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SOLAR CYCLE VARIATION OF THE SOLAR CORONA SHAPE: A NEW OUTLOOK

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Solar coronal streamers constitute a closed belt around the Sun which serves as a base of the heliospheric current sheet. Such a new view of the solar corona structure leads one to regard in a new light the evolution of coronal form during the sunspot cycle. So far the general coronal shape is quantitatively described with Ludendorff's parameter characterizing the flattening of the corona in the heliographic frame of reference. Such a description is formal and has no physical meaning. Modification of Ludendorff's technique is proposed which consists of substitution of the heliographic frame of reference by a heliomagnetic one. The principal plane of the heliomagnetic coordinate system is that of the magnetic dipole equator which coincides with the geometric mean plane of the solar corona. The new quantity of observed coronal flattening depends on two parameters: (1) the angle between the line of sight and the coronal mean plane, and (2) an angular spread of coronal streamer belt in 3-D space. The above considerations justify a need to revise all available eclipse data concerned with the solar corona shape. Some results of such a revision are presented in this paper.

KEY WORDS Solar corona, solar eclipses

1 CLASSICAL PROCEDURE BY LUDENDORFF

It is well known that the shape of the solar corona is subject to significant variations during the solar activity cycle. The conventional way of quantitatively describing the general coronal form is the procedure first suggested by Ludendorff almost 70 years ago (Ludendorff, 1928). Let us recall its substance. Let a set of whitelight corona isophotes obtained from a solar eclipse observation be available. For each of the isophotes we determine the flattening index, \( \varepsilon \), by the formula:

\[
\varepsilon = \frac{d_e - d_p}{d_p} = \frac{d_e}{d_p} - 1.
\]

Here, \( d_e \) is the mean of the equatorial diameter of an isophote and two of its diameters at angles \( \pm 22.5^\circ \) to the equatorial direction; \( d_p \) is the analogous quantity for the polar direction. Naturally, the heliographic frame of reference is used with the
polar axis being the Sun's rotation axis. Mean equatorial distance of an isophote from the solar disk centre, \( r_e = d_e/2 \), is measured in solar radii.

Usually, the flattening index increases linearly from the limb to some distance \( r_e = r_m \); the value \( r_m \) varies from eclipse to eclipse within the range of \( \approx 1.4 \) to \( \approx 2.2 \). In such a case the increase of \( \varepsilon \) between \( r_e = 1 \) and \( r_e = r_m \) can be fitted well with a function:

\[
\bar{\varepsilon} = a + b(r_e - 1).
\]

The relation of both parameters, \( a \) and \( b \), to sunspot number was analysed primary for 13 eclipses from 1893 to 1927. Later, Ludendorff (1934) found that it is more convenient to deal with the sum \( (a + b) \) instead of independent analysis of parameters \( a \) and \( b \). The above sum evidently presents the value of \( \bar{\varepsilon} \) at \( r = 2 \). Ever since the quantity of \( (a + b) \) has been used as a quantitative characteristic of the general form of the solar corona (Ludendorff's parameter). A graph of the dependence of \( (a + b) \) on the 11 year solar cycle phase, \( \Phi \), is usually considered. \( \Phi \) is calculated by

\[
\Phi = (T - m)/|M - m|,
\]

where \( T \) is the time of eclipse, and \( M \) and \( m \) are the times of the solar cycle maximum and minimum nearest to \( T \), respectively. Ludendorff's (1934) graph was based on results for 17 eclipses. Subsequently, a number of authors added more and more points to the graph of \( (a + b) = f(\Phi) \).

To date, the most comprehensive graph is the diagram by Loucif and Koutchmy (1989) with data for 40 eclipses. We can now add nine more points making the total number of eclipses 49. The data concerned with new eclipses are listed in Table 1. The supplemented plot is presented in Figure 1. Black circles are consistent with data from Loucif and Koutchmy (1989). We do not indicate eclipse years for the data (which is done as a rule) in order to avoid overloading the figure. New data for nine eclipses (crossed circles) are marked by the year of observation.

To determine the cycle phase for eclipses of 1990, 1991 and 1994 we took January, 1996 as the time of sunspot minimum. By the way, the current behaviour of solar activity yields some hints that the sunspot minimum has already occurred.

### Table 1. New data on Ludendorff's parameter

<table>
<thead>
<tr>
<th>No.</th>
<th>Eclipse</th>
<th>( \Phi )</th>
<th>( a + b )</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1941, 21 Sep.</td>
<td>-0.35</td>
<td>0.34</td>
<td>Zeltser and Markov (1949)</td>
</tr>
<tr>
<td>2</td>
<td>1974, 20 Jun.</td>
<td>-0.26</td>
<td>0.18</td>
<td>Waldmeier (1974)</td>
</tr>
<tr>
<td>3</td>
<td>1976, 23 Oct.</td>
<td>+0.09</td>
<td>0.36</td>
<td>Waldmeier and Weber (1977)</td>
</tr>
<tr>
<td>4</td>
<td>1977, 12 Oct.</td>
<td>+0.38</td>
<td>0.24</td>
<td>Waldmeier and Weber (1978)</td>
</tr>
<tr>
<td>5</td>
<td>1979, 26 Feb.</td>
<td>+0.76</td>
<td>0.15</td>
<td>Waldmeier and Weber (1979)</td>
</tr>
<tr>
<td>6</td>
<td>1984, 22 Nov.</td>
<td>-0.27</td>
<td>0.35</td>
<td>Loucif and Koutchmy (1989)</td>
</tr>
<tr>
<td>7</td>
<td>1990, 22 Jul.</td>
<td>-0.85</td>
<td>0.12</td>
<td>Koutchmy et al. (1992)</td>
</tr>
<tr>
<td>8</td>
<td>1991, 11 Jul.</td>
<td>-0.69</td>
<td>0.00</td>
<td>Vanyarkha et al. (1993);</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Markova and Belik (1995);</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Sykora et al. (1995)</td>
</tr>
<tr>
<td>9</td>
<td>1994, 3 Nov.</td>
<td>-0.18</td>
<td>0.27</td>
<td>Markova and Belik (1995);</td>
</tr>
</tbody>
</table>
Figure 1  Ludendorff's parameter of coronal shape versus sunspot cycle phase. Black circles are consistent with data from Loucif and Koutchmy (1986). Crossed circles signify new additional values for nine marked eclipses. Black square represent the \((a + b)\) value for the eclipse of 1954 according to Wallenquist (1957).

For the eclipse of 1941 we have calculated the \((a + b)\) parameter from isophotes presented by Zeltser and Markov (1949). Values of \((a + b)\) for the eclipses of 1974–1994 are given in papers cited in Table 1. For the 1954 eclipse at the epoch of deep solar minimum, Loucif and Koutchmy (1989) use a value of \(a + b = 0.28\). Meanwhile, Waldmeier (1955) and Wallenquist (1957) found for that eclipse much higher values of Ludendorff's parameter: 0.39 and 0.35, respectively. Wallenquist's value appears to be the most reliable one; we have plotted it on Figure 1 as well (black square).

So, we have the most complete data on the photometric coronal shape in the classical meaning presented in Figure 1.

2 MODIFIED PROCEDURE: USING THE HELIOMAGNETIC FRAME

The distribution of points in Figure 1 shows a gradual increase of coronal flattening in going from solar maximum to minimum and a similar gradual decrease with ascending solar activity from minimum to maximum. Just such an outlook is generally accepted. According to Loucif and Koutchmy (1989), the mean value of \((a + b)\)
varies from 0.06 near the sunspot maximum to 0.27 near minimum; hence, the drop equal to 0.21. However, we see a very large scatter of points at all phases of the solar cycle except maybe the sunspot maximum. For example, an enlarged fragment of the general diagram is presented in Figure 2 for the range of $\Phi$ relevant to the first half of the descending part of the sunspot cycle. We see that monotonic growth of flattening does not occur. Instead we have a random distribution of points over the whole area within $(a + b)$ from 0 to $\approx 0.25$. Probably at intermediate phases of the solar cycle Ludendorff's parameter can take any value between the minimum and maximum one. In other words, there is not a unique dependence of Ludendorff's parameter on solar cycle phase, and the relation of $(a + b)$ with $\Phi$ outlined by the average curve (Loucif and Koutchmy, 1989) is illusory.

The above features of behaviour of Ludendorff's parameter become clear in the light of recent views on the solar corona structure. We know now that coronal helmet streamers constitute a closed belt around the Sun which serves as the base of the heliospheric current sheet (HCS). So the outer solar corona composed of helmet streamers can be taken as a surface in 3-D space. We have found that during most
of the 11 year sunspot cycle the above surface is largely flattened, and the outer corona with its streamers should resemble a galaxy. Such a conclusion induced the author to formulate a concept of the flat solar corona (Gulyaev, 1992). In the general case, the said surface is fitted well by the hyperbolic paraboloid (Gulyaev, 1994b).

The HCS base configuration is outlined by the neutral line of the radial magnetic field on the source surface. The orientation of the HCS and coronal streamer belt in 3-D space depends on the dipole component of the source surface magnetic field. It follows from the above, that the equator of the magnetic dipole on the source surface determines the location of the geometric mean plane of the corona.

The angle between the magnetic dipole axis and the Sun’s rotation axis varies with sunspot cycle from $\approx 0$ at the cycle minimum to $\approx 90^\circ$ at the maximum (see, e. g. Hoeksema, 1991). Naturally, the inclination of the coronal mean plane to the solar equator changes just like that. Now we can easily imagine that with a fairly steep inclination to the equator, the corona can demonstrate to an observer during solar rotation a full set of classical coronal forms: from minimum corona if it is oriented edge-on towards the Earth to maximum corona when after the one quarter of rotation it is turned face-on. A striking instance of edgewise orientation of the flat, steeply tilted corona occurred during the total solar eclipse of July 11, 1991 when observers were impressed by the unusual shape of the corona which was difficult to explain using the standard method (see, e. g. Gulyaev, 1994a).

In the light of the above it is clear that at intermediate phases of the solar cycle Ludendorff’s parameter can take any value whatever the actual phase of the solar cycle; thus the vast spread of points in the diagram of $(a + b) = f(\Phi)$ is inevitable. So the routine use of Ludendorff’s technique is only a formal procedure which lacks physical meaning. However the technique has a sound physical basis as soon as we discard the solar equatorial plane as a principal plane of the coordinate system, replacing it by the mean plane of corona, i.e. the plane of the magnetic dipole equator. In other words, we should substitute the heliographic frame of reference with the heliomagnetic one. Modified Ludendorff’s parameter in the heliomagnetic frame has quite a clear physical meaning since it depicts the real concentration of coronal material towards the heliospheric current sheet.

Previously we have presented spectacular examples of efficiency of turning to the heliomagnetic system for the eclipses of 1991 and 1974 (Vanyarkha et al., 1993). Unfortunately, we have restricted space to reproduce these results in the present paper.

The modified Ludendorff’s parameter $(a + b)^*$ fastened on the magnetic equator direction at the solar disk should evidently depend on two values: (1) the angle $\gamma$ between the line of sight and the plane of the magnetic dipole equator, and (2) a measure of deviation of the actual coronal configuration from the mean plane, i.e. an angular spread of the coronal streamer belt. We have shown earlier (Gulyaev, 1992) that the RMS deviation of the HCS base from the mean plane does not exceed $15^\circ$ during most of the sunspot cycle. For this reason and also in view of the insufficient quantity of data available so far we will not analyse the effects of
the second value in this paper but confine ourselves to the relation between \((a + b)^*\) and \(\gamma\).

To determine the direction of the magnetic equator at the Sun’s disk and calculate the angle \(\gamma\), we need data on the configuration of the HCS base or the neutral line at the source surface. We have suitable data from 1972 to the present; relevant references are given in Table 2 (column 4, Ref. 1). Omitting intermediate mathematical considerations, the final formula for the angle \(\gamma\) between the line of sight and the magnetic equator plane is as follows:

\[
\sin \gamma = \cos \psi \sin B_0 + \sin \psi \cos B_0 \cos (\lambda_0 - L_0). \tag{4}
\]

Here, \(\psi\) is the angle between the magnetic dipole axis and the Sun’s rotation axis, \(B_0\) is the heliographic latitude of the solar disk center, \(\lambda_0\) is the Carrington longitude of the magnetic pole nearest to the north heliographic pole, and \(L_0\) is the Carrington longitude of the solar disk center.

For the period 1972–1994 we have isophotes of solar corona for 12 eclipses. We will not consider eclipses of 1979, 1980, 1981 and 1990 when the corona had a shape close to the maximum type. Besides, we have not yet succeeded in obtaining certain results for the eclipse of 1994. The remaining seven eclipses are listed in Table 2. Column 6 contains references to sources of isophotes which have been used for calculation of \((a + b)^*\) values (column 5).

To produce, if only a rough, qualitative picture of the expected variation of the parameter \((a + b)^*\) depending upon the angle \(\gamma\), we consider the following idealized model. Let us assume that an isophote defining the \((a + b)\) parameter for the minimum corona is in the form of an ellipse (this is close to reality) with the major semi-axis \(A = 2\) and minor semi-axis \(B = 1.54\) \((A\) and \(B\) are in solar radii). We take the ratio \((A - B)/B = 0.3\) as a measure of the coronal flattening at \(r = 2\) which is close to the mean value of Ludendorff’s parameter for the minimum corona. Let us consider now a flattened ellipsoid of revolution with semi-axes of \(A\) and \(B\) arbitrarily oriented in space. The plane of symmetry of the ellipsoid normal to the

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### Table 2. Modified Ludendorff’s parameters concerned with the heliomagnetic frame of reference

<table>
<thead>
<tr>
<th>No.</th>
<th>Eclipse</th>
<th>(\gamma)</th>
<th>Ref. 1</th>
<th>((a + b)^*)</th>
<th>Ref. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1972, 10 Jul.</td>
<td>11°</td>
<td>(1)</td>
<td>0.285</td>
<td>(4)</td>
</tr>
<tr>
<td>2</td>
<td>1973, 30 Jun.</td>
<td>21</td>
<td>(1)</td>
<td>0.26</td>
<td>(5)</td>
</tr>
<tr>
<td>3</td>
<td>1974, 20 Jun.</td>
<td>7</td>
<td>(1)</td>
<td>0.25</td>
<td>(6)</td>
</tr>
<tr>
<td>4</td>
<td>1976, 23 Oct.</td>
<td>0</td>
<td>(2)</td>
<td>0.36</td>
<td>(7)</td>
</tr>
<tr>
<td>5</td>
<td>1977, 12 Oct.</td>
<td>18</td>
<td>(2)</td>
<td>0.25</td>
<td>(8)</td>
</tr>
<tr>
<td>6</td>
<td>1983, 11 Jun.</td>
<td>17</td>
<td>(2)</td>
<td>0.25</td>
<td>(9)</td>
</tr>
<tr>
<td>7</td>
<td>1991, 11 Jul.</td>
<td>18</td>
<td>(3)</td>
<td>0.22</td>
<td>(10)</td>
</tr>
</tbody>
</table>

minor axis can be taken as an analogue of the equatorial plane. The cross-section of our ellipsoid by the plane of the sky passing through its centre represents an ellipse with major semi-axis \( A = 2 \) and minor semi-axis \( B' \); the value of \( B' \) is dependent on the angle \( \gamma \) between the line of sight and the equatorial plane of the ellipsoid. It is clear that \( B' \) varies within the limits of \( B' = B = 1.54 \) at \( \gamma = 0 \) (minimum corona) to \( B' = A = 2 \) (maximum corona).

Accepting the above ellipsoid cross-section as an image of the isophote defining the \((a + b)^*\) parameter versus \( \gamma \), we can readily find suitable values of \((a + b)^*\). Results are presented in Figure 3 with a solid line. The results of calculation of actual \((a+b)^*\) values for seven eclipses are represented by black circles. Incidentally, during all the seven eclipses the corona was oriented almost edgewise towards the Earth \( (\gamma \leq 20^\circ) \). This is why all the points proved to be concentrated unfortunately in one narrow range of \( \gamma \), so we cannot trace the run of \((a+b)^*\) throughout the entire range of variation of \( \gamma \). Nevertheless, we see that all the points are concentrated close to the model curve.

Subsequently we are going to extend the above analysis to other eclipses that have occurred during this century. This will become possible when data on the HCS configuration for past epochs become available. Such possibilities already exist in principle. Among various feasible techniques, we discuss elsewhere a method for restoring the HCS configuration of the past using geomagnetic data (Gulyaev and Vanyarkha, 1995).

In conclusion, we note that Sykora et al. (1995) have also drawn inferences on the need to revise the conventional view of coronal shape evolution allowing for the heliomagnetic frame of reference.
References