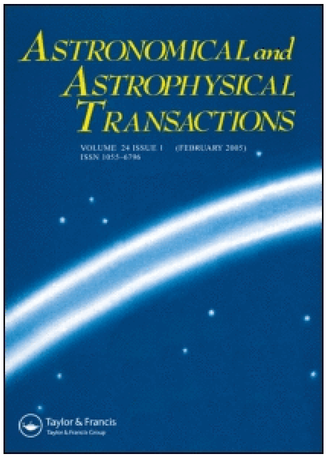

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LARGE-AMPLITUDE LANGMUIR ACOUSTIC WAVES IN A RELATIVISTIC PLASMA

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The region where large-amplitude Langmuir acoustic waves occur in a relativistic plasma is investigated using the pseudopotential method. It is shown that their occurrence depends sensitively on parameters, such as wave velocity, electrostatic potential and relativistic effect. The region of occurence increases as the relativistic effect increases. The range of the electrostatic potential increases when the velocity of drifting electrons approaches the wave velocity. New findings of large-amplitude Langmuir acoustic waves in a plasma with relativistic drifting electrons, are predicted. This theory is applicable to the results from space observations.

KEY WORDS Langmuir acoustic wave, relativistic drifting electrons, the pseudopotential method

1 INTRODUCTION

From recent space observations it has been noted that high-speed electrons play a major role in the formation of non-linear wave structures. Assuming that electron energies purely kinetic, electrons must be relativistic in the solar atmosphere and the Earth's magnetosphere (Scarf *et al.*, 1984a, b). A velocity of 5.0 keV electrons equivalent to $0.13c$ (~ 44000 km s^{-1} is found. When electron velocities take the order of 0.01–$0.1c$, where is the velocity of light, we consider their effects on non-linear plasma waves (Nejoh, 1987, 1992, 1994a). When the electron velocity approaches that of light, the non-linear waves exhibit particular properties. In fact, interplanetary space and the Earth's magnetosphere encompass a rich variety of plasma physical processes and non-linear wave phenomena (Nejoh, 1992b, 1994b; Mejoh and Sanuki, 1994, 1995). In the situation where a non-linear wave with high-speed electrons propagates in interplanetary space, solitary wave events are observed by satellities (Temerin *et al.*, 1982; Kintner, 1983). Energetic electron streams with an energy range from 1.4 to 45 keV are frequently observed in the upstream of the Earth's bow shock and high-speed solar wind (Parks *et al.*, 1981; Anderson *et al.*, 1981), and are strongly related to Langmuir acoustic waves (Marsch, 1985). It has been pointed out that the broad-band electrostatic noises detected in the upstream of the Earth's bow shock are due to Langmuir acoustic waves where ion and/or electron beams exist (Marsch, 1985). Energetic events such as $0.1 \leq T_i/T_e < 10^4$ are observed in the radiation brets (Vette, 1970) and in high-speed solar wind streams (Bame *et al.*, 1977), where $T_i(T_e)$ is the ion (electron) temperature. Thus, nonlinear wave modes such as solitary waves are of vital importance in the study of space plasmas.

Langmuir acoustic waves in plasmas have been studied by several authors (Lashmore–Davies and Martin, 1973; Sah and Goswani, 1994). Langmuir acoustic waves can propagate in a plasma where wave velocity is much less than the thermal velocity of hot ions (Yu and Shukla, 1983). Many efforts have been made to obtain stationary wave solutions for solitary waves associated with Langmuir acoustic waves in ummagnetized plasmas. However, little of the theoretical work on these topics has been done on a plasma with relativistic drifting electrons and thermal ions.

In this paper, we make an attempt to investigate theoretically the existence of large-amplitude Langmuir acoustic waves under the influence of relativistic drifting electrons in a plasma consisting of cold electrons and thermal ions. We also demonstrate the region of occurence large-amplitude Langmuir acoustic waves and study its dependence on the relativistic effect, electrostatic potential, and so on.

The layout of this paper is as follows. In Section 2, we present the basic equations for a plasma with relativistic drifting electrons and thermal ions and derive a pseudopotential for Langmuir acoustic waves. In Section 3, we define the conditions of occurrence of Langmuir acoustic waves and illustrate the dependency of this region of occurrence on several parameters. The last section is devoted to concluding discussions.

2 DERIVATION OF THE PSEUDOPOTENTIAL

We consider a plasma with cold relativistic drifting electrons and non-drifting thermal ions. The plasma is unmagnetized, collisionless and ionization free. We assume that the phase velocity of the waves is much less than the thermal velocity of ions so that ion inertia can be ignored (Yu and Shukla, 1983). The cold drifting electrons are governed by fluid equations because the relativistic electrons are in a state of equilibrium when the Langmuir acoustic waves propagate. This implies that $\kappa T_i \gg m_e v_e^2$, where κ, T_i, m_e, v_e denote the Boltzmann constant, the ion temperature, the electron mass and electron velocity, respectively. It is assumed that the background non-drifting isothermal ions are described by a Boltzmann distribution. For one-dimensional propagation of low-frequency oscillation, the continuity equation and the equation of motion for relativistic electrons are described as

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x} n_e v_e = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}\right) \gamma_e v_e - \frac{\partial \phi}{\partial x} = 0. \tag{2}$$

The system of equations is closed with the help of Poisson's equation

$$\frac{\partial^2\phi}{\partial x^2} = n_e - n_i, \tag{3}$$

with

$$n_i = \exp(-\phi),$$

where

$$\gamma_e = \left[1 - (v_e/c)^2\right]^{-1/2}. \tag{4}$$

In Equations (1)–(5), n_e, n_i are the electron and ion density, respectively, normalized by the equilibrium background electron density n_0; v_e and c are the drift velocity of electrons and the velocity of light normalized by the electron acoustic speed $(\kappa T_i/m_e)^{1/2}$. The potential ϕ, time t and space coordinate x are normalized by $(\kappa T_i/e)$, the electron plasma frequency $\omega_{\rm pe}^{-1} = (\varepsilon_0 m_e/n_e 0 e^2)^{1/2}$, and the ion Debye length $(\varepsilon_0 \kappa T_i/n_{i0}e^2)^{1/2}$, respectively.

In order to solve Eqs. (1)–(5), we introduce the variable $\xi = x - Mt$, which is the moving frame with velocity M. Integrating Eqs. (1) and (2) and using the boundary conditions, $\phi \to 0$, $n_e \to 1$, $v_e \to v_0$ at $\xi \to \infty$, we obtain

$$n_e = \frac{v_0 - M}{v_e - M}. \tag{5}$$

Integrating Eqs. (2) and using the same boundary conditions, we get the equation

$$-M\left(\frac{v_e}{\sqrt{1-\frac{v_e^2}{c^2}}} - \frac{v_0}{\sqrt{1-\frac{v_0^2}{c^2}}}\right) + c^2\left(\frac{1}{\sqrt{1-\frac{v_e^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}\right) - \phi = 0. \tag{6}$$

In the weakly relativistic case, i.e. $\gamma_e \to 1 + v_e^2/2c^2$, from Eqs. (5) and (6) we obtain the equation

$$n_e = \frac{1}{\sqrt{\frac{1+\frac{2\phi}{(v_0-M)^2}}{1-3\left(\frac{M}{c}\right)^2\left(1+\frac{M}{2(v_0-M)}\right)}}}. \tag{7}$$

We reduce Poisson's equation to

$$\frac{\partial^2\phi}{\partial^2\xi^2} = \frac{1}{\sqrt{\frac{1+\frac{2\phi}{(v_0-M)^2}}{1-3\left(\frac{M}{c}\right)^2\left(1+\frac{M}{2(v_0-M)}\right)}}} - \exp(-\phi) = -\frac{\partial U}{\partial \phi}, \tag{8}$$

where U denotes the pseudopotential. Integration of Eq. (8) gives the Energy integral

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \tag{9}$$

where

$$V(\phi) = U - W = 1 - \exp(-\phi) - (v_0 - M)^2 \times \sqrt{1 - 3\left(\frac{M}{c}\right)^2 \left(1 + \frac{M}{2(v_0 - M)}\right)} \left(\sqrt{1 + \frac{2\phi}{(v_0 - M)^2}} - 1\right) - W. \quad (10)$$

The oscillatory solution of the large-amplitude non-linear Langmuir acoustic waves exists when the following two conditions are satisfied:

(1) the potential U has a minimum value W_{min} at $\phi = 0$;

(2) the maximum potential W_{max} is satisfied when $W_{\text{max}} = U(\phi_c)$, where $\phi_c = -\frac{(v_0 - M)^2}{2}$.

Non-linear Langmuir acoustic waves exist provided that the constant energy W exceeds a minimum energy W_{min} and W is less than the maximum energy W_{max}. We obtain the large-amplitude Langmuir waves by an arbitrary choice of W in the range

$$0 < W < 1 - \exp\left(-\frac{(v_0 - M)^2}{2}\right) - 0.414(v_0 - M)^2 \times \sqrt{1 - 3\left(\frac{M}{c}\right)^2 \left(1 + \frac{M}{2(v_0 - M)}\right)}. \quad (11)$$

We used the boundary condition $V(\phi) \to 0$ at $\phi \to 0$.

It should be noted that U is real if

$$\phi > -\frac{(v_0 - M)^2}{2}, \quad (12)$$

and

$$\frac{M}{c} < \frac{1}{\sqrt{3\left(1 + \frac{M}{2(v_0 - M)}\right)}}. \quad (13)$$

Equation (13) determines the range of the relativistic effect of Langmuir waves.

We next consider the region of occurrence of large-amplitude Langmuir acoustic waves.

3 THE REGION OF OCCURRENCE OF LARGE-AMPLITUDE LANGMUIR ACOUSTIC WAVES

We show bird's eye views of the pseudopotential $-V(\phi)$ when parameter $(v_0 - M)^2 = 0.05$, where $v_0 = 1.36$, $M = 1.10$, and $(v_0 - M)^2 = 0.2$, where $v_0 = 1.58$, $M = 1.13$, in Figures 1 and 2, respectively. Figures 1 and 2 illustrate the dependence of the pseudopotential $-V(\phi)$ on the electrostatic potential ϕ and relativistic effect M/c. In the case of $(v_0 - M)^2 = 0.2$, where $v_0 = 1.58$, $M = 1.13$, we illustrate the

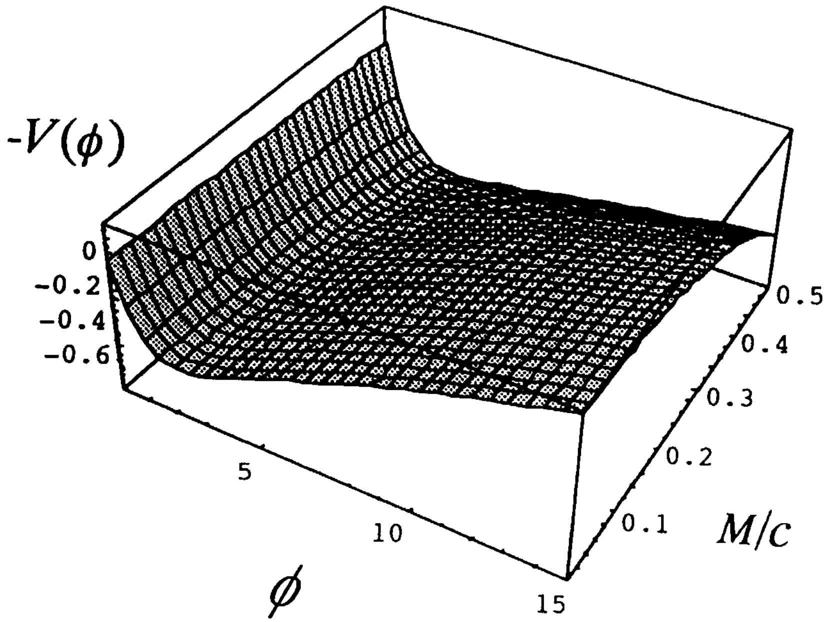

Figure 1 A bird's eye view of the pseudopotential under the conditions of $(v_0 - M)^2 = 0.05$ where $v_0 = 1.36$ and $M = 1.10$.

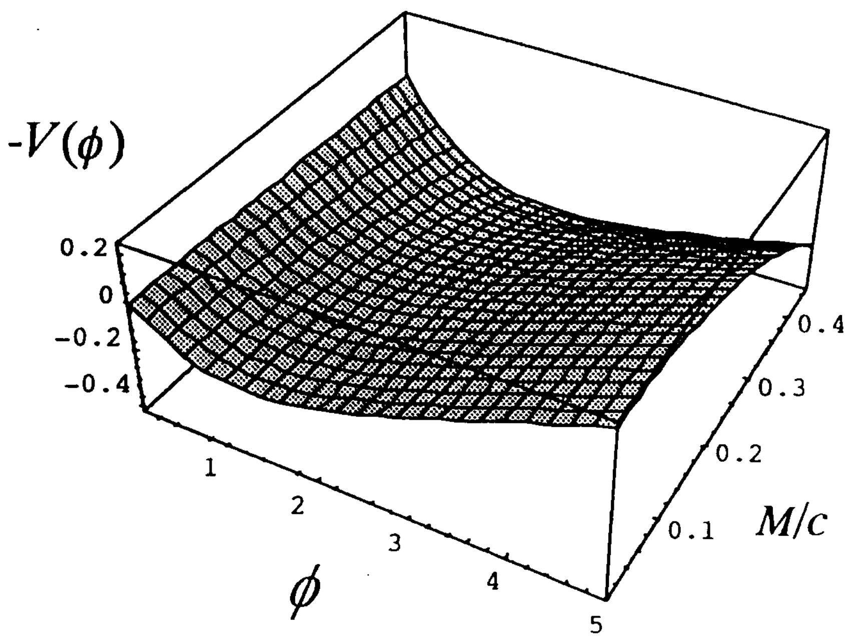

Figure 2 A bird's eye view of the pseudopotential under the conditions of $(v_0 - M)^2 = 0.20$ where $v_0 = 1.58$ and $M = 1.13$.

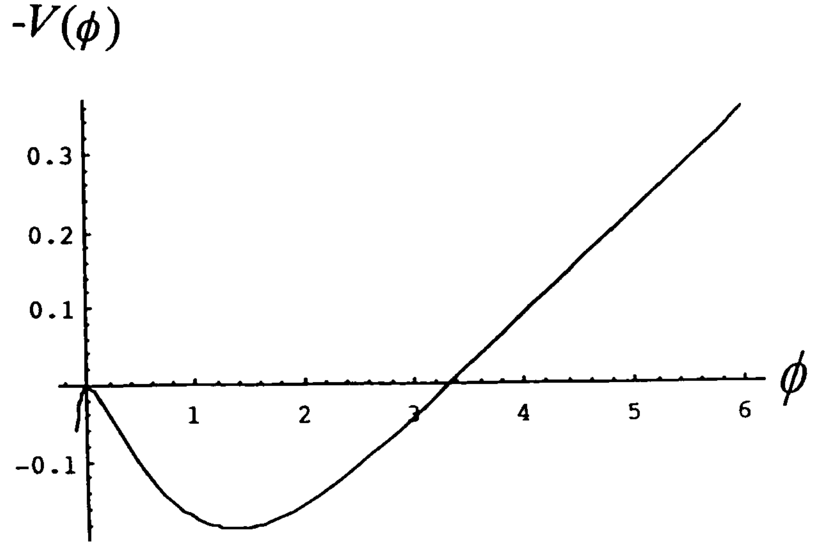

Figure 3 A pseudopotential curve of large-amplitude Langmuir acoustic waves for $(v_0 - M)^2 = 0.20$ where $v_0 = 1.58$, $M = 1.13$ and $M/c = 0.05$.

pseudopotential for $M/c = 0.05$ and 0.3 in Figures 3 and 4, respectively. We show the region of occurrence of large-amplitude Langmuir acoustic waves depending on the relativistic effect in the ϕ–M_c plane in the case of $(v_0 - m)^2 = 0.07$, where $v_0 = 1.37$, $m = 1.11$ and $(v_0 - M)^2 = 0.2$, where $v_0 = 1.58$, $M = 1.13$, respectively, in Figures 5 and 6. The large-amplitude Langmuir acoustic waves propagate in the lower region bounded by the curve but do not exist in the upper region. Figure 7 also illustrates the region of occurrence of large-amplitude Langmuir acoustic waves in the $\phi - (v_0 - M)^2$ plane for the fixed of $M/c = 0.05$. The large-amplitude Langmuir acoustic waves occur in the lower region of the curve. In addition, we show the region of propagation of large-amplitude Langmuir acoustic waves in the $M/c - (v_0 - M)^2$ plane in Figure 8. Langmuir acoustic waves also exist in the lower region of the curve.

From Figures 1–4, we can infer the following points:

(1) In the range of $\phi < 0$, the pseudopotential is always negative. In this case, since the potential well is not formed, Langmuir acoustic waves do not propagate.

(2) If $0 < \phi < 3.35$, the pseoudopotential forms the potential well. In the well, Langmuir acoustic waves can propagate.

(3) In the range of $\phi > 3.35$, the pseudopotential is always positive. In this case the well is not formed.

We also show the following.

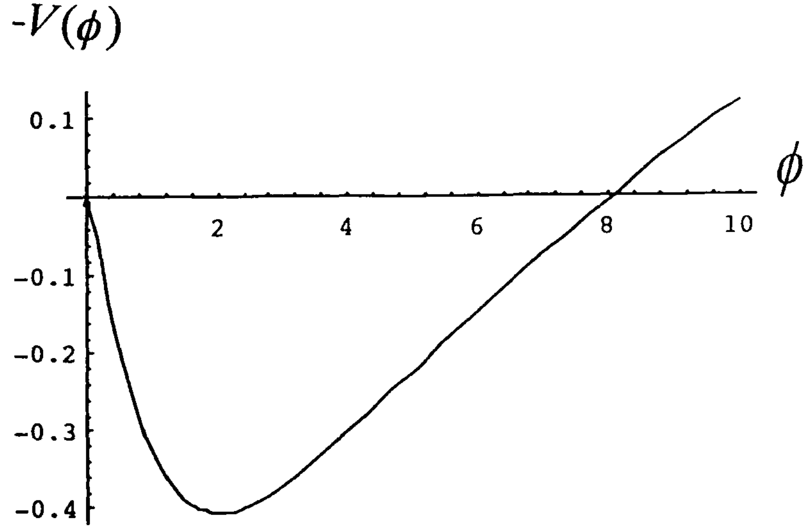

Figure 4 A pseudopotential curve of large-amplitude Langmuir acoustic waves for $(v_0 - M)^2 = 0.20$ where $v_0 = 1.58$, $M = 1.13$ and $M/c = 0.30$.

(4) The region of occurrence of Langmuir acoustic waves increases as both the electrostatic potential and relativistic effect increase, as shown in Figures 5 and 6. The amplitude of the Langmuir acoustic waves increases as the relativistic effect increases.

(5) For the fixed value of M/c, the region of the electrostatic potential narrows as the value of $(v_0 - M)^2$ increases, as seen in Figure 7. When v_0 approaches M, the amplitude of the Langmuir acoustic waves increases.

(6) As shown in the $M/c - (v_0 - M)^2$ plane of Figure 8, we understand that the region of occurrence of Langmuir acoustic waves increases as the parameter $(v_0 - M)^2$ increases for the fixed M/c and the relativistic effect increases for the fixed $(v_0 - M)^2$.

It turn out that large-amplitude Langmuir acoustic waves can propagate under the proper conditions mentioned above.

4 CONCLUDING DISCUSSION

The non-linear wave structures of large-amplitude Langmuir acoustic waves are investigated in a collisionless unmagnetized plasma. We present the region of occurrence of large-amplitude Langmuir acoustic waves on the basis of fluid equations for a plasma with relativistic drifting electrons and non-drifting thermal ions.

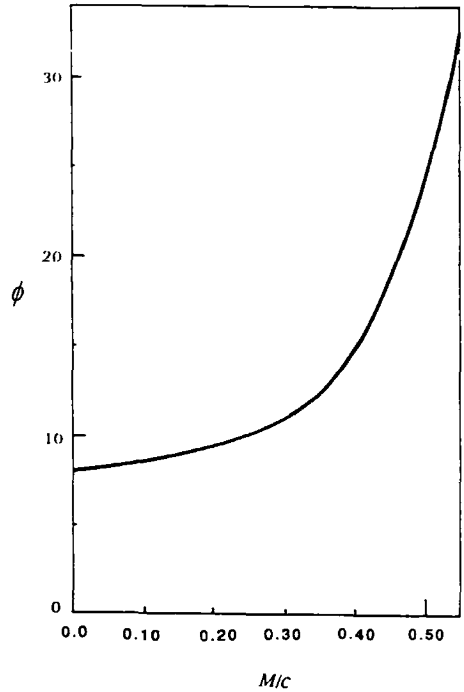

Figure 5 The region of occurrence of large-amplitude Langmuir acoustic waves in the $\phi - M/c$ plane, in the case of $(v_0 - M)^2 = 0.07$ where $v_0 = 1.36$ and $M = 1.12$. The large-amplitude Langmuir acoustic waves occur in the lower region of the curve.

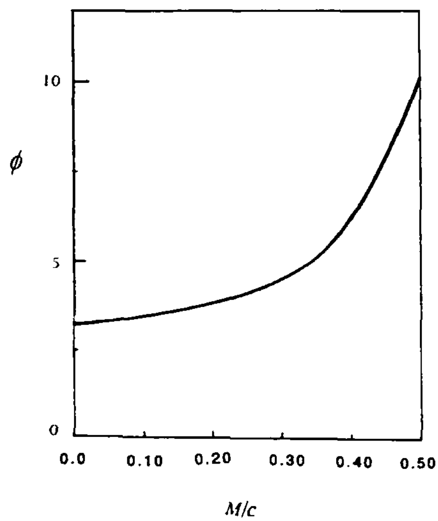

Figure 6 The region of occurrence of large-amplitude Langmuir acoustic waves in the $\phi - M/c$ plane, in the case of $(v_0 - M)^2 = 0.20$ where $v_0 = 1.58$ and $M = 1.13$. The large-amplitude Langmuir acoustic waves occur in the lower region of the curve.

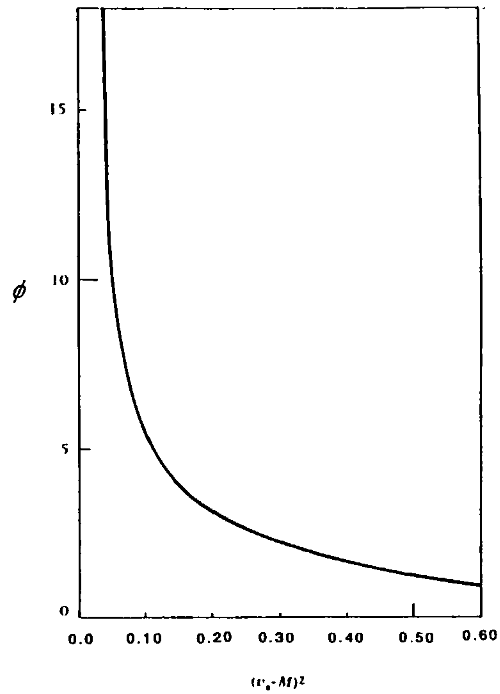

Figure 7 The region of occurrence of large-amplitude Langmuir acoustic waves in the $\phi - M/c$ plane, for the fixed value of $M/c = 0.05$. The large-amplitude Langmuir acoustic waves can propagate in the lower region of the curve.

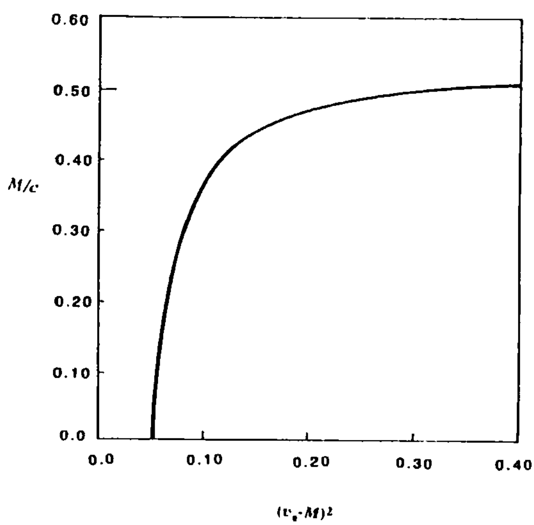

Figure 8 The region of occurrence of large-amplitude Langmuir acoustic waves in the $M/c - (v_0 - M)^2$ plane. The Langmuir acoustic waves can propagate in the lower region of the curve.

We investigated the conditions of occurrence for the stationary wave solutions of the Langmuir waves, by analyzing the structure of the pseudopotential, as illustrated in Figures 1–8. The results are briefly summarized as follows:

(1) the occurrence of conditions for large-amplitude Langmuir acoustic waves depend sensitively on the relativistic effect of drifting electrons, electrostatic potential and the velocity of the wave;

(2) the range of the electrostatic potential increases as the relativistic effect increases and the parameter $(v_0 - M)^2$ decreases;

(3) the region of occurrence increases as the relativistic effect increases and parameter $(v_0 - M)^2$ increases;

(4) the amplitude of Langmuir acoustic waves increases as the relativistic effect increases.

The present investigation predicts new findings on large-amplitude non-linear Langmuir acoustic waves in plasmas. In actual situations, Langmuir acoustic wave events associated with relativisic high-speed electrons are frequently observed in interplanetary space and the Earth's magnetosphere. Hence, the present studies enable us to understand the properties of large-amplitude Langmuir acoustic waves in space plasmas where relativistic high-speed electrons exist. Although we do not refer to any specific observations, the present theory is applicable to the analysis of large-amplitude Langmuir acoustic shock and solitary waves with high-speed electrons which may occur in space plasmas.

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