yx} &= -\frac{w'^2}{w^2}ig - \frac{w'^2}{w^2}\frac{ig}{w'}(v_x k_x + v_y k_y) + \frac{w'^2}{w^2}\frac{(\epsilon_\perp - 1)}{w'}(v_x k_y + v_y k_x) \\ &+ \frac{w'2}{w^2}\frac{v_x v_y}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2], \end{aligned} \tag{7}$$

$$\begin{aligned} \epsilon_{xz} &= \frac{w'^2}{w^2}\frac{v_x k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'}{w}\frac{k_x v_z}{w}(\epsilon_\perp - 1) + ig\frac{w'}{w}\frac{v_z k_y}{w} \\ &+ \frac{w'^2}{w^2}\frac{v_x v_z}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2], \end{aligned} \tag{8}$$

$$\begin{aligned} \epsilon_{zx} &= \frac{w'^2}{w^2}\frac{v_x k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'}{w}\frac{k_x v_z}{w}(\epsilon_\perp - 1) - ig\frac{w'}{w}\frac{v_z k_y}{w} \\ &+ \frac{w'^2}{w^2}\frac{v_x v_z}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2], \end{aligned} \tag{9}$$

$$\begin{aligned}\epsilon_{yz} &= \frac{w'^2}{w^2}\frac{v_y k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'}{w}\frac{k_y v_z}{w}(\epsilon_\perp - 1) - ig\frac{w'}{w}\frac{v_z k_x}{w} \\ &+ \frac{w'^2}{w^2}\frac{v_y v_z}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2],\end{aligned} \tag{10}$$

$$\begin{aligned}\epsilon_{zy} &= \frac{w'^2}{w^2}\frac{v_y k_z}{w'}(\epsilon_\parallel - 1) + \frac{w'}{w}\frac{k_y v_z}{w}(\epsilon_\perp - 1) + ig\frac{w'}{w}\frac{v_z k_x}{w} \\ &+ \frac{w'^2}{w^2}\frac{v_y v_z}{w'^2}[(k_x^2 + k_y^2)\epsilon_\perp + k_z^2\epsilon_\parallel - k^2].\end{aligned} \tag{11}$$

As can be seen from the previous results, the anisotropies of the system become more complex than in the rest frame due to plasma displacement. Moreover, the dielectric tensor of permittivity in the laboratory coordinate system forms a hermitic matrix ($\epsilon = \epsilon^H$). Then, we try to transform the tensor ϵ^{-1} and write it in the invariant representation. Proceeding this way and using the diada operation, the inverse dielectric tensor of permittivity is written as (Feodorov, 1956):

$$\epsilon^{-1} = a + b(C_1\, C_2^* + C_2\, C_1^*) \tag{12}$$

where

$$C_1 = \left(\frac{a_1}{\sqrt{2}}, -i\frac{a_1}{\sqrt{2}}, a_3\right) \quad C_2 = \left(-\frac{a_1}{\sqrt{2}}, i\frac{a_1}{\sqrt{2}}, a_3\right),$$

$$a_1 = \sqrt{\frac{\alpha_2 - \alpha_1}{\alpha_3 - \alpha_1}} \quad a_3 = \sqrt{\frac{\alpha_3 - \alpha_2}{\alpha_3 - \alpha_1}}.$$

$a = \alpha_2$, $b = (\alpha_3 - \alpha_1)/2$, C_1 and C_2 are the axes of tensor ϵ^{-1}, α_1, α_2, α_3 are the eigen values of inverse tensor ϵ^{-1}.

Further, assuming that the plasma velocity direction is along the x-axis, the magnetic field is located along the z-axis, and the propagation of the electromagnetic waves is taken to occur in the xz-plane, we have derived the eigen values and axes for the inverse dielectric tensor $\epsilon - 1$ written in the diada-representation (equation 12). We have carried out these calculations and have obtained the following results:

$$\epsilon^{-1} = a + b(C_1\, C_2^* + C_2\, C_1^*)$$

with

$$C_1 = \left(\frac{a_1}{\sqrt{1+A^2}}, i\frac{a_1 A}{\sqrt{1+A^2}}, a_3\right) \quad C_2 = \left(-\frac{a_1}{\sqrt{1+A^2}}, -i\frac{a_1}{\sqrt{1+A^2}}, a_3\right),$$

$$a_1 = \sqrt{\frac{\alpha_2 - \alpha_1}{\alpha_3 - \alpha_1}} \quad a_3 = \sqrt{\frac{\alpha_3 - \alpha_2}{\alpha_3 - \alpha_1}},$$

$$A = -\frac{\epsilon_{yy} - \epsilon_{xx} + \sqrt{(\epsilon_{yy} - \epsilon_{xx})^2 + 4G^2}}{2G},$$

$$G = \frac{w'^2}{w^2} g \left(1 + \frac{vk}{w'}\right) \quad B = \sqrt{(\epsilon_{yy} - \epsilon_{xx})^2 + 4G^2},$$

$$\alpha_1 = \frac{1}{2}(\epsilon_{xx} + \epsilon_{yy} + B) \quad \alpha_2 = \frac{1}{2}(\epsilon_{xx} + \epsilon_{yy} - B) \quad \alpha_3 = \epsilon_{zz}.$$

The eigen values and eigen vectors of the dielectric tensor ϵ^{-1} are as follows:

$$\epsilon^{-1}(C_1 - C_2) = \lambda_1(C_1 - C_2),$$

$$\lambda_1 = \alpha_2 - \frac{(\alpha_3 - \alpha_1)}{2}(1 - C_1 C_2^*),$$

$$\epsilon^{-1}(C_1 C_2) = \lambda_2(C_1 C_2),$$

$$\lambda_2 = \alpha_2,$$

$$\epsilon^{-1}(C_1 + C_2) = \lambda_3(C_1 + C_2),$$

$$\lambda_3 = \alpha_2 + \frac{\alpha_3 - \alpha_1}{2}(1 + C_1 C_2^*).$$

Working simultaneously with the general expression for Frenel's dispersion equation (Ginsburg, 1967) and with the dielectric tensor in its invariant representation equation (12), it is not difficult to deduce the relationship for the refractive index, expressed in the invariant representation:

$$\frac{1}{n_\pm^2} = a + b[(\mathbf{n}C_1)(\mathbf{n}C_2^*) \pm \sqrt{(\mathbf{n}C_1)^2(\mathbf{n}C_2)^2}], \tag{13}$$

the probable directions for the magnetic field in the electromagnetic waves will be defined by relations of the following type:

$$H_+ \| \left[\frac{(\mathbf{m}C_1)}{\sqrt{(\mathbf{m}C_1)^2}} + \frac{(\mathbf{m}C_2)}{\sqrt{(\mathbf{m}C_2)^2}}\right], \tag{14a}$$

$$H_- \| \left[\frac{(\mathbf{m}C_1)}{\sqrt{(\mathbf{m}C_1)^2}} - \frac{(\mathbf{m}C_2)}{\sqrt{(\mathbf{m}C_2)^2}}\right], \tag{14b}$$

where $\mathbf{m} = n\mathbf{n}$, n is the refractive index and $\mathbf{n}$ is the unitary vector along the path of the wave.

The previous expression gives general solutions to the dispersion equation for plasma which has two optical axes (concurrence of the refractive index along these directions).

3 DISPERSION IN MOVING PLASMA

Using equation (13) and the results from the last section, we have derived an expression for the refractive index in magnetoactive plasma:

$$\frac{1}{n_{\pm}^2} = a + b\left[-\frac{a_1^2 A^2}{(1+A^2)} - \frac{a_1^2}{(1+A^2)}n_z^2 + a_3^2 n_x^2 \right.$$
$$\left. \pm \sqrt{\left(\frac{a_1^2 A^2}{(1+A^2)} + \frac{a_1^2}{(1+A^2)}n_z^2 + a_3^2 n_x^2\right)^2 - \frac{A a_1^2 a_3^2 n_x^2 n_z^2}{(1+A^2)}}\right]. \quad (15)$$

With the assumption that the direction of the propagation of the wave makes a right angle to the magnetic field direction ($\theta = \frac{\pi}{2}$), the last equation becomes:

$$\frac{1}{n_+^2} = \alpha_3 = \frac{1}{\epsilon_{zz}} = 1 - \frac{w_p^2}{w^2} \quad (16a)$$

and

$$\frac{1}{n_-^2} = a - 2b\frac{a_1^2 A^2}{(1+A^2)}. \quad (16b)$$

The negative solution n_- after some simple transformations, acquires the following form:

$$w_b^2(n_-^2 - 1) + (n^2 w^2 + w_p^2 - w^2)\left[\frac{w_p^2}{w^2} - (1 - n_-\frac{v}{c})^2\right] = 0. \quad (17)$$

Taking into account the neglectably small value of the plasma velocity to the light velocity ratio ($v^2 \ll c^2$), this equation implies that:

$$2\frac{v}{c}n^3 - \left[1 - \frac{(w_b^2 + w_p^2)}{w^2}\right]n^2 - 2\frac{v}{c}\left[1 - \frac{w_p^2}{w^2}\right]n + \left[\left(1 - \frac{w_p^2}{w^2}\right)^2 - \frac{w_b^2}{w^2}\right] = 0. \quad (18)$$

If the magnetic field is neglected, equation (18) coincides with the expression for the isotropic moving plasma, already obtained (Lopez, 1995):

$$2\frac{v}{c}n^3 - \left(1 - \frac{w_p^2}{w^2}\right)n_-^2 - 2\frac{v}{c}\left(1 - \frac{w_p^2}{w^2}\right)n_- + \left(1 - \frac{w_p^2}{w^2}\right)^2 = 0.$$

The solution of equation (18), in the case of rare plasma when the refraction index has values near unity $n \approx 1$, can be written in the following form:

$$n_-^2 = 1 - \frac{w_p^2(1 - \frac{w_p^2}{w^2} - \frac{v}{c})}{w^2(1 - \frac{w_b^2}{w^2} - \frac{w_p^2}{w^2} - \frac{2v}{c})}. \quad (19)$$

For longitudinal propagation along the external magnetic field ($\theta = 0$), our calculations for the refraction index are:

$$n_+^2 = 1 - \frac{w_p^2}{w(w + w_b)}$$

$$n_-^2 = 1 - \frac{w_p^2}{w(w - w_b)}, \tag{20}$$

this direction represents an optical axis. The general motion of plasma has no influence on the propagation of the electromagnetic waves, the wave is refracted according to the ordinary refraction law.

We have derived equations which describe the propagation of electromagnetic waves in moving plasma. It is possible to apply the present results for determination of the angular displacement in the position of radio sources, when radiation passes through any moving plasma located in the path of waves between the source and observer. Measuring the angular displacement with the help of modern techniques of interferometry, the spectra for the electron density, magnetic field and plasma velocity, can be established. In this direction, we have made some estimations and have calculated the angular deviation for the radiation coming from radio sources and crossing through the magnetoactive plasmas in the solar corona and solar wind. These results will be presented in a subsequent paper which is being prepared.

References

Bastian, T. S. (1995) *Astrophys. J.* **426**, 774.
de Pater, I. and Ip, W.-H. (1984) *Astrophys. J.* **283**, 895.
Feodorov, F. I. (1956) *Opt. Spectrosc.* **1**, 926.
Ginsburg, V. L. (1967) *Propagation of Electromagnetic Waves in Plasma*, Moscow, Nauka (in Russian).
Gnedin, Y. N. and Lopez, E. D. (1995) *Sol. Phys.* (in press).
Lopez, E. D. (1995) *Radiofizika* (in press).
Wright, C. S. and Nelson, G. J. (1979) *Icarus* **38**, 123.