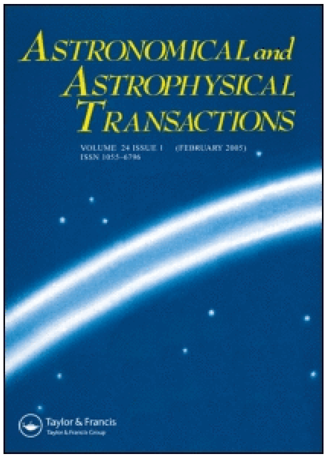

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Nonlinear dynamo waves in the incompressible medium when the dissipative coefficients depend on temperature

A. D. Pataraya [a]; T. A. Pataraya [b]
[a] Abastumany Astrophysical Observatory, Tbilisi, Republic of Georgia
[b] Tbilisi State University, Tbilisi, Republic of Georgia

Online Publication Date: 01 July 1997
To cite this Article: Pataraya, A. D. and Pataraya, T. A. (1997) 'Nonlinear dynamo waves in the incompressible medium when the dissipative coefficients depend on temperature', Astronomical & Astrophysical Transactions, 13:2, 127 - 135
To link to this article: DOI: 10.1080/10556799708208122
URL: http://dx.doi.org/10.1080/10556799708208122

Astronomical and Astrophysical Transactions, 1997,
Vol. 13, pp. 127–135
Reprints available directly from the publisher
Photocopying permitted by license only

NONLINEAR DYNAMO WAVES IN THE INCOMPRESSIBLE MEDIUM WHEN THE DISSIPATIVE COEFFICIENTS DEPEND ON TEMPERATURE

A. D. PATARAYA[1] and T. A. PATARAYA[2]

[1] *Abastumany Astrophysical Observatory, Kazbegi Str. 2., Tbilisi, 380060, Republic of Georgia*

[2] *Tbilisi State University, Tbilisi, 380028, Republic of Georgia*

(Received December 8, 1995)

The nonlinear α–ω dynamo waves existing in the incompressible medium in the case when the dissipative coefficients of turbulence depend on the temperature is studied in this paper. The latter incorporates investigation of α–ω solar nonlinear dynamo waves when only the first harmonics of magnetic induction components are included, for the velocity the only second harmonics are presented for pressure zero. In general, α–ω nonlinear dynamo waves exist during a definite period of time, this period depends on the dynamo number and other parameters of the medium. The amplitude of the second harmonics of the velocity and pressure are sufficiently small to be ignored. In the case when we ignore the second harmonics in the nonlinear equation, the turbulence magnetic diffusion coefficient increases with the temperature while the coefficient of turbulence viscosity reduces, and during the definite interval of time the value of the dynamo number is greater than 1. Under these conditions the stationary solution for the nonlinear equation for the dynamo wave's amplitude exists. This means that the magnetic field excites sufficiently. The amplitude of the dynamo waves moves oscillatorily and reaches a stationary state. Using these results we can explain the existence of Maunder's minimum.

KEY WORDS Nonlinear α–ω dynamo waves, turbulence dissipative coefficients, stationary solution, excitement

1 INTRODUCTION

It is known (Parker, 1979; Priest, 1982), that to excite (α–ω) dynamo waves it is necessary to take into the account the coefficient of magnetic diffusion turbulence η, which is of the order of $\eta = vL$, where L is the moving length. This magnitude is equal to the local scale of altitude for the solar convection zone and is proportional to temperature. Accordingly we can assume that $\eta = \eta_0(T/T_0)^{n_1}$, where T and T_0 are the temperature of the excited and the unexcited medium. Analogously, we can consider that for the kinematics of the viscosity $v = v_0(T/T_0)^{n_2}$, for the coefficient of temperature conductivity $\chi = \chi_0(T/T_0)^{n_3}$ and for the $\alpha = \alpha_0(T/T_0)^{n_4}$. In case the plasma is entirely un-inductive and non-turbulent $n_1 = -1.5$; $n_2 = 2.5$; $n_3 = 2.5$. The aim of this work is to solve the problem of the nonlinear dynamo and waves in the incompressible medium when the dissipative coefficients of turbulence depend on the temperature. The calculations are done in the local bartesian coordinate system with the origin at the centre of the sun.

In the first section the nonlinear equations for dynamo waves are given and analytical and numerical investigations of the equations are performed. In the second section the adopted results for the Sun are discussed.

2 INVESTIGATION OF THE NONLINEAR DYNAMO WAVE EQUATIONS

We investigate dynamo waves with the help of magnetohydrodynamic equations in the present paper. We assume that the medium is uncompressed and conductive, the turbulence is dissipative and coefficients depend on temperature. The calculations are done in the local bartesian orthogonal stationary coordinate system with the origin at the centre of the Sun. The z-axis is directed locally orthogonal to the solar surface, y is directed to the North pole, locally placed along the tangent of the meridian and the x-axis direction is a long the West (toroidally).

The main equation of induction is given in Priest (1982):

$$\partial \mathbf{B}/\partial t + (\mathbf{v}\nabla)\mathbf{B} = (\mathbf{B}\nabla)\mathbf{v} - \operatorname{curl}(\eta \operatorname{curl}\mathbf{B}) + \operatorname{curl}(\alpha B_x \mathbf{i}_x). \tag{1}$$

The second one is the equation of continuity:

$$\operatorname{div}\mathbf{v} = 0, \quad \rho = \text{const}. \tag{2}$$

The next is the equation of motion:

$$\frac{\partial \operatorname{curl}\mathbf{V}}{\partial t} + \operatorname{curl}[\operatorname{curl}\mathbf{v}, \mathbf{v}] = 1/4\pi\rho(\operatorname{curl}[\operatorname{curl}\mathbf{B}, \mathbf{B}] + \operatorname{curl}\mathbf{F}). \tag{3}$$

The last equation is an equation of energy:

$$\partial p/\partial t + (\mathbf{v}\nabla)p = 1/4\pi(\gamma - 1)\eta(\operatorname{curl}\mathbf{B})^2 + (\gamma - 1)\operatorname{div}(\chi\nabla p) + (\gamma - 1)H. \tag{4}$$

Here $\mathbf{B}$ and $\mathbf{v}$ are the vectors of magnetic induction and velocity respectively, ρ is density, p is pressure, γ is a relative heat, $\mathbf{i}_x$ is the unit vector along the x-axis, B_x is the x component of the magnetic induction. The last equation is the equation of state of an ideal gas $p = (R/\mu)\rho T$, where T is temperature, R is the constant, μ is the average atomic mass, χ is a coefficient of thermal conductivity.

In the equations (3) and (4) $\mathbf{F}$ and H are the force of viscosity and effective viscosity dissipation respectively. From Priest (1982) we can consider:

$$F_j = \rho \sum_{k=1}^{3} \frac{\partial}{\partial \mathbf{X}_k}\left(\mathrm{v}\frac{\partial \mathbf{V}_i}{\partial \mathbf{X}_k}\right), \tag{5}$$

$$H = \rho \mathrm{v} \frac{1}{2} \sum_{i,k=1}^{3} \left(\frac{\partial \mathbf{V}_i}{\partial \mathbf{X}_k} + \frac{\partial \mathbf{V}_k}{\partial \mathbf{X}_i} \right) \tag{6}$$

where $x_1 = x$, $x_2 = y$, $x_3 = z$, $v_1 = v_x$, $v_2 = v_y$, $v_3 = v_z$.

The coefficients η, v χ, γ depend on temperature (or on pressure as $T/T_0 = (p/p_0)$:

$$\eta = \eta_0 (p/p_0)^{n_1} \tag{7}$$

$$\mathrm{v} = \mathrm{v}_0 (p/p_0)^{n_2}, \quad \chi = \chi_0 (p/p_0)^{n_3} \tag{8}$$

$$\alpha = \alpha_0 (p/p_0)^{n_4}. \tag{9}$$

Here η_0, v_0, χ_0, α_0 are constant values.

The equations (1–4) can be solved with the help of perturbation theory. All the functions take the following form: $f = f_0 + f_1'$. Here f_0 is the unperturbed term and f_1' is the perturbed function. Let us consider, that the unperturbed density $\rho = \rho_0$, pressure p_0, the unperturbed quantity of the magnetic field is equal to zero and the velocity v_0 has only x components:

$\mathbf{v}_0 = (v_{x0} + v_{xy} y + v_{xz} z)\mathbf{i}_x$, where v_{x0}, v_{xy}, v_{xz} are constant quantities.

In accordance with the equation (4) we investigate the non-perturbed state (gravity is ignored):

$$\partial p_0/\partial t + (\mathbf{v}_0 \nabla) p_0 = \mathrm{div}(\chi_0 \nabla p_0) + (\gamma - 1) H_0, \tag{10}$$

where

$$H_0 = \rho_0 \mathrm{v}_0 (v_{xy}^2 + v_{xz}^2). \tag{11}$$

Let estimate the characteristic length L and time t_x when the non-perturbed pressure is changed. The estimation is done in the convected region, where $p_S = 8 \times 10^{11}$ din cm^{-1}, $\rho_0 = 10^{-2}$ gr cm^{-3}, $\chi \cong \mathrm{v}_0 = 10^{12}$ cm^2 s^{-1}, $|V_{xy}| \cong |V_{xz}| \cong \Omega$, Ω is the frequency of the Sun's rotation, $|V_{xy}| R_Q \leq 10^{10}$ cm s^{-1}, R_Q is the radius of the Sun.

First of all we will determine the characteristic length L. If $\partial p_0/\partial t = 0$, $|\nabla p_0| \cong p_0/L$. In accordance with equation (11) we will get: $L = 10^6 R_Q$. When $(\nabla p_0) = 0$, and $\partial p_0/\partial t \cong p_0/t_x$, $t_x \cong 10^5 t_0$ ($t_0 = 22$ year). In this conditions we can suggest that p_0 depends weakly on time and coordinates. In this case we can use p_0 as a constant in the equations for perturbed quantities.

For the perturbed terms of the equations (1–4) we will get:

$$\begin{aligned} \partial \mathbf{B}'/\partial t + (\mathbf{v}_0 \nabla)\mathbf{B}' &= -(\mathbf{v}'\nabla)\mathbf{B}' + (\mathbf{B}'\nabla)\mathbf{v}' + \eta_0 \mathrm{curl}\,[(1 + p'/p)^{n_1} \mathrm{curl}\,\mathbf{B}'] \\ &+ \mathrm{curl}\,[\alpha_0 (1 + p/p_0)^{n_4} B_x \mathbf{i} \mathbf{S}_x]; \end{aligned} \tag{12}$$

$$\begin{aligned} \partial(\mathrm{curl}\,\mathbf{v}'/\partial t + (\mathbf{v}_0 \nabla)\mathrm{curl}\,\mathbf{v}' &= (\mathrm{curl}\,\mathbf{v}_0 \nabla)\mathbf{v}' + (\mathrm{curl}\,\mathbf{v}'\nabla)\mathbf{v}_0 + (\mathrm{curl}\,\mathbf{v}'\nabla)\mathbf{v}' \\ &- (\mathbf{v}'\nabla)\mathrm{curl}\,\mathbf{v}' + \mathrm{curl}\,[\mathrm{curl}\,\mathbf{B}', \mathbf{B}']/4\pi\rho \\ &+ \mathrm{curl}\,\mathbf{F}'; \end{aligned} \tag{13}$$

$$\begin{aligned} \partial p'/\partial t + (\mathbf{v}_0\nabla)p' &= -(\mathbf{v}'\nabla)p' + (\gamma-1)\eta_0(1+p'/p_0)^{n_1}(\mathrm{curl}\,\mathbf{B}')^2/4\pi \\ &+ \chi_0\mathrm{div}[(1+p'/p_0)^{n_3}\nabla(p'/p_0)] + (\gamma-1)(H-H_0). \quad (14) \end{aligned}$$

These equations are right only for the characteristic times t' and characteristic length L which satisfies the following conditions: $t' \ll 10^5 t_0$ ($t_0 = 22$ year), $L \ll 10^6 R_Q$.

The perturbation terms in (12–14) we can consider as:

$$B_x = B_0 u_2 \exp(i\varphi) + C.C., \quad (15)$$

$$B_y' = B_0\Delta^{-1}k_z\omega u_4 \exp(i\varphi) + C.C., \quad (16)$$

$$B_z' = -B_0\Delta^{-1}k_y\omega u_4 \exp(i\varphi + u_1) + C.C., \quad (17)$$

$$p'/p_0 = u_0 + u_6\exp(i2\varphi) + u_6^*\exp(-i2\varphi), \quad (18)$$

$$V_x' = u_8\exp(i2\varphi) + C.C., \quad (19)$$

$$V_y' = u_{10}\exp(i2\varphi) + C.C., \quad (20)$$

$$V_z' = u_{10}\exp(i2\varphi) + C.C., \quad (21)$$

$$\varphi = k_y y + k_z z + \delta\omega\int_0^t (1+u_0)^{n_4/2}dt. \quad (22)$$

Here k_y, k_z are y, z components of the wave-number; $\omega = \sqrt{|\alpha_0\Delta|/2}$ is the frequency of the linear dynamo waves; $\Delta = k_z v_{xy} - k_y v_{xz}$; $\delta = 1$, when $\alpha_0\Delta > 0$, $\delta = -1$ when $\alpha_0\Delta < 0$, C.C. means complex conjuction. B_0 is a constant, which is determined as a value of perturbed magnetic induction at the time $t = 0$. u_0 is a real function of the variable τ, i.e. Im $u_0 = 0$, and in the equation (18) u_6^* means the complex conjuction of the function u_6.

The components of magnetic induction include a characteristic phase of oscillation φ, which as we can see from the equation (22), depends on ω – the frequency of the linear oscillation of dynamo waves. Taking into the account these conditions, we can suggest that the oscillations of the perturbed magnetic induction include only the first harmonics of the dynamo waves. According to the equation (18) the perturbed pressure consists of the terms with zero and second harmonics of dynamo waves, but the perturbed velocity components from (19–21) contain the second harmonics of the dynamo waves only. We can also consider that in the equations in (12–14) the terms of third-order higher harmonics are negligibly small and are ignored. In our discussion we consider those terms which contain u_6, u_8, $\ldots u_{16}$ only in the first order.

In accordance with the latest discussion we divide our investigation into two parts: in the (a) section nonlinear dynamo waves are investigated only with the zero order harmonics of pressure, the second harmonics of velocity and pressure are not taken into the account. In section (b) investigation is included of the nonlinear dynamo waves as with the zero-order harmonics of pressure, the second harmonics of velocity and pressure also. As we can see from equations (15–17), B_0u_2 is an amplitude of the x component of magnetic induction, and B_0u_4 is an amplitude of the function $(\mathbf{B}'\nabla)v_x/\omega$. We must say that all the perturbed values are homogeneous in the x direction and we can substitute in the equation (12–14) $\delta/\delta x = 0$ (Priest, 1985).

(a) We investigate dynamo waves not taking into account the second harmonics of dynamo waves for the perturbed velocity and pressure. In this case the following equations are adopted:

$$du_0/d\tau = C_s L_1(|u_2|^2 + d_0|u_4|^2) + p_m C_2(L_2 - N^{-1/2}), \quad (23)$$

$$du_2/d\tau = u_4 - [L_1 + i\delta(1+u_0)^{n_4/2}]u_2, \quad (24)$$

$$du_4/d\tau = i2\delta(1+u_0)^{n_4}u_2 - [L_1 + i\delta(1+u_0)^{n_4/2}]u_4. \quad (25)$$

Here $\tau = \omega t$, $C_s = 2(\gamma-1)\beta^{-1}$, $C_2 = \gamma(\gamma-1)M_{T1}^2$, $\beta = 4\pi p_0 B_0^{-2}$,

$$M_{T1} = V_1 V_T^{-1}, \quad V_1 = K^{-1}(V_{xy}^2 + V_{xz}^2)^{1/2},$$

$V_T = (\gamma p_0 \rho_0^{-1})^{1/2}$ is the sound velocity in the unperturbed medium, $p_m = v_0\eta_0^{-1}$ is the magnetic number of Prandtl (Priest, 1982),

$$d_0 = \omega^2 k^2 \Delta^{-2}, \quad k^2 = k_x^2 + k_y^2, \quad L_1 = (1+u_0)^{n_1}N^{-1/2}, \quad L_2 = (1+u_0)^{n_2}N^{-1/2},$$

$N = \omega^2\eta^{-2}k^4$ is the dynamo number (Priest, 1982).

First of all we investigate the equations (23–25) in the case $n_2 = 0$ and $n_4 = 0$ and with the following initial conditions: when $\tau = 0$, Re $u_2 = 1$, Im $u_2 = 0$, Re $u_4 = 1$; Im $u_4 = \delta$, $u_0 = 0$. Using all these conditions we will obtain:

$$u_2 = [\Phi(u_0)]^{1/2}, \quad (26)$$

$$u_4 = (1+i\delta)[\Phi(u_0)]^{1/2}, \quad (27)$$

$$\tau = C_{10}^{-1}N^{1/2}\int_0^{u_0}(1+\xi)^{-n_1}[\Phi(\xi)]^{-1}d\xi \quad (28)$$

$$\Phi(u_0) = 1 + 2C_{10}^{-1}\{N^{1/2}(1-n_1)^{-1}[(1+u_0)^{1-n_1} - 1] - u_0\}, \quad (29)$$

where $C_{10} = C_1(1+2d_0)$, $N < 1$, $u_0 > 0$.

Now we have to find the point of time t_1, ($\tau_1 = \omega t_1$), when u_2 reaches its maximal value $(u_2)_1 = u_{2\,\max}$, and $(u_0)_1 = u_{01}$.

At that moment $(L_1)_1 = L_{10} = 1$,

$$\left.\frac{d^2u_2}{d\tau^2}\right|_1 = -2n_1 N^{-1/2n_1}[\Phi(u_{01})]^{1/2}, \quad (30)$$

$$t_1 = \omega^{-1}C_{10}^{-1}N^{1/2}\int_0^{u_{01}}(1+\xi)^{-n_1}[\Phi(\xi)]^{-1}d\xi, \quad (31)$$

where $u_{01} = N^{1/(2n_1)} - 1$.

In accordance with equation (30), u_2 reaches its maximal value when $n_1 > 0$.

From equations (26) and (28) when $u_2 = 1$, we can consider that the moment of time $t_2 > 0$ ($t_2 = \omega t_2$) can be given as:

$$t_2 = \omega^{-1} C_{10}^{-1} N^{1/2} \int_0^{u_{02}} (1+\xi)^{-n_1} [\Phi(\xi)]^{-1} d\xi, \tag{32}$$

here u_{02} is a solution of the following algebraic equation

$$N^{1/2}(1-n_1)^{-1}[(1+u_{02})^{1-n_1} - 1] - u_{02} = 0. \tag{33}$$

When $n_1 = 1$, equation (33) can be given as:

$$N^{1/2} \ln(1+u_{02}) - u_{02} = 0, \tag{34}$$

when $n_2 = 2$,

$$u_{02} = N^{1/2} - 1. \tag{35}$$

Let investigate equations (26–32) when $N \approx 1$. In this case we obtain:

$$u_{2\max} = (1+\alpha_n)^{1/2}, \tag{36}$$

$$t_1 = [n_1 C_{10} \omega^2 (1+\alpha_n)]^{-1/2} \ln[(1+\alpha_n)^{1/2} + \alpha_n^{1/2}], \tag{37}$$

$$t_2 \approx 2t_1. \tag{38}$$

Here $\alpha_n = (N-1)^2/(4n_1C_{10})$, $n_1 > 0$, $N > 1$.

Now we investigate the stationary solution of the equations (23–25). We mark the stationary value of the functions as: $u_2 = u_{20}$, $u_4 = u_{40}$, $u_0 = u_{00}$.

Under these conditions we obtain:

$$u_{40} = (1+i\delta)u_{20}(1+u_{00})_4^{n/2}, \tag{39}$$

$$u_{00} = -1 + N^{1/(2n_1-n_4)}, \tag{40}$$

$$C_1(1+u_{00})^{n_4/2}[1+2d_0(1+u_{00})^{n_4}]|u_{20}|^2 = \mathrm{Pm} C_2 N^{-1/2}[1-(1+u_{00})n_2]. \tag{41}$$

In accordance with the equations (40,41), for the existence of stationary solutions it is necessary to satisfy the following inequality:

$$1 > N^{n_2/(2n_1-n_4)}. \tag{42}$$

The inequality (42) can be satisfied with the following conditions:

$$(1)\ N > 1, \quad n_2(2n_1 - n_4) < 0, \tag{43}$$

or

$$(2)\ N < 1, \quad n_2/(2n_1 - n_4) > 0. \tag{44}$$

Let investigate the stationarity of the solution of the equation (23–25) with the help of perturbation theory for the nonlinear wave. If the perturbed value is proportional to $\exp(q\tau)$, for the * we get the fourth-order algebraic equation. The discussion of this equation shows us that Re $q < 0$ (Korn and Korn, 1968) if we satisfy the inequality:

$$n_2 < 0, \tag{45}$$

$$2n_1 - n_4 > 0. \tag{46}$$

So $n_2/(2n_1 - n_4) < 0$ and in accordance with the (43) $N > 1$.

When $n_4 = 0$ the criterion of stability gets the following form:

$$n_2 < 0, \quad n_1 > 0,$$

$$(1 + |n_2|/n_1)^{2n_1 - n_4)/|n_2|} > N > 1. \tag{47}$$

(b) Now we investigate the nonlinear dynamo waves taking into account the second harmonics for the velocity and pressure. In this case the equations (12–14) with consideration of (15–23) give the following form:

$$\begin{aligned}
du_0/d\tau &= C_1L_1D_2 - 0.5C_1n_1L_1(1+u_0)^{-1}(D_1u_6^* + D_1^*u_6) \\
&+ p_mC_2N^{-1/2}[(1+u_0)^{n_2} - 1], && (48)\\
du_2/d\tau &= u_4 - (L_1 + i\delta/H_2 + n_1L_1(1+u_0)^{-1}u_2^*u_6, && (49)\\
du_4/d\tau &= i2\delta u_2 - (L_1 + i\delta)u_4 + n_1L_1(1+u_0)^{-1}u_4^*u_6, && (50)\\
du_6/d\tau &= -0.5C_1L_1D_1 + C_1L_1n_1(1+u_0)^{-1}D_2u_6 - 4p_m\gamma p_r^{-1}L_3u_6 \\
&+ p_mC_2n_2L_2(1+u_0)^{-1}u_6 + i4\gamma p_mL_2V_T^{-2}V_2u_8 - i2\delta u_6, && (51)\\
du_8/d\tau &= -(4p_mL_2 + i2\delta)u_8 + i2n_2p_mV_2L_2(1+u_0)^{-1}u_6. && (52)
\end{aligned}$$

Here $V_2 = (k_yV_{xy} + k_zV_{xz})K^{-2}$, $L_3 = (1+u_0)^{n_3}N^{-1/2}$, $p_r = v_0/\chi_0$ is a Prandtl number (Priest, 1982),

$$D_1 = u_2^2 + d_0u_4^2, \tag{53}$$

$$D_2|u_2|^2 + d_0|u_4|^2. \tag{54}$$

First of all we will investigate the self-oscillations of the amplitude u_0, u_6 and u_8 when the magnetic induction is ignored. It means that $u_2 = u_4 = d = 0$. The oscillation u_0, u_6 and u_8 do not depend on each other. If in the equation (49) $u_2 = u_4 = 0$ and one considers $u_0 = 0$ and the perturbed term $u_0' \approx \exp(q_1t)$, in this case

$$q_1 = \eta_0K^2n_2C_2p_m \tag{55}$$

if $n_2 > 0, q_1 > 0$.

Now we will discuss the oscillation of u_6 and u_8 when $u_2 = u_4 = \delta = 0$ with the help of the equations (51–52). If the nonperturbed terms are equal to zero u_6 and u_8, and u_6', $u_8' \approx \exp(q_2t)$ for the q_2 we will get:

$$q_2 = 0.5p_m\eta_0K^2\{n_2C_2 - 4(n_2^{-1}+1) \pm [(n_2C_2 - 4n_2^{-1} + 4)^2 - 32n_2\gamma M_{T2}^2]^{1/2}\}. \tag{56}$$

Here $M_{T2} = V_2V_T^{-1}$.

In accordance with (56) when $n_2 \leq 0$, Im $q_2 = 0$.

When $n_2 = 1$ and $n_2 > 0$, $\mathrm{Im}\, q \neq 0$ if the following inequality is correct:

$$V_T^2 > 32^{-1}\gamma n_2(\gamma - 1)V_2^2 V_I^{-2}. \tag{57}$$

When $n_2 = 0$, $\mathrm{Re}\, q_2 < 0$.

If $n_2 > 0$ and $C_2 > 4n_2^{-1}(n_2^{-1} + 1)$, in this case $\mathrm{Re}\, q > 0$.

Let look for the stationary solution of the equations (48–52). Denote the stationary solution of the function in this way: $u_2 = u_{20}$, $u_4 = u_{40}$, $u_6 = u_{60}$, $u_8 = u_{80}$, and $u_0 = u_{00}$. For these values we will obtain the following dependences:

$$u_{40} = L_{10}^{-1}(i\delta + \delta_1(2L_{10} - 1)^{1/2})u_{20}, \quad |u_{40}|^2 = 2L_{10}^{-2}|u_{20}|^2, \tag{58}$$

$$\begin{aligned} u_{60} &= n_1^{-1}L_{10}^{-2}u_{20}^2|u_{20}|^{-2}(1 + u_{00})[L_{10}^2 - \delta_1(2L_{10} - 1)^{1/2} \\ &+ i\delta(L_{10} - 1)], \end{aligned} \tag{59}$$

$$\begin{aligned} &[2\delta_1(2L_{10} - 1)^{1/2} + 4d_0][L_{10}^2 + 1 - 2\delta_1(2L_{10} - 1)^{1/2}](2 - n_2C_3) + n_1\beta L_1^2 \\ &\times\gamma(\gamma - 1)p_m M_{T1}^2[L_{10} - 1 + 2d_0(1 - \delta_1(2L_{10} - 1)^{1/2})](L_{20} - N^{-1/2}) = 0, \end{aligned} \tag{60}$$

$$\begin{aligned} &n_1(\gamma - 1)|u_{20}|^2 L_{10}[L_{10} - 1 + 2d_0(1 - \delta_1(2L_{10} - 1)^{1/2})] \\ &= \beta(1 + u_{00})(L_{10}^2 + 1 - 2\delta_1(2L_{10} - 1)^{1/2})(2 - n_2C_3), \end{aligned} \tag{61}$$

$$\begin{aligned} &p_m L_{10}[L_{10} - 1 + 2d_0(1 - \delta_1(2L_{10} - 1)^{1/2})](4L_{30}(\gamma p_r)^{-1} - n_2L_2\delta_4) \\ &= (2 - n_2C_3)[L_{10}(L_{10}^2 + 2 - 3\delta_1(2L_{10} - 1)^{1/2}) \\ &+ 2d_0(L_{10}^2 + L_{10} + 2 - 4\delta_1(2L_{10} - 1)^{1/2})]. \end{aligned} \tag{62}$$

Here δ_1 is one of the following numbers: $+1$ and -1.

$$C_3 = 2\gamma p_m^2 L_{20}^2(1 + u_{00})^{-1}M_{T2}^2(1 + 4p_m^2L_{20}^2)^{-1}, \tag{63}$$

$$C_4 = \gamma(\gamma - 1)M_{T1}^2 - 2C_3, \tag{64}$$

$$L_{10} = (1 + u_{00})^{ni}N^{-1/2}, \quad i = 1, 2, 3. \tag{65}$$

For the existence of the stationary solution of the equations (48–52) it is necessary that $L_{10} \geq 0.5$. We look for the stationary solutions with the following conditions:

$$|u_{20}| \gg 1, \quad |u_{40}| \gg 1, \quad |u_{60}| < 1.$$

3 DISCUSSION

(1) The asymptotic solutions for the amplitudes of the dynamo waves obtained in the equations (26–29) when the coefficients of viscousity and α are considered to have constant values ($n_2 = 0$, $n_4 = 0$) are given. It is shown that the amplitude of the magnetic field reaches its maximal value (the magnetic field gets stronger) when $n_1 > 0$, the period of time t, during which the magnetic field reaches its maximal value (38) was found. The equation for the dependence of the magnetic field on the dynamo number is obtained. We can say exectly, that the magnetic

field strengthens when the dynamo number $N > 1$. In the case $N \approx 1$, the rate of strengthening of the magnetic field (ratio of the amplitude of the magnetic field to its meaning when $t = 0$) is proportional to $(N-1)^2\beta_0$ where $\beta_0 = 4\pi p_0/B_0^2$ is for the unperturbed medium. We can see that the strengthening of the magnetic field is high when $(N-1)^2\beta_0 \gg 1$.

(2) In the case when $n_1 \neq 0$, $n_2 \neq 0$ and $n_4 \neq 0$, the stationary solution of the equation for the amplitude of the magnetic field and pressure is given. The conditions of stability of this solution can be shown as:

$$n_1 > 0, \quad n_2 < 0, \quad 2n_1 - n_4 > 0, \quad N_1 > N > 1.$$

Here $N_1 = (1 + |n_2|n_1)^{2n_1 - n_4)/|n_2|}$. If N is near to N_1 the perturbed amplitude oscillatorily approaches the stationary state.

The period of oscillation $T > 5(n_1 p_m)^{-1/2}[(n_1 + |n_2|)/n_1]^{|n_2|/[2(n_1+1)]}$ can last several hundred years. With these oscillations the existence of the Maunder minimum can be described.

In the case when we take into account the second harmonics of the dynamo waves for the velocities and pressure, we will get the following results:

(1) The stationary solution is obtained after the solution of the equation (48–52) and is given in (58–62). These solutions exist if $n_2 < 0$, $(1 + u_{00})^{n_1} N^{-1/2} \geq 0.5$, $(1 + u_{00})^{n2} > 0$.

u_{00} is the stable meaning of the zero order harmonic of the pressure.

(2) It can be shown, that the stationary solutions of the equations (48–52) are stable if the Prandtl number $p_r = v_0/\chi_0 \leq 2$.

Acknowledgements

We would like to thank the Soros Foundation for the financial support necessary for our theoretical research.

References

Parker, E. N. (1982) *Cosmical Magnetic Fields*, Moscow (in Russian).
Priest, E. R. (1985) *Solar Magnetohydrodynamics*, Moscow (in Russian).
Korn, G. A. and Korn, T. M. (1984) *Mathematical Handbook*, Moscow (in Russian).