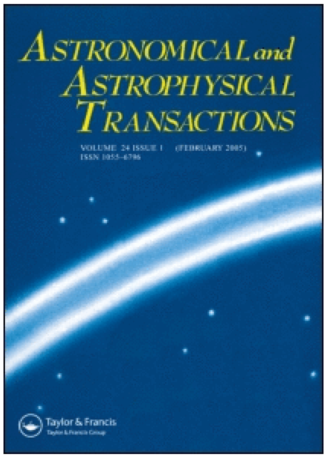

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AN EXCITEMENT OF THE GRAVITATIONAL AND ACOUSTIC WAVES IN THE SOLAR ATMOSPHERE CAUSED BY THE INHOMOGENEOUS FLOW

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The excitement of the gravitational and acoustic waves in the Solar atmosphere is investigated in this paper. The atmosphere is assumed to be ideally isothermally stratified and the waves excited by the inhomogeneous wind. The amplitude reaches its maximal value at a certain moment. The maximal amplitude of the gravitational wave is larger than that of the acoustic waves.

KEY WORDS Gravitational and acoustic waves, frequency

1 INTRODUCTION

There are several type of waves in the solar atmosphere. Among them we can distinguish acoustic and gravitational waves. It is known (Gibson, 1973; Priest 1982), that in the solar photosphere and in the lower layers of the chromosphere that oscillations with five-minute periods always exist. These oscillations are considered to belong to acoustic and gravitational waves. According to some assumptions (Gibson, 1973; Priest, 1982) the appearance of these waves has been explained. A new mechanism of excitement of the acoustic and gravitational waves, caused by the flow in the vertical plane with shear is presented in this paper. This kind of inhomogeneous flow can be observed in the higher horizontal regions of the Solar granules and supergranules.

The investigation of excitement of acoustic and gravitational waves in the Solar atmosphere is presented in this paper. We consider that the existence of such waves is caused by the inhomogeneity of the wind field. The inhomogeneity is directed perpendicular to the direction of the wind velocity. In this case we take account of inhomogeneity only in the first-order coordinates. This problem is solved with the help of ideal gas-dynamic equations in the case of stratification of the Solar atmosphere for a constant unperturbed temperature, with at the same time the wave-number, amplitude and frequency dependent on time. These problems have been investigated in the following works (Goldstein, 1931; Taylor, 1931; Gossard and Hooke, 1975; Pedlosky, 1975; Gill, 1982) for a constant value of frequency and wave-number, and dependence of the amplitude on the z coordinate.

This paper consists of three sections. The first one gives the main equations, investigated in the linear approximation. In the second section the analytical results and numerical calculations of the linear equations for acoustic and gravitational waves are given. The results are discussed in the third section.

1.1 The Equation for the Gravitational and Acoustic Waves

We investigate the gravitational and acoustic waves in the Earth's atmosphere with the help of the ideal gas-dynamic equations which have the following form (Gibson, 1975):

$$\rho[\partial \mathbf{v}/\partial t + (\mathbf{v}\nabla \mathbf{v}] = -\nabla p + \mathbf{g}\rho, \tag{1}$$

$$\partial\rho/\partial t + \operatorname{div}(\rho\mathbf{v}) = 0, \tag{2}$$

$$[\partial/\partial t + (\mathbf{v}\nabla]p - \gamma p \operatorname{div}\mathbf{v} = 0. \tag{3}$$

Here ρ is density, p is pressure, $\mathbf{v}$ is velocity, g is the gravitational acceleration.

Let discuss these equations in the bartesian coordinate system. The z-axis is directed vertical by to the Solar surface, and the y and x-axes are directed along the surface.

The equations (1–3) are investigated in the linear approximation. We consider the environment as an ideal gas, which obeys the laws of ideal gases:

$$p = (R/M)\rho T,$$

where R is the universal gas constant, T is the temperature of the environment. We can present all the functions which appear in the equations (1–3) in the following way: $f = f_0 + f_1 \exp(i\varphi + \delta z/H)$, where $\varphi = k_x x + K_y y + K_z z - k_x v_{0x} t - \int_0^T \omega_1 d\tau'$. Here f_0 is the unperturbed term of the function, and f_1 is the perturbed term, $\delta = 1$ for the perturbed velocity and $\delta = -1$ for the perturbed pressure and density; $K_y = k_y - k_x v_{xy} t$, $K_z = k_z - k_x v_{xz} t$. The dependence of the P_0 unperturbed terms of pressure and density – ρ on the z variable can be shown to be:

$$p_0 = p_{00} \exp(-z/H), \quad \rho_0 = \rho_{00} \exp(-z/H). \tag{4}$$

The value of the unperturbed temperature is constant and equal to T_0. The velocity consists of only the x component:

$$v_0 = (v_0 x + v_{xy} y + v_{xz} z) x_0, \tag{5}$$

where v_{0x}, v_{xy}, v_{xz} are constant, and x_0 is a unit vector along the x-axis.

After the linearization of the equations (1–3) we derive the following linear equations for perturbed terms:

$$\begin{aligned} dX_0/d\tau &= i\omega_1 X_0 - (v_2X_1 + v_3X_2) - iK_1X_3, & (6)\\ dX_1/d\tau &= i\omega_1 X_1 - iK_2X_3, & (7)\\ dX_2/d\tau &= i\omega_1 X_2 + (1/2 - iK_3)X_3 - X_4, & (8)\\ dX_3/d\tau &= i\omega_1 X_3 + (1-\gamma/2)X_2 - i\gamma(X_0K_1 + K_2X_1 + K_3X_2), & (9)\\ dX_4/d\tau &= i\omega_1 X_4 + X_2/2 - i\gamma(X_0K_1 + K_2X_1 + K_3X_2), & (10) \end{aligned}$$

Here $X_0 = v_{x1}/\omega_0 H$, $X_1 = v_{y1}/\omega_0 H$, $X_2 = v_{z1}/\omega_0 H$, $X_3 = p_1/p_0$, $X_4 = \rho_1/\rho_0$. v_{x1}, v_{y1}, v_{z1} are amplitudes, x, y, z are components of the perturbed velocity of the medium, p_1 and ρ_1 are amplitudes of the pressure and density.

In the equations (6–10) $K_1 = k_xH$, $K_2 = k_yH - k_xHv_2\tau$, $K_3 = k_zH - k_zv_3\tau$, $\omega_0 = (g/H)^{1/2}$, $\omega_1 = \omega/\omega_0$, $v_2 = v_{xy}/\omega_0$, $v_3 = v_{xz}/\omega_0$ and ω is a time-dependent frequency, $\omega_0 = (g/H)^{1/2}$, $\tau = \omega_0 t$.

1.2 The Investigation of the Main Equations

Let us investigate the equations (6–10) analytically. When $\nu_2 = \nu_3 = 0$, we obtain the system of homogeneous first-order differential equations with constant coefficients. In the case when $\omega_1 = \omega_{10}$ we obtain:

$$\omega_{10} = \{\gamma(1/8 + K_0^2/2) \pm [\gamma^2(1/8 + K_0^2/2)^2 - (\gamma-1)K_{10}^2]^{1/2}\}^{1/2}. \qquad (11)$$

Here $K_0^2 = K_1^2 + K_{20}^2 + K_{30}^2$, $K_{10}^2 = K_1^2 + K_{20}^2$, $K_{20} = k_yH$, $K_{30} = k_zH$.

In equation (11) the positive sign means the frequency of the linear acoustic waves and the negative sign means the frequency of the linear gravity wave (Gibson, 1973).

If $\partial/\partial\tau = 0$, the determinant of the equations (6–10) Δ has the following form:

$$\begin{aligned} \Delta &= i\omega_1[\omega_1^4 - \omega_1^2(\gamma/4 + \gamma K^2) + K_\perp^2(\gamma-1)] + K_1K_2V_2[-\gamma\omega_1^2 + (\gamma-1)]\\ &+ V_3K_1[-K_3\gamma + i(2-\gamma)/2]\omega_1^2, \quad K_\perp^2 = K_1^2 + K_2^2, \quad K^2 = K_\perp^2 + K_3^2. \quad (12) \end{aligned}$$

Now we discuss the equation $\Delta = 0$ if $\omega_1 = \omega_{11} + iq$, $|q| \ll \omega_1$. We obtain the equation for q:

$$\begin{aligned} q &= 0.5\omega_{11}^{-2}K_1[2\omega_{11}^2 - 0.25\gamma - \gamma K^2]^{-1}\{K_2v_2[(\gamma-1) - \gamma\omega_{11}^2]\\ &+ \nu_3[-K_3\gamma + i(2-\gamma)/2]\omega_{11}^2\}, \qquad (13)\\ &\omega_{11}^4 - \omega_{11}^2(\gamma/4 + \gamma K^2) + K_\perp^2(\gamma-1) = 0. \end{aligned}$$

When Re $q > 0$, the waves are unstable.

In the case $v_2 = 0$, the acoustic waves are unstable when $K_1K_3v_3 < 0$, and the same happens with the gravitational waves when $K_1V_3K_3 > 0$.

If $V_3 = 0$, the unstable acoustic waves occur if $K_1K_3v_2 < 0$, and for gravitational waves the following in equality must be satisfied: $K_1K_3v_2 > 0$.

Now we investigate the equations (6–10) asymptotically using the $\tau \to \infty$ approximation when $\omega_1 = 0$.

In order to present the equations (6–10) in an operator form, we will obtain (Korn and Korn, 1968):

$$(dX/d\tau) = \hat{A}_1 X + \hat{A}_0 X.$$

The X consists of a column with five components and $\hat{A}_1$ and $\hat{A}_0$ are five-by-five matrices.

The asymptotic solution of this equation has the following form:

$$X = \tau^s\{X_0 + X_1\tau^{-1} + X_2\tau^{-2} + \ldots\}\exp(0.5\mu_2\tau^2 + \mu_1\tau),$$

and the results will be the following:

$$\mu_2 = \pm i[\gamma K_1^2(\tau_2^2 + v_3^2)], \tag{14}$$

$$\mu_1 = \gamma K_1(K_{20}v_2 + K_{30}v_3)\mu_2^{-1}, \tag{15}$$

$$S = \{-\mu_1^2 - \gamma K_0^2 - (2-\gamma)^2/4\gamma + K_1^2 v_3^2(\gamma - 1)/\mu_2^2 + i\mu_2 K_1\}(2\mu_2)^{-1}. \tag{16}$$

As well as the equations (14–16), the equations (6–10) has an asymptotic ($\tau \to \infty$) solution ($v_2 \neq 0$):

$$\mu_2 = 0, \tag{17}$$

$$\mu_1 = \pm[(\gamma - 1)v_2^2\gamma^{-1}(v_2^2 + v_3^2)^{-1}]^{1/2}, \tag{18}$$

$$S = \mu_1 v_3(K_3 v_2 - K_2 v_3)(K_1\nu_2)^{-1}(v_2^2 + v_3^2)^{-1}. \tag{19}$$

In the case $\mu_2 = \mu_1 = 0$ the asymptotic solution of the equations (6–10) are:

$$S = -1, \quad \text{when } v_2 \neq 0, v_3 = 0, \tag{20}$$

$$S = -1/2 \pm A_1, \quad \text{when } v_3 \neq 0, v_2 = 0. \tag{21}$$

Here $A_1 = [1/4 - (\gamma - 1)\gamma^{-1}K_{\perp 0}^2 K_1^{-2} v_3^{-2}]^{1/2}$.

With the help of the equations (14–21) we can find the whole asymptotic solution of the equations (6–10) in the case when $\tau \to \infty$. It can be shown that the amplitudes in the asymptotic solutions do not increase. In the equations (6–10) the assumption $\omega_1 = 0$, $\nu_{xy} = 0$ and $\gamma \to \infty$ (an incompresible medium) the following equation is obtained:

$$\frac{d^2X_{41}}{d\xi^2} + K_{\perp 0}^2 K_1^{-2} R[\xi^2 - (b-1)](1 - \xi^2)^{-2} X_{41} = 0, \tag{22}$$

where $\xi = K_3/\alpha$, $\alpha^2 = K_{\perp 0}^2 + 1/4$, $R = V_3^{-2}$ is a Richardson number, $b = K_1^2\alpha^{-2}R^{-1}$. In the equation (22) the function X_{41} is connected with X_4 as follows:

$$X_4 = X_{41}(1 + \xi^2)^{-1/2}\exp(-i0.5\alpha^{-1}\tan^{-1}\xi). \tag{23}$$

When $b > 1$, we can mention, that $\xi = \pm(b-1)^{1/2}$ will be a turning point of the equation (22). We solve the equation (22) in case when $R \gg 1$ ($b \ll 1$) with the help of the WKB method (Korn and Korn, 1968):

$$\begin{aligned} X_4 &= \{X_{40}[\cos(s_0 s_2) + 0.5 s_0^{-1}(\xi_0 + i\alpha^{-1})(1+\xi_0^2)^{-1/2}\sin(s_0 s_2)] \\ &- \dot{X}_{40}(1+\xi_0^2)^{1/2}(s_0 K_1 \nu_{xz})^{-1}\sin(s_0 s_2)\}[(1+\xi_0^2)(1+\xi^2)]^{1/4} \\ &\times \exp[-is_1/(2\alpha)]. \end{aligned} \tag{24}$$

Here $\xi_0 = \xi|_{t=0} = k_z H/\alpha$, $X_{40} = X_4|_{t=0}$, $\dot{X}_{40} = (dX_4/dt)|_{t=0}$, $s_0 = K_{\perp 0} K_1^{-1} R^{1/2}$,

$$\begin{aligned} s_1 &= \tan^{-1}\xi - \tan^{-1}\xi_0, & (25) \\ s_2 &= \sinh^{-1}\xi - \sinh^{-1}\xi_0. & (26) \end{aligned}$$

2 DISCUSSION OF THE RESULTS

The following main results obtained in the paper are:

(1) The investigation of the equations (14–21) show that the equations (14–16) are for the acoustic and (17–21) for the gravitational waves. Accordingly the frequency for both the acoustic and gravitational waves can be given as:

$$\omega = -\partial\varphi/\partial t, \tag{27}$$

where

$$\varphi = k_x x + K_y y + K_z z - k_x + v_{0x} - \int_0^T \omega_{11} d\tau', \tag{28}$$

$$\omega_{11} = (A_3 \pm A_2)^{1/2}, \tag{29}$$

where $A_3 = \gamma/8 + \gamma K^2/2$, $A_2 = \{[A_3^2 - (\gamma-1)K_\perp^2]^{1/2}\}^{1/2}$, the positive sign being for the acoustic waves and the negative for the gravitational waves.

According to the equations (27, 28) we will get the following:

$$\omega = k_x v_{x0} + k_x v_{xy} y + k_x v_{xz} z + \omega_{11}\omega_0. \tag{30}$$

With the help of (30) the y and z components of the group velocity $d\omega/dk_y$ and $d\omega/dk_z$ can be determined:

$$\begin{aligned} d\omega/dk_y &= \pm K_y H^2 \omega_0 (2A_2)^{-1}[\gamma\omega_{11} + (1-\gamma)\omega_{11}^{-1}], & (31) \\ d\omega/dk_z &= \pm\gamma K_z H^2 \omega_0 \omega_{11}(2A_2)^{-1}. & (32) \end{aligned}$$

Here the positive sign is in accordance with the acoustic waves and the negative with gravitational. In the case $v_{xy} k_x k_y > 0$ and $v_{xz} k_x k_y > 0$ the group velocities change their sign.

(2) Considering the assumption that $d\varphi/dk_z = 0$ can datermine the wave amplitude $z = z(t)$, we will obtain:

$$z = -(k_x v_{x0} + \omega_{11}\omega_0)(k_x v_{xz})^{-1}, \tag{33}$$

$d\varphi/dk_y = 0$ determines $y = y(t)$:

$$y = -(k_x v_{x0} + \omega_{11}\omega_0)(k_x v_{xy})^{-1}. \tag{34}$$

In the condition $t \to \infty$ for acoustic waves $|z| \to \infty$, $|y| \to \infty$, and for gravitational waves

$$z \to v_{x0}/v_{xz}, \quad y \to v_{x0}/v_{xy} - \omega_0(k_x v_{xy})^{-1}(1-\gamma^{-1})^{1/2}.$$

(3) The results given in the present work can describe the existence of the gravitational waves in the lower part of the Solar chromosphere, where at the higher spheres of supergranula there exists the horizontal flow $v_{x0} \neq 0$, $v_{xz} \neq 0$ (Gibson, 1975). If we consider, that $v_{xz} \approx 10^{-2}$ s^{-1}, $k_z/k_x \approx 10^2$, in this case $t_* \approx 10^4$ s. During this interval of time the gravitational wave can excite, leave the region of the higher supergranula (the width of it is of the order of 10^3 km), and get into the lower level of the Solar chromosphere.

Now let estimate the amplitude of the z component of the velocity X_2 when $t = t_*$. According to the equations (6–10) in the $\gamma \to \infty$ case $X_2 = 2X_4$. If $k_z H = 5$, $|k_y H| \approx |k_x H| \ll 1$, we can show that $\xi_0 = 20$. Following the equation (24) at the $t = t_*$ moment of time, $|X_2| \approx \xi_0^{3/2} \approx 10^2$. This number is compared to the observed values.

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