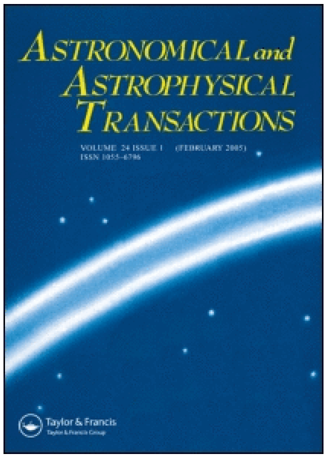

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POSSIBLE APPLICATION OF GRAPHS TO GALACTIC EVOLUTION

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Variations in the contributions of the main galactic components (i.e. gas, substars, stars and degenerate stars) to the mass of the Galaxy in the process of its evolution are considered.

KEY WORDS Galactic cosmogony, theory of graphs, stellar statistics, galactic components

Dynamical, physical and chemical processes in the Galaxy can be analyzed by means of observed statistical characteristics of its main components. These statistical characteristics are expected to be different within the frameworks of the theories of continuous and discrete star formation in the Galaxy.

In the given work we have made an attempt, using elements of the theory of graphs and the theory of active phases, to develop an algorithm for computing the ratio of the main components to be found in the Galaxy. The following data argue in favor of the theory of active phases: the age distribution of stars and star clusters; the difference between the age of the halo and the disk, the discreteness of subsystems (discontinuities of the space distribution parameters of stellar populations [1]), and a step-like form of the red dwarf mass spectrum [2].

We proceed from the classical scheme of evolution. In accordance with this scheme, the protogalactic gaseous nebula preceded the stars and substars of the first generation (their age coincides with that of the stellar phase of Galaxy). Here the ratio of the relic gas having cosmological chemical composition made up $G_{11} = 1$.

Star formation in the Galaxy is usually presented as a process of cascade fragmentation of the relic galactic nebula into the smaller and smaller fragments with minimum mass comparable with that of substars. At the same time, according to the analysis, the formation of fragments of the Jeans mass comparable to that of large spherical clusters took place.

There are usually groups of stars to originate and disintegrate in the process of evolution. The largest star clusters whose masses are comparable with the Jeans mass of the primary fragmentation of the Galaxy, must have "fallen" into the center of the Galaxy because of the dynamic stellar friction [3] and formed its nuclear. Intermediate and small stellar clusters must dissipate. The older the cluster is the greater mass must be observed. Young clusters whose masses come to several hundred solar masses can be observed now. Those with smaller masses must disintegrate since the moment of the last star formation. Considering these processes as unified succession of the cascade fragmentation we can be describe the change of the main statistical characteristics of the Galaxy in the process of evolution on the base of discrete star formation.

Let the set of probabilities of the key events defining evolution in the Galaxy be as follows:

(a) a^i_{rs}, the ratio of matter masses remained after star cluster formation ($i = 0$) and spent on star formation ($i = 1$);

(b) v^j_{rs}, the ratio of the cluster mass directed towards the galactic center because of dynamic stellar friction [3], $j = 1$, and preserved in stellar clusters ($j = 2$) until the end of r-th phase of star formation;

(c) g^{k1}_{rsi}, the ratio of matter mass remained after the formation of stars and substars in the galactic nucleus, field and clusters ($k = 1$–4);

(d) g^{k2}_{rsi}, g^{k3}_{rsi}, the relative quantities of matter spent to star and substar formation, respectively (k is analogous to that in (c);

(e) w^{kj}_{rsi}, the ratio of matter consumed by a star ($j = 2$) of the r-th generation during its lifetime on the main sequence. $j - 1$ corresponds to the number of stars remained after the final stage of the next star formation. Some of them remain on the main sequence (p^{k1}_{rsi}), others (p^{k2}_{rsi}) lose their envelopes and turn into degenerate stars g^{k1}_{rsi} and the gas component g^{k2}_{rsi};

(f) p^{kj}_{rsi}, the ratio of stars with $m \leq m_{rsi}$ ($m > m_{rsi}$), whose lifetime on the sequence is more, $j = 1$ (less $j = 2$), than the period of the next star formation.

Index i in the probabilities g, w, p, q indicates their belonging to the i-th level.

Then, on arranging the probabilities of the set X in the junctions of graphs and subgraphs in such a way that the equation $\sum x = 1$ holds true, it is possible to form oriented graphs $\sigma_{rs}(X, U)$ which describe sequence of determined events in the Galaxy (see Figures 1 and 2). These sequence can be determined from the chains consisting of arcs which enter into the set U. Figure 1 shows the graph $\sigma_{rs}(X, U)$, where f^k_{rs} are apexes of subgraphs that describe cascade fragmentation of the nuclear ($k = 1$), of the field ($k = 2, 3$) and of the clusters ($k = 4$). In Figure 2 the number of levels depends upon their belonging to the population k of the Galaxy at different stages of evolution.

The first stage of star formation ($r = 1$) turned out to be described by means of a single graph $\sigma_{11}(X, U)$, the second one by means of two incoherent graphs $\sigma_{21}(X, U)$ and $\sigma_{22}(X, U)$, the third one by means of three graphs $\sigma_{31}(X, U)$, $\sigma_{32}(X, U)$ and $\sigma_{33}(X, U)$, etc.

Here $s = 1, 2, 3 \ldots$ describes "conditionally"† the evolution of the components of the Galaxy on the base of s – multipleconverted substance.

In order to compute relative masses of basic components at the end of the r-th star formation it is necessary to choose the sequences of events interest, write down

† "Conditionally, because, in fact, gaseous matter in the Galaxy is mixed.

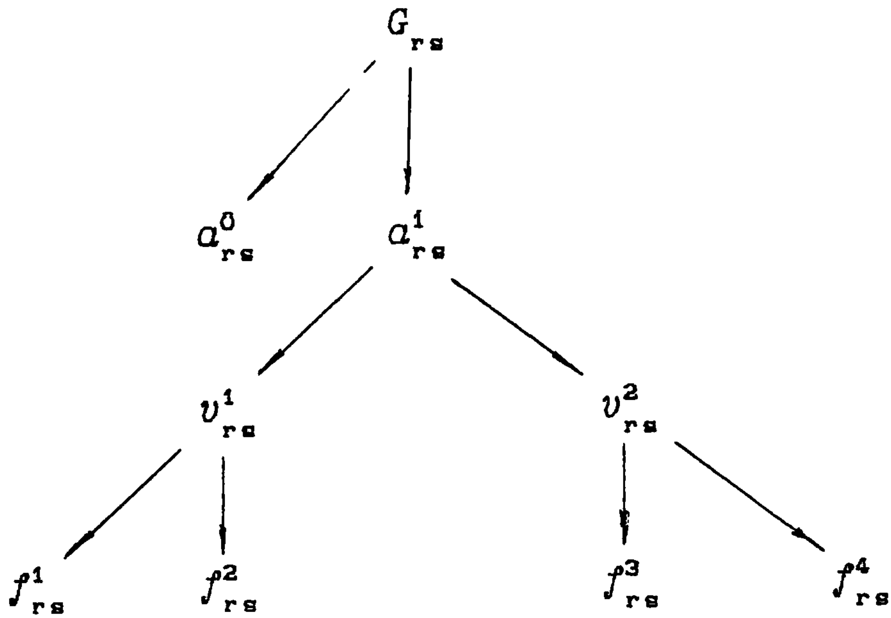

Figure 1

their directions and calculate their moduli. By the moduli we the product of values that enter into the set which include all the probabilities of the given chain.

Since there are 2 arcs to leave the vertex of the subgraph $\sigma_{rs}(X, U) - \sigma_{rs}$, it is necessary to compute to determine the total gas ratio. These chains can be represented in the following forms:

$$\alpha_{rs} = a_{rs} \sum_{j=1}^{2} v^j_{rs} \sum_{k=2j-1}^{2j} f^{1k}_{rs} g^{k1}_{rs}, \tag{1}$$

$$\beta_{rs} = a_{rs} \sum_{j=1}^{2} v^j_{rs} \sum_{k=2j-1}^{2j} f^{1k}_{rs} g^{k3}_{rs} \sum_{i=1}^{2} w^{ki}_{rs} p^{k(i+1)}_{rs} q^{k(i+1)}_{rs}, \tag{2}$$

where $p^{k(i+1)}_{rs} \equiv 1$, $q^{k(i+1)}_{rs} \equiv 1$, at $i \geq 2$.

Chains that correspond to the ends of events [formation of substars (γ_{rs}), stars (δ_{rs}) and degenerated stars (ε_{rs})] can be represented in the following form:

$$\gamma_{rs} = a_{rs} \sum_{j=1}^{2} v^j_{rs} \sum_{2j-1}^{2j} f^{1k}_{rs} g^{k2}_{rs}, \tag{3}$$

$$\delta_{rs} = a_{rs} \sum_{j=1}^{2} v^j_{rs} \sum_{2j-1}^{2j} f^{1k}_{rs} g^{k3}_{rs} w^{k1}_{rs} p^{k1}_{rs}, \tag{4}$$

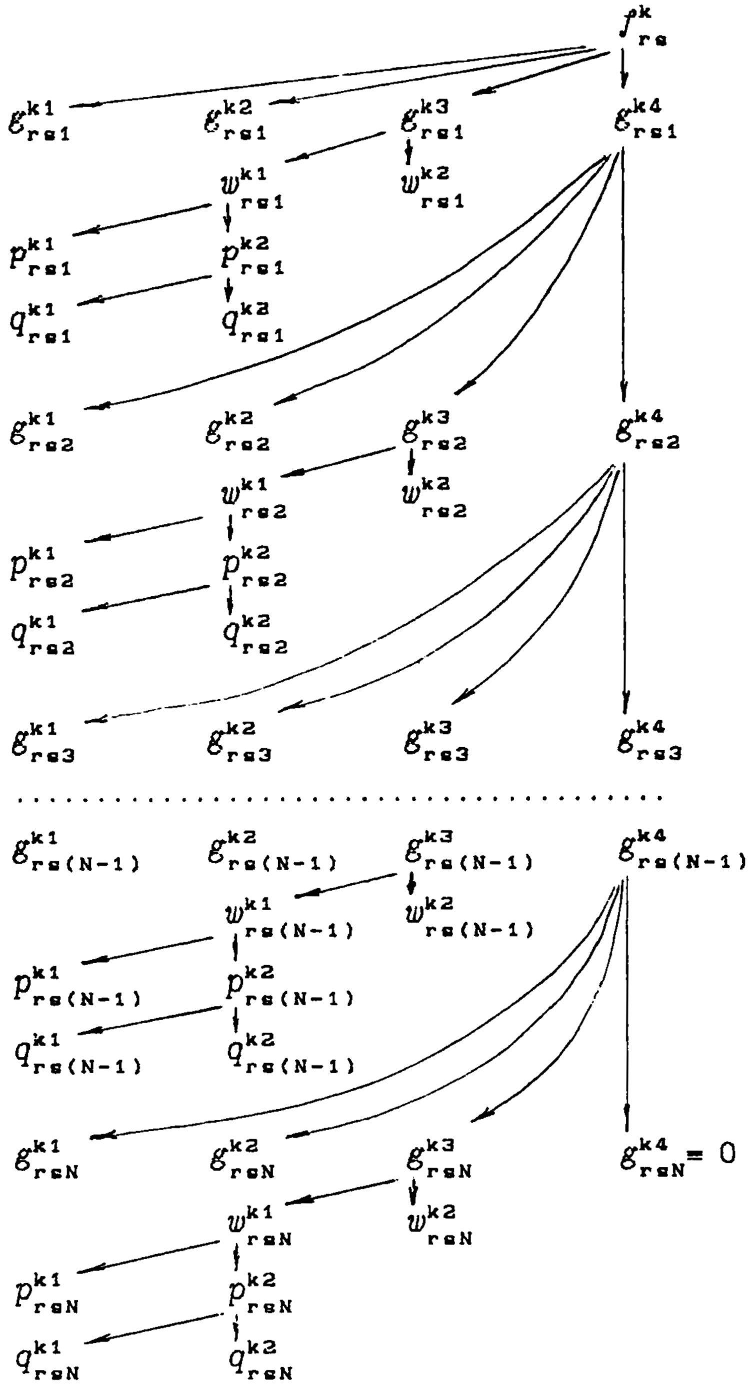
f^{k}_{rs}
g^{k1}_{rs1} g^{k2}_{rs1} g^{k3}_{rs1} g^{k4}_{rs1}
w^{k1}_{rs1} w^{k2}_{rs1}
p^{k1}_{rs1} p^{k2}_{rs1}
q^{k1}_{rs1} q^{k2}_{rs1}
g^{k1}_{rs2} g^{k2}_{rs2} g^{k3}_{rs2} g^{k4}_{rs2}
w^{k1}_{rs2} w^{k2}_{rs2}
p^{k1}_{rs2} p^{k2}_{rs2}
q^{k1}_{rs2} q^{k2}_{rs2}
g^{k1}_{rs3} g^{k2}_{rs3} g^{k3}_{rs3} g^{k4}_{rs3}
$g^{k1}_{rs(N-1)}$ $g^{k2}_{rs(N-1)}$ $g^{k3}_{rs(N-1)}$ $g^{k4}_{rs(N-1)}$
$w^{k1}_{rs(N-1)}$ $w^{k2}_{rs(N-1)}$
$p^{k1}_{rs(N-1)}$ $p^{k2}_{rs(N-1)}$
$q^{k1}_{rs(N-1)}$ $q^{k2}_{rs(N-1)}$
g^{k1}_{rsN} g^{k2}_{rsN} g^{k3}_{rsN} $g^{k4}_{rsN} = 0$
w^{k1}_{rsN} w^{k2}_{rsN}
p^{k1}_{rsN} p^{k2}_{rsN}
q^{k1}_{rsN} q^{k2}_{rsN}

Figure 2

$$\varepsilon_{rs} = a_{rs} \sum_{j=1}^{2} v_{rs}^{j} \sum_{2j-1}^{2j} f_{rs}^{1k} g_{rs}^{k3} w_{rs}^{k1} p_{rs}^{k2} q_{rs}^{k1}. \tag{5}$$

The contribution of the gas component (G), of the substar component (S), that of stars (Z) and that of degenerate stars (D) after n of star formation can be described by the following matrix:

$$\eta = \begin{bmatrix} Y_{11} & 0 & \dots & 0 & \dots & 0 \\ Y_{21} & Y_{22} & \dots & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ Y_{r1} & Y_{r2} & \dots & Y_{rs} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ Y_{t1} & Y_{t2} & \dots & Y_{ts} & \dots & Y_{tt} \end{bmatrix} \tag{6}$$

where $\{G_{rs},\ S_{rs},\ Z_{rs},\ D_{rs}\} \in Y_{rs} \in Y$, $1 \le r \le t$, $1 \le s \le t$, $t = n+1$ for G component since the initial content of gas in the protogalaxy was G_{11}, and at the end of the first star formation it was $G_1 = G_{21} + G_{22}$, and $t = n$ for the components S, Z and D.

The contribution of the components entering the set Y (by the end of any stage of star formation) can be evaluated from the following matrix:

$$\eta \cdot X^0 = \begin{bmatrix} Y_1 \\ Y_2 \\ \dots \\ Y_r \\ \dots \\ Y_t \end{bmatrix}, \tag{7}$$

where $Y_r = \sum_{i=1}^{s} Y_{ri}$, and X^σ is the unit matrix.

The analytical form of Y_{ri} for the galactic components (except for the gas) is

$$Y_{ri} = G_{ri} y_{ri}, \tag{8}$$

where $\{\gamma_{rs},\ \delta_{rs},\ \varepsilon_{rs}\} \in y_{rs}$.

Values G_{ri} can be calculated in a similar manner, but it is necessary to take into account the condition for the gas given above: $t = n+1$ and also the vertex of the subgraph G_{rs} into two arcs.

The Galaxy is supposed to be going through the 4-th stage of star formation [1]; $n = 4$. Then the expression for G_{ri} can be written explicitly:

$$G_{11} \equiv 1, \tag{9}$$

$$G_{21} = G_{11}\alpha_{11} = \alpha_{11}, \tag{10}$$

$$G_{22} = G_{11}\beta_{11} = \beta_{11}, \tag{11}$$

$$G_{31} = G_{21}\alpha_{21} = G_{11}\alpha_{11}\alpha_{21} = \prod_{i=1}^{2} \alpha_{i1}, \tag{12}$$

$$G_{32} = G_{21}\beta_{21} + G_{22}\alpha_{22} = G_{11}\alpha_{11}\beta_{21} + G_{11}\beta_{11}\alpha_{22} = \alpha_{11}\beta_{21} + \alpha_{22}\beta_{11}, \tag{13}$$

$$G_{33} = G_{22}\beta_{22} = G_{11}\beta_{21}\beta_{22} = \prod_{i=1}^{2} \beta_{ii}, \tag{14}$$

$$G_{41} = G_{31}\alpha_{31} = \prod_{i=1}^{3} \alpha_{i1}, \tag{15}$$

$$G_{42} = G_{31}\beta_{31} + G_{32}\alpha_{32} = \beta_{31}\prod_{i=1}^{2} \alpha_{i1} + \alpha_{32}(\alpha_{11}\beta_{21} + \alpha_{22}\beta_{11}), \tag{16}$$

$$G_{43} = G_{32}\beta_{32} + G_{33}\alpha_{33} = \beta_{32}(\alpha_{11}\beta_{21} + \alpha_{22}\beta_{11}) + \alpha_{33}\prod_{i=1}^{2} \beta_{ii}, \tag{17}$$

$$G_{44} = G_{33}\beta_{33} = \prod_{i=1}^{3} \beta_{ii}, \tag{18}$$

$$G_{51} = G_{41}\alpha_{41} = \prod_{i=1}^{4} \alpha_{i1}, \tag{19}$$

$$\begin{aligned} G_{52} &= G_{41}\beta_{41} + G_{42}\alpha_{42} \\ &= \beta_{41}\prod_{i=1}^{3} \alpha_{i1} + \alpha_{42}[\beta_{31}\prod_{i=1}^{2} \alpha_{i1} + \alpha_{32}(\alpha_{11}\beta_{21} + \alpha_{22}\beta_{11})], \end{aligned} \tag{20}$$

$$\begin{aligned} G_{53} &= G_{42}\beta_{42} + G_{43}\alpha_{43} \\ &= \beta_{42}[\beta_{31}\prod_{i=1}^{2} \alpha_{i1} + \alpha_{32}(\alpha_{11}\beta_{21} + \alpha_{22}\beta_{11})] \\ &+ \alpha_{43}[\alpha_{33}\prod_{i=1}^{2} \beta_{ii} + \beta_{32}(\alpha_{11}\beta_{21} + \alpha_{22}\beta_{11})], \end{aligned} \tag{21}$$

$$\begin{aligned} G_{54} &= G_{43}\beta_{43} + G_{44}\alpha_{44} \\ &= \beta_{43}[\alpha_{33}\prod_{i=1}^{2} \beta_{ii} + \beta_{32}(\alpha_{11}\beta_{21} + \alpha_{22}\beta_{11})] + \alpha_{44}\prod_{i=1}^{3} \beta_{ii}, \end{aligned} \tag{22}$$

$$G_{55} = G_{44}\beta_{44} = \prod_{i=1}^{4} \beta_{ii}. \tag{23}$$

As has been mentioned above, the galactic components are distributed in the nucleus, in the field and the clusters of the Galaxy.

In the order to calculate their contribution to populations N, F and C, let present the chains of graph $\sigma_{rs}(X, U)$ as the following sums:

$$\alpha_{rs}^{N} + \alpha_{rs}^{F} + \alpha_{rs}^{C} = \alpha_{rs}, \tag{24}$$

$$\beta_{rs}^{N} + \beta_{rs}^{F} + \beta_{rs}^{C} = \beta_{rs}, \tag{25}$$

$$\gamma_{rs}^{N} + \gamma_{rs}^{F} + \gamma_{rs}^{C} = \gamma_{rs}, \tag{26}$$

$$\delta_{rs}^{N} + \delta_{rs}^{F} + \delta_{rs}^{C} = \delta_{rs}, \tag{27}$$

$$\varepsilon_{rs}^{N} + \varepsilon_{rs}^{F} + \varepsilon_{rs}^{C} = \varepsilon_{rs}, \tag{28}$$

Here the indices indicate definite population.

The ratio of gas and any value entering expressions (24–28) can be calculated by means of the graph $\sigma_{rs}(X, U)$.

Hence, the contribution of any component to population L for the r-th star formation can be presented the following form:

$$G_{(r-1)s}^{L}, \quad Y_{rs}^{L} = G_{rs}^{L} y_{rs}^{L}. \tag{29}$$

As a result we obtain the present-day distribution of the components in the Galaxy:

in the nucleus

$$N = \sum_{i=1}^{5} G_{i5}^{N} + \sum_{i=1}^{4} S_{i4}^{N} + \sum_{i=1}^{4} Z_{i4}^{N} + \sum_{i=1}^{4} D_{i4}^{N}, \tag{30}$$

in the field

$$F = \sum_{i=1}^{5} G_{i5}^{F} + \sum_{i=1}^{4} S_{i4}^{F} + \sum_{i=1}^{4} Z_{i4}^{F} + \sum_{i=1}^{4} D_{i4}^{F}, \tag{31}$$

in the clusters

$$C = \sum_{i=1}^{5} G_{i5}^{C} + \sum_{i=1}^{4} S_{i4}^{C} + \sum_{i=1}^{4} Z_{i4}^{C} + \sum_{i=1}^{4} D_{i4}^{C}, \tag{32}$$

In this paper we have suggested a general idea concerning the applicability of the theory of the galactic cosmogony. This mathematical technique might be quite effective for the investigation of the structure and evolution of the Galaxy. Further investigations would it possible to solve the problems of galactic matter, multiple system dissemination in the Galaxy, etc.

In particular, this approach is effective for the calculation of the number of planetary and substellar systems [4].

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