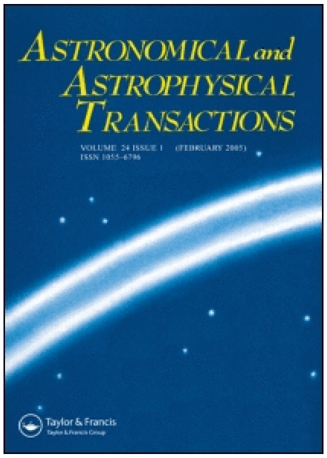

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Formation and stability of the galactic shock waves

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Session 6 : Stellar Evolution and Normal Galaxies

FORMATION AND STABILITY OF THE GALACTIC SHOCK WAVES

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The stability of the shocked and the shock-free gas flows in the gaseous galactic disc is discussed. It is shown that the periodic shock-free supersonic flow between the spiral gravitational potential wells is unstable due to the parametric instability and thus should be re-organized into the shocked flow. The steady-state shock at the rear side of the well (relative to the flow) is subject to the local instability. This instability is suppressed if the shock is at the front side of the well.

KEY WORDS **Hydrodynamic instability, shock waves, galaxy dynamics**

Each element of a gas, orbiting the centre of a flat spiral galaxy, periodically crosses the spiral arms and experiences the periodic variation of the gravitational field. Since gas hits the arms supersonically, shock waves may form (and do form). It is widely believed that a galactic shock wave arises if the strength of the spiral gravitational field exceeds a certain critical value (Roberts, 1969; Shu *et al.*, 1973). For a weaker field the flow should remain smooth. In what follows I try to cast some doubt on the truth of this commonly accepted opinion. Using simple arguments I show that a smooth flow needs only to be periodic and supersonic to be unstable.

Let us consider an evolution of the small-amplitude perturbations in a periodic gas flow via the gravitational potential wells. In the case where the shock-free flow is dynamically unstable, the flow must reconstruct itself, so it is likely that shocks will be formed. To simplify the analysis I neglect the effects of multi-dimensionality and rotation, and consider 1-D, planar, steady-state, undisturbed perfect gas flow, described by function $\mathbf{f}(x) = [\rho_0(x), \mathbf{v}_0(x), \mathbf{p}_0(x)]$; the notations are standard. The undisturbed flow is spatially periodic, that is, $\mathbf{f}(x) = \mathbf{f}(x+a)$, as well as the potential $\Psi(x) = \Psi(x+a)$. The linear perturbation is 3-D and can be written as $\delta\mathbf{f} = [\delta\rho(x), \delta\mathbf{v}(x), \delta\tilde{p}(x)]e^{-i\omega t + iky}$, where y is a coordinate transverse to the flow direction. Solving five coupled linearized hydrodynamic equations with periodic boundary conditions $\delta\mathbf{f}(0) = \delta\mathbf{f}(a)$, one finds the wave function $\delta\mathbf{f}(x)$ and an eigenvalue ω, which is in general complex.

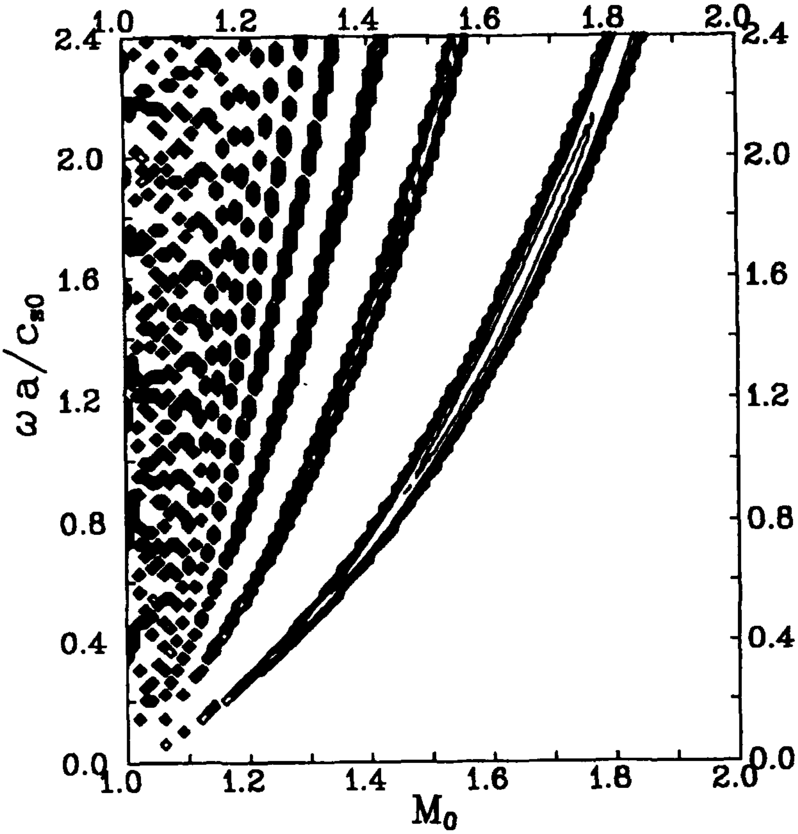

Figure 1 Contours of constant maximum transition coefficient $|q|$. Contour levels extend from $|q| = 1.01$ with increments of 0.05. Coefficients are calculated as a grid of 60 X 80. The left-hand side of the plot suffers insufficient resolution.

To clarify the physical nature of the instability I shall delay for the present the particular solution to the boundary problem and consider the general solution for the infinite sequence of wells. Suppose the perturbation before the nth well can be expanded in five normal modes — two adiabatic modes, two vortex modes, and an entropic one, with amplitudes $\mathbf{c_n} = (c_1, \ldots, c_5)_n$. The amplitudes of waves emerging from the well are related linearly to those entering the well: $\mathbf{c_{n+1}} = \hat{\mathbf{B}}\mathbf{c_n}$, where $\hat{\mathbf{B}}$ is the transition matrix. The solution to this algebraic equation can be expressed as a superposition of exponents, that is, $\mathbf{c_n} = \sum_{i=1}^{5} \mathbf{A_i} q_i^n$, $n = 0, 1, 2, \ldots$, where $\mathbf{A_i}$ are some amplitudes and q_i are the transition coefficients. The perturbation will increase with n if there is any $|q_j| > 1$.

One can easily show that three of $|q_i|$ are identically unity (they meet the interaction of entropic and vortex modes with the acoustic ones), whereas the other two $|q_i|$ may differ from 1 but cannot be both concurrently less than 1.

Figure 1 presents a contour plot of the maximum absolute value of $|q_i|$ with frequency on the vertical axis and inflow Mach number M_0 on the horizontal axis. The contours are spaced between 1.01 and the maximum value 1.4 with an increment of 0.05. The well was chosen in the form of

$$\Psi(x) = \Psi_0 \sin^2\left(\frac{x\pi}{a}\right), \quad \Psi_0/c_{s0}^2 = -2, \tag{1}$$

where c_{s0} is the sound speed of the inflow. Gas is taken as monatomic ($\gamma = 5/3$) and the normal impingement of the waves ($k = 0$) is assumed.

As seen, the coefficient $|q|_{\max}$ equals 1 everywhere, except in narrow bands (where $|q|_{\max} > 1$) issuing from the centre of coordinates. Here, an astute reader is certain to recognize them as the well-known Bragg zones of opacity for the waves in periodic structures. Within these zones there is a close coupling between two acoustic modes, one of which has a negative energy. The energy exchange between the modes leads to the concurrent growth of their amplitudes and hence to instability. Remarkably, that instability exists at given $M_0 > 1$.

Turning back to the boundary problem one can roughly estimate the corresponding dimensional increments of instability as

$$\mathrm{Im}\,\omega \approx \frac{(M_0 - 1)c_{s0}}{a} \ln |q|_{\max}.$$

If we take $|q|_{\max} = 1.4$ as is in Figure 1, we find that amplitude increases twice after one revolution in a bisymmetrical system.

The plot does not change significantly with varying k and $\Psi(x)$, and only $|q|_{\max}$ falls to 1 with $\Psi \to 0$.

These simple arguments appear to indicate the absence of a critical potential for the shock wave formation, in contrast to the standard concept (Roberts, 1969). The transition of linear perturbations to the non-linear regime and formation of shocks should occur with necessity, though this process may take more time the weaker the deviations of the potential are. To give more exact quantitative estimates multidimensional calculations are desirable.

Another problem that merits detailed study is the dynamic stability of the galactic shock waves. Of the known instabilities for shock waves two types stand out as being much more studied — the instability of accelerating shock waves and thermodynamic instabilities caused by the specific equation of state of gas.

Fridman and Khoruzhij (private communication, 1993) proposed a new resonance mechanism of instability in the inhomogeneous flow. Their idea is as follows. Suppose a steady-state shock wave is in the potential well (Figure 2). A sound wave i_0 hits the shock surface; the front generates a decayed reflected sound wave r_0 along with the entropic and vortex waves (as is known, a shock wave in a homogeneous flow is stable (D'jakov, 1954; Erpenbeck, 1962) and, particularly, reflects the acoustic waves falling from behind with decay); a part of the sound wave leaves the well as a transmitted wave t_0, and it is reflected partially back as the wave i_1. Since only one outgoing wave is far from the well, it looks like a spontaneous radiation of sound by the shock. The hypothetical FK instability

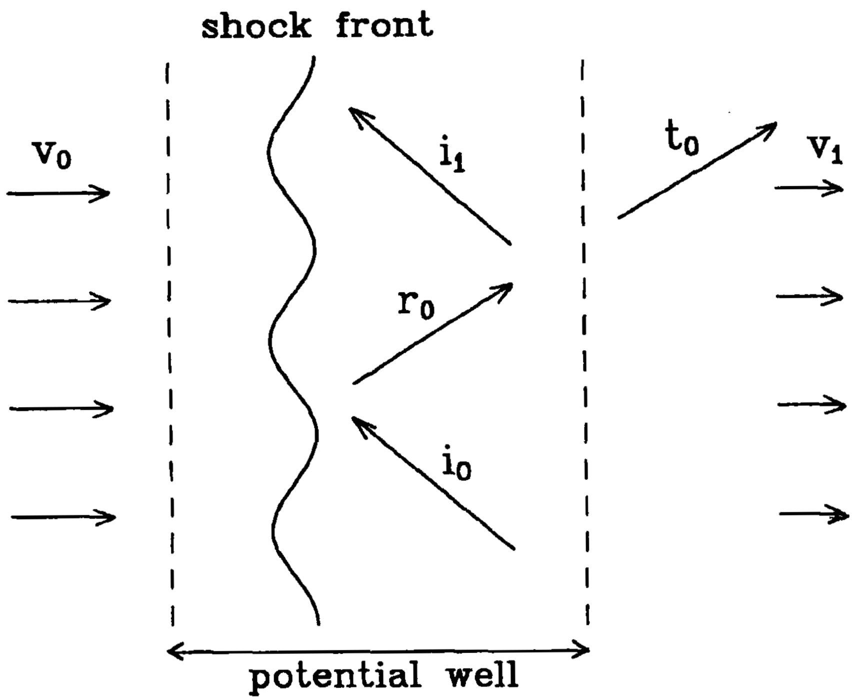

Figure 2 Schematic illustration of multiple wave reflections within the potential well.

rests on the assumption that the secondary reflection at the rear side of the well proceeds with amplification due to the interplay between the acoustic and entropic waves.

To verify the FK hypothesis I carried out a linear analysis for the steady-state shocked flow through the potential well. The primary purpose of stability analysis is finding the intrinsic modes of the shock front, that is, waves radiating by front (strictly speaking, by the system "front-well"). It is convenient, however, to solve a more general problem of front response to the perturbations incident to it. It happens that ignoring the secondary reflections in the post-shock flow allows us to reduce the problem to the analytically soluble one.

The most interesting and, perhaps unexpected, finding was that the shock front amplifies reflected sound waves in the case where pre-shock pressure gradient coincides with the direction of flow

$$\nabla p_0 \cdot \mathbf{v}_0 > 0, \tag{2}$$

and damps otherwise. This happens because the shock front "feels" the inhomogeneity of the flow and thereby the reflection coefficient at the front differs from that for the homogeneous case. Within the region of frequencies, where the overreflection takes place, there always exists a root, say ω_∞, at which the reflection coefficient

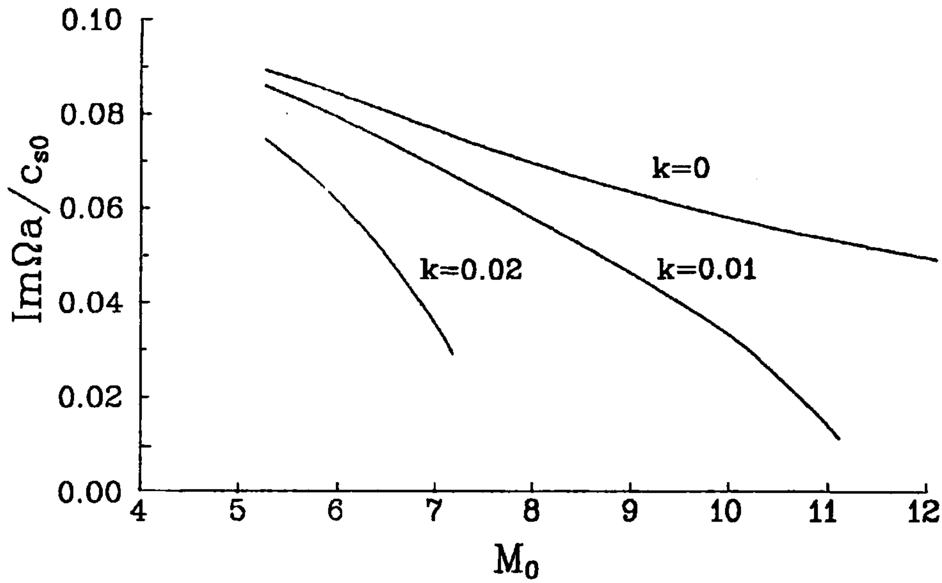

Figure 3 Increment of instability as a function of the Mach number shown for the perturbations with different wavenumbers k. The amplitude of the well is $\Psi/c_{s0}^2 = -1$. The shock front is located at the rear side of the well, potential gradient in this point is $\Psi' a/c_{s0}^2 = 0.17$.

turns to infinity. The existence of this root means that the shock would be unstable even if there were no secondary reflections in the inhomogeneous post-shock flow, and so resonance is not an essential feature of this instability.

Numerical integration, allowing for the secondary reflection, gives the exact root with frequency close to ω_∞. For a few oblique sound waves this frequency is purely imaginary. The condition for instability (1) is fulfilled at the rear side of the well. The corresponding increments for the modes with different pre-shock Mach numbers M_0 are displayed in Figure 3. These curves show that the instability primarily manifests itself as a growing displacement of the front (the mode $k = 0$), so that the front moves away leaving a shock-free flow in the well. The modes rippling the front would be less pronounced. At the same time for the front standing at the front side of the well the condition for overreflection is not fulfilled and no unstable modes were found numerically. The results found are in perfect agreement with numerical hydrodynamic experiments (Kovalenko and Levy, 1992). A more detailed description of the results will be given in (Kovalenko and Lukin, 1996).

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