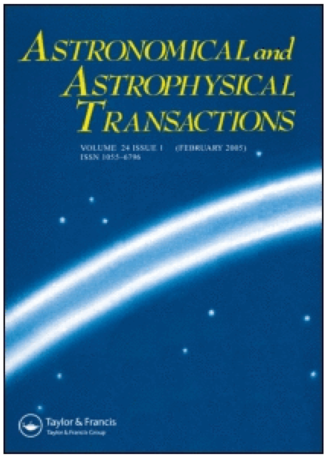

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THE VARIATION OF PHYSICAL CONSTANTS AND RED SHIFT IN 6D COSMOLOGICAL MODELS

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On the basis of the obtained general class of solutions of 6D vacuum Einsteins equations and using the well-known results for the 6D geometric model of electroweak and gravitational interactions [1], the variations of the fundamental physical constants and their influence on the observed red shift of galaxies were investigated. Considering extra components of the 6D metric as reproducing the properties of matter in the 4D curved space-time, the effective quantities of matter were obtained. In particular, the states of dust and radiation were shown. The corresponding 5D models were also investigated.

1 INTRODUCTION

The present work was carried out within the frame of the 6-dimensional geometric model of electroweak and gravitational interactions [1, 2] in which the classical 5D Kaluza theory is generalized [3]. In this theory, both electromagnetic and Z-boson fields are described by components σ_{M5} and σ_{M6} of 6D metric σ_{MN}. In this paper the capital Latin letters assume the values 0, 1, 2, 3, 5, 6 and the Greek letters, the 4D values 0, 1, 2, 3. Charged W-bosons are conditioned to depend on the extra dimensions of these metric components. Higgs scalar bosons are introduced in the metric by considering a conformal factor which is expressed in terms of σ_{55} (or σ_{66}) metric components.

Toroidal topology was used in this theory, i.e. compactness of additional dimensions is assumed. The particular cases of this 6D theory are Einstein's theory of general relativity and the well-known Weinberg–Salam model of electroweak interactions.

It is important to remark that an essential feature of this 6D theory is that a series of fundamental physical constants can be described in terms of σ_{55} and σ_{66} metric components. Thus, for electric charge e, interaction constant with Z-boson $\bar{g}$, Weinberg angle Θ_W and Z-boson mass m_z, the following expressions have been obtained:

$$e = \frac{\sqrt{k}\hbar\tilde{\alpha}}{c}\frac{\lambda^5 T^6}{\sqrt{(\lambda^5)^2 + (T^6)^2}}, \quad \bar{g} = \frac{4\sqrt{k}\hbar\tilde{\alpha}}{c}\sqrt{(\lambda^5)^2 + (T^6)^2},$$

$$\sin\Theta_W = \frac{\lambda^5}{\sqrt{(\lambda^5)^2 + (T^6)^2}}, \quad m_z = \frac{4\sqrt{5}\hbar\tilde{\alpha}(b_0\eta_0)}{c}\sqrt{(\lambda^5)^2 + (T^6)^2}, \tag{1}$$

where k is the Newtonian gravitational constant, $\hbar$ is the Planck constant, $\tilde{\alpha}$ is the parameter defining the radius of compactness of extra dimensions, $b_0\eta_0$ is massive factor, λ^N and T^N are components of the orthonormal 6D vectors, orthogonal to metric $g_{\mu\nu}$ of the 4D space-time section ($g_{\mathrm{MN}}\lambda^N = g_{\mathrm{MN}}T^N = 0$). The last ones are determined by 1+1+4 – splitting procedure from 6D metric

$$\sigma_{\mathrm{MN}} = g_{\mathrm{MN}} - \lambda_M\lambda_N - T_M T_N. \tag{2}$$

In special gauge procedure analogous to the 2-multiple chronometrical one in the general theory of relativity, the contravariant components of vectors λ^N and T^N have the following form:

$$\lambda^M = 0, \quad \lambda^5 = \frac{\sqrt{-\sigma_{66}}}{\sqrt{\sigma_{55}\sigma_{66} - \sigma_{56}^2}}, \quad \lambda^6 = \frac{\sigma_{56}}{\sqrt{-\sigma_{66}(\sigma_{55}\sigma_{66} - \sigma_{56}^2}},$$

$$T^M = 0, \quad T^5 = 0, \quad T^6 = \frac{1}{\sqrt{-\sigma_{66}}}. \tag{3}$$

The covariant four-dimensional components of the metric tensor are:

$$g_{\alpha\beta} = \sigma_{\alpha\beta} - \frac{\sigma_{\alpha 5}\sigma_{\beta 6}\sigma_{55} + \sigma_{\alpha 5}\sigma_{\beta 5}\sigma_{66}}{\sigma_{55}\sigma_{66} - \sigma_{56}^2} + \frac{\sigma_{56}(\sigma_{6\alpha}\sigma_{5\beta} + \sigma_{5\alpha}\sigma_{6\beta})}{\sigma_{55}\sigma_{66} - \sigma_{56}^2},$$

$$\lambda_\alpha = \frac{\sigma_{56}\sigma_{6\alpha} - \sigma_{66}\sigma_{5\alpha}}{\sqrt{-\sigma_{66}}\sqrt{\sigma_{55}\sigma_{66} - \sigma_{56}^2}}, \quad T_\alpha = \frac{\sigma_{6\alpha}}{\sqrt{\sigma_{66}}}. \tag{4}$$

2 HOMOGENEOUS AND ISOTROPIC 6D COSMOLOGICAL MODELS

In the present paper, keeping within the bounds of homogeneous and isotropic (in three-dimensional sense) 6D cosmological models, possible consequences due to variable values of physical constants are analysed. With this object, a series of corresponding exact solutions were found for the six-dimensional Einstein's equations:

$${}^6R_{\mathrm{MN}} - \frac{1}{2}\sigma_{\mathrm{MN}}^6 R = \tilde{\varkappa}T_{\mathrm{MN}}, \tag{5}$$

where $\tilde{\varkappa}$ is the generalized Einstein gravitational constant, T_{MN} is the 6D stress-energy tensor of nongeometrical matter. The metric was searched in the form

$$dI^2 = \varphi^2\{a^2[d\chi^{0^2} - d\chi^{1^2} - F^2(d\Theta^2 + \sin^2\Theta d\phi^2)] - d\chi^{5^2} - \Psi^2 d\chi^{6^2}\}, \tag{6}$$

where $\sigma_{55} = -\varphi^2 \cdot \sigma_{66} = -\varphi^2\Psi^2$ are the extra diagonal metric components, the function $F^2(\chi^1)$ acquires three forms: $F^2 = \mathrm{sh}^2\chi^1$ for hyperbolic space section, $F^2 = {\chi^1}^2$ for Euclidean space and $F^2 = \sin^2\chi^1$ for 3D Riemaniann space section. There are three unknown functions $\varphi(\chi^0)$, $a(\chi^0)$, $\Psi(\chi^0)$ depending on the time-like coordinate χ^0, which are yet to be defined. In our previous articles, a series of exact solutions were found for a dust type source [4, 5] and for multidimensional vacuum [6].

An important aim of this paper is to find vacuum solutions for six-dimensional Einstein equations (5). For the metric (6), using equations (5), we obtain the following equations:

$$3(\dot a/a)^2 + 10(\dot\varphi/\varphi)^2 + 12\dot a\dot\varphi/a\varphi + 3(\dot\Psi/\Psi)[\dot a/a + (4/3)\dot\varphi/\varphi] + 3\eta = 0, \tag{7}$$

$$2\ddot a/a - (\dot a/a)^2 + 12(\dot\varphi/\varphi)^2 + 4\dot a\dot\varphi/a\varphi + (\dot\Psi/\Psi)(\dot a/a + 4\dot\varphi/\varphi) + \ddot\Psi/\Psi + 4\ddot\varphi/\varphi + \eta = 0, \tag{8}$$

$$3\ddot a/a + 2(\dot\varphi/\varphi)^2 + 8\dot a\dot\varphi/a\varphi + 4\ddot\varphi/\varphi + 2(\dot\Psi/\Psi)(\dot a/a + 2\dot\varphi/\varphi) + \ddot\Psi/\Psi + 3\eta = 0, \tag{9}$$

$$3\ddot a/a + 2(\dot\varphi/\varphi)^2 + 8\dot a\dot\varphi/a\varphi + 4\ddot\varphi/\varphi + 3\eta = 0, \tag{10}$$

where parameter η may take three values, namely, $\eta = +1$ for a 3D space with constant and positive curvature, $\eta = -1$ for a 3D Lobachevsky space, $\eta = 0$ for a 3D Euclidean space. The overdot $(\dot{\ })$ denotes total derivative.

From equations (7)–(10) on substituting $\xi = \varphi\Psi$ and using the condition

$$\frac{\dot\xi}{\xi} = \alpha\frac{\dot\varphi}{\varphi}, \tag{11}$$

where α is a factor of proportionality, we get the following general class of vacuum solutions:

The flat models $(\eta = 0)$

$$a = a_0(c \pm \sqrt{\lambda}\chi^0)^{[\sqrt{\lambda}\pm(\alpha+3)]/2\sqrt{\lambda}}, \quad \varphi = \varphi_0(c \pm \sqrt{\lambda}\chi^0)^{\mp 1/\sqrt{\lambda}},$$
$$\Psi = \Psi_0(c \pm \sqrt{\lambda}\chi^0)^{\mp(\alpha-1)/\sqrt{\lambda}}, \quad \lambda^2 = \alpha^2 + \frac{2}{3}\alpha + 1, \quad c = \text{const}. \tag{12}$$

The open models $(\eta = -1)$.

$$a = a_0 \mathrm{sh}\,\Theta^{1/2}(\tanh\Theta/2)^{\pm(\alpha+3)/2\sqrt{\lambda}}, \quad \varphi = \varphi_0(\tanh\Theta/2)^{\mp 1/\sqrt{\lambda}},$$
$$\Psi = \Psi_0(\tanh\Theta/2)^{\mp(\alpha-1)/\sqrt{\lambda}}, \quad \Theta = c - 2\chi^0, c = \text{const}. \tag{13}$$

The close models $(\eta = +1)$.

$$a = a_0\cos\beta^{1/2}[\tan(\beta/2 + \pi/4)]^{\mp(\alpha+3)/2\sqrt{\lambda}}, \quad \varphi = \varphi_0[\tan(\beta/2 + \pi/4)]^{\pm 1/\sqrt{\lambda}},$$
$$\Psi = \Psi_0[\tan(\beta/2 + \pi/4)]^{\pm(\alpha-1)/\sqrt{\lambda}}, \quad \beta = c + 2\chi^0, \quad c = \text{const}. \tag{14}$$

Here the parameter α may take any real value.

We shall restrict our considerations to the case of cosmological models with flat space section (for the open and close models, calculations may be carried out in a similar way).

It should be emphasized that χ^0 is only a time-like parameter from which the physical time τ is calculated like in the Friedmann model, by considering the scale factor. However, here in the six-dimensional theory, there are two possible ways of choosing the time. Firstly, we choose the function $a(\chi^0)$ as a scale factor; then for the flat model (12) we get

$$\tau = \frac{\pm 2a_0}{3\sqrt{\lambda} \pm (\alpha+3)}(c \pm \sqrt{\lambda}\chi^0)^{[3\sqrt{\lambda}\pm(\alpha+3)]/2\sqrt{\lambda}}, \quad \sqrt{\lambda} \pm (\alpha+3) > 0. \tag{15}$$

In the second possibility, the scale factor is chosen in the form $a(\chi^0)\varphi(\chi^0)$ and consequently

$$\tilde{\tau} = \frac{\pm 2a_0\varphi_0}{3\sqrt{\lambda} \pm (\alpha+1)}(c + \sqrt{\lambda}\chi^0)^{[3\sqrt{\lambda}\pm(\alpha+1)]/2\sqrt{\lambda}}, \quad \sqrt{\lambda} \pm (\alpha+1) > 0. \tag{16}$$

From expressions (1), using (12) and taking into account (15), we obtain the following laws for the variation of physical constants:

$$\begin{gathered} e = e_0(1+\tilde{\varphi}_0^{-1}T^{-\gamma})^{-1/2}, \quad \sin\Theta_W = (1+\tilde{\varphi}_0 T^{\gamma})^{-1/2}, \\ \overline{g} = \overline{g}_0(1+\tilde{\varphi}_0 T^{\gamma})^{1/2}, \quad m_z = m_{z0}(1+\tilde{\varphi}_0 T^{\gamma})^{1/2}, \end{gathered} \tag{17}$$

where e_0, $\overline{g}_0$, m_{z0} are constant quantities. Hereafter we denote:

$$\gamma = \pm 4(\alpha-1)/[3\sqrt{\lambda}\pm(\alpha+3)], \; \tilde{\varphi}_0 = \varphi_0^{2(1-\alpha)}\Psi_0^{-2}, \; T = \pm[3\sqrt{\lambda}\pm(\alpha+3)]\tau/2a_0. \tag{18}$$

Thus, it follows from (17) that for the expanding model we have an increasing electric charge e, which asymptotically approaches the constant value e_0 as $\tau \to \infty$. At the same time, the Weinberg angle asymptotically decreases from the value $\pi/2$ at $\tau = 0$ to zero as $\tau \to \infty$. The behaviour of $\overline{g}$ and m_z is similar to each other, their values increase from constants $\overline{g}_0$ and m_{z0} at $\tau = 0$ and approach infinity as $\tau \to \infty$.

3 CONTRIBUTION OF THE VARIATION OF CONSTANTS TO THE RED SHIFT OF SPECTRAL LINES

In the 4D general theory of relativity, the evolution of universe is described by the Friedmann models. For instance, corresponding to the experimentally obtained values of the Hubble red shift of the spectral lines, such solutions are chosen which describe the expanding model at least for some stage of evolution. At the same time, it is postulated that all physical constants remain invariant in time. Here in the six-dimensional theory, we cannot use such assumption and consequently the interpretation of the observed red shift should be reconsidered. Two factors are proposed to contribute to the red shift. First, like in Einstein's theory, the Doppler receding effect of the galaxies $(\Delta\nu/\nu)_D$ and second, the displacement effect of spectral lines due to different values of physical constants $(\Delta\nu/\nu)_{v.c.}$ at the moments when the photon is emitted and absorbed.

Thus, the total effect $(\Delta\nu/\nu)_\Sigma$ of relative displacement of spectral lines may be written as follows:

$$(\Delta\nu/\nu)_\Sigma = (\Delta\nu/\nu)_D + (\Delta\nu/\nu)_{v.c.} \tag{19}$$

To estimate $(\Delta\nu/\nu)_{v.c.}$, it is necessary to substitute the electric charge expression from (17) into the well-known formula for the values of energy levels in hydrogen-like atoms. After the corresponding calculations, we get the following expression:

$$(\Delta\nu/\nu)_{v.c.} = 2\gamma \frac{\tilde{\varphi}_0^{-1} T^{-\gamma}}{1 + \hat{\varphi}_0^{-1} T^{-\gamma}} \cdot \frac{\Delta\tau}{\tau}. \tag{20}$$

From this expression, using (15) we find that for models with $\alpha > 1$ in the expanding case, $(\Delta\nu/\nu)_{v.c.}$ contributes a positive value to the total effect (19), i.e. it increases the Doppler red shift, so that in the early stages of evolution of such universes an intensified red shift must be observed. For essentially later stages of evolution, $(\Delta\nu/\nu)_{v.c.}$ does not contribute appreciably.

For the models with $\alpha < 1$, there is no contribution from $(\Delta\nu/\nu)_{v.c.}$ in the early stages of the universe, but for large values of τ this contribution becomes negative and the violet shift begins to be significantly large, which results in the weakening of the red shift. Thus, in principle there exist expanding models of universe with fixed values of the parameter $\alpha < 1$, for which a total violet shift occurs for the later stages of evolution.

In the case of contracting models, from (20) and considering (15) it is not difficult to see that there do not exist universes for values of $\alpha > 0$. For $\alpha < -\frac{3}{2}$, there are possible universe models in the early stages for which $(\Delta\nu/\nu)_{v.c.}$ increase the usual displacement effect, so that it results in an intesified red shift. At the later stages of such universes, the contribution from $(\Delta\nu/\nu)_{v.c.}$ is very small and it may be ignored.

For $\alpha = 1$, we have $\gamma = 0$. For such universes, the variation in the physical constants does not contribute to the total displacement of spectral lines.

It must be mentioned that at the present time, the age of the universe is estimated from the asumption that the red shift is conditioned by the Doppler effect exclusively. It is known that this assumption leads to an insufficient age of the universe compared to that of the Earth's crust. If one considers the variation of physical constants in the 6D theory, then it is possible to increase the age of the universe in the flat model studied here. It may be shown that such a possibility also takes place for other homogeneous isotropic six-dimensional cosmological models.

4 A PHYSICAL INTERPETATION OF GEOMETRIC SCALAR FIELDS

It should be remarked that today there exist at least two principal points of view on interpreting the extra diagonal components of 6D metric:

(1) They may be understood as new geometric scalar fields which exist with both electromagnetic and gravitational fields,

(2) They may be treated as some kind of objects, so that properties of matter in a curved space-time are geometrically described.

The first aproach is more known. It was used in the works of E. Schmutzer [7], P. Jordan [8, 9], W. Pauli [10], and other authors. In our paper [5], this method was employed too; the components σ_{55} and σ_{66} of 6D metric were treated as squared potentials of hypothetical scalar fields.

It is possible to describe realistic models of the universe by the second method as well. This method characterizes the works of P. Wesson [11, 12, 13], Ponce de Leon [14], V. Belinsky [15], and other authors.

The effects described here, in general depend on the used interpretation of the extra metric components. However, to describe a realistic cosmology as in the present case, the second interpretation is just essential. For this reason, we shall consider in more detail the second method keeping within the bounds of 6D vacuum flat cosmological models already found (12).

It must be observed that, in the left side of the Einstein's equation (7), it is possible to distinguish between the four-dimensional terms and the others, which are contributions of higher dimensions. According to the proposed method, the part conditioned by contributions from the 5th and the 6th should be transfered to the right side of equation (7), and afterwards considered as the effective density of matter M_{eff}. Repeating the same procedure for equation (8), we get the expression for the effective pressure P_{eff}. Finally, considering (11) for the effective matter density and pressure from (7) and (8), the expressions follow:

$$\begin{aligned} \varkappa M_{\text{eff}} &= -2(2\alpha+3)(\dot\varphi/\varphi)^2 - 3(\alpha+3)\dot a\dot\varphi/a\varphi, \\ \varkappa P_{\text{eff}} &= \alpha(\alpha+1)(\dot\varphi/\varphi)^2 + (\alpha+3)\dot a\dot\varphi/a\varphi + (\alpha+3)\ddot\varphi/\varphi. \end{aligned} \tag{21}$$

Now, by substituting the functions $\varphi(\chi^0)$ and $a(\chi^0)$ from 6D flat models of universe already found (12) into (21), it follows

$$\begin{aligned} M_{\text{eff}} &= \frac{[3\sqrt{\lambda} \pm (\alpha+3)]^2}{4\varkappa a_0^2\varphi_0^2}(c \pm \sqrt{\lambda}\chi^0)^{-[3\sqrt{\lambda}\pm(\alpha+1)]/\sqrt{\lambda}}, \\ P_{\text{eff}} &= \frac{\alpha^2 - 2\alpha - 3 \pm (\alpha+3)\sqrt{\lambda}}{2\varkappa a_0^2\varphi_0^2}(c \pm \sqrt{\lambda}\chi^0)^{-[3\sqrt{\lambda}\pm(\alpha+1)]/\sqrt{\lambda}}. \end{aligned} \tag{22}$$

In this case, the equation of state has the form

$$P_{\text{eff}} = \frac{\alpha^2 - 2\alpha - 3 \pm (\alpha+3)\sqrt{\lambda}}{\alpha^2 + \frac{10}{3}\alpha + 5 \pm (\alpha+3)\sqrt{\lambda}} \cdot \frac{M_{\text{eff}}}{3}. \tag{23}$$

From (22) it follows that $M_{\rm eff}$ is always positive for any value of α. The values of α should be chosen so that $P_{\rm eff}$ is positive.

It is easy to see that in the expanding case, the value of $\alpha = 0$ corresponds to dust state:

$$P_{\rm eff} = 0, \quad M_{\rm eff} = \frac{12}{\varkappa a_0^2 \varphi_0^2}(c + \chi^0)^{-4}, \tag{24}$$

and the value $\alpha = -\frac{3}{2}$, to the radiation one:

$$P_{\rm eff} = \frac{M_{\rm eff}}{3}, \quad M_{\rm eff} = \frac{27}{4\varkappa a_0^2 \varphi_0^2}(C + \frac{3}{2}\chi^0)^{-4}. \tag{25}$$

5 THE TRANSITION TO 5D COSMOLOGICAL MODELS

One of the features of the obtained 6D cosmological models is that by straightforward substitution of the value $\alpha = 0$ into (12)–(14), we obtain solutions which satisfy the 5D vacuum equations analogous to (7)–(9):

$$3(a\varphi)^{-2}[(\dot{a}/a)^2 + \eta + 2(\dot{\varphi}/\varphi)^2 + 3\dot{a}\dot{\varphi}/a\varphi] = 0, \tag{26}$$
$$(\alpha\varphi)^{-2}[2\ddot{a}/a - (\dot{a}/a)^2 + \eta + 3\dot{a}\dot{\varphi}/a\varphi + 3\ddot{\varphi}/\varphi] = 0, \tag{27}$$
$$(a\varphi)^{-2}[\ddot{a}/a + \eta + 2\dot{a}\dot{\varphi}/a\varphi + \ddot{\varphi}/\varphi] = 0. \tag{28}$$

For this case, the solution is

$$\varphi = \varphi_0(c \pm \chi^0)^{\mp 1}, \quad a = a_0(c \pm \chi^0)^{\frac{1\pm 3}{2}}, \tag{29}$$

and it corresponds to 5D homogeneous and isotropic cosmological models. The effective density and pressure for these models are:

$$M_{\rm eff} = \frac{3(5 \pm 3)}{2\varkappa a_0^2 \varphi_0^2}(c \pm \chi^0)^{-(3\pm 1)},$$
$$P_{\rm eff} = \frac{3(-1 \pm 1)}{2\varkappa a_0^2 \varphi_0^2}(c \pm \chi^0)^{-(3\pm 1)}. \tag{30}$$

The equation of state can be written in the form

$$P_{\rm eff} = \frac{-1 \pm 1}{5 \pm 3} M_{\rm eff}. \tag{31}$$

In the expanding case, from (30) and (31) one gets the state of dust:

$$P_{\rm eff} = 0, \quad M_{\rm eff} = \frac{12}{\varkappa a_0^2 \varphi_0^2}(c + \chi^0)^{-4}. \tag{32}$$

Similarly to (15)–(16), in the 5D case we have the following expressions for the physical time:

$$\tau = \frac{a_0}{3}(c + \chi^0)^3 + \tau_0, \tag{33}$$

and

$$\tilde{\tau} = \frac{a_0 \varphi_0}{2}(c + \chi^0)^2 + \tilde{\tau}_0. \tag{34}$$

where τ_0 and $\tilde{\tau}_0$ are constants of integration. For calculations, the expanding case was considered.

It is not difficult to show that the effective density (32) changes as $M_{\text{eff}} = 3/\tilde{\tau}^2$, where $\tilde{\tau}$ is taken from (34).

After the corresponding calculations, through the time $\tilde{\tau}$ from (34), the 5D analogue of metric (5) takes the form

$$dI^2 = d\tilde{\tau}^2 - \tilde{\tau}(d\chi^2 + dy^2 + dz^2) - \tilde{\tau}^{-1} d\chi_5^2, \tag{35}$$

which is in complete agreement with the analogous five-dimensional Friedmann–Robertson–Walker (FRW) model shown by Wesson [16, 17] to be a particular solution of metric

$$ds^2 = e^{\nu} dt^2 - e^{w}(d\chi^2 + dy^2 + dz^2) - e^{\mu} d\chi_5^2. \tag{36}$$

The metric (35) has a space part growing in the same way as that of the four-dimensional FRW model with the equation of state of radiation. To show this, in the equation (26) we make the following substitution:

$$a^2 \varphi^2 = e^{\beta}, \quad \varphi^2 = e^{\gamma}. \tag{37}$$

Then the equation (26), using (37) and (29), acquires the form

$$\frac{3}{4} e^{-\beta}(\dot{\beta}^2 + \dot{\beta}\dot{\gamma}) = 0. \tag{38}$$

Whence, here in correspondence with the interpretation, the following expression for the effective density is obtained

$$M_{\text{eff}} = 3/4\tilde{\tau}^2, \tag{39}$$

where the expression (34) was implemented. By the same procedure, from equation (27) we obtain the effective pressure to be

$$P_{\text{eff}} = 1/4\tilde{\tau}^2. \tag{40}$$

From expressions (39) and (40), in this case the equation of state of radiation is satisfied: $P_{\text{eff}} = M_{\text{eff}}/3$.

Hence, in the five-dimensional case as well as in the six-dimensional one, the extra components of metric (geometric scalar fields) are reproducing the properties of matter, which in the four-dimensional theory is introduced from exterior to the right side of Einstein equation in form of stress-energy tensor.

It should be mentioned that a family of solutions for the metric (36) was found by Ponce de Leon [14]. Among them, there is a solution for the model with the equation of state of dust [16]: $M = 4/3T^2$, $P = 0$, where $T = \chi^5 t$ is proper time. It should be indicated that due to the presence of the 5th coordinate in these expressions, a series of difficulties arise as was mentioned by Wesson [16]. In fact, in spite of definition of proper time T, the questions about compactness of the five-extra dimension and its physical interpretation remains open.

6 CONCLUSIONS

In this paper, six-dimensional cosmological models and the six-dimensional geometric theory [1] of electroweak interactions were used to show the variation of fundamental physical constants and its influence on the observed red shift of spectral lines of light from distant sources. This necessarily leads to reconsidering the usual interpretation of the red shift of galaxies.

Here we also considered the point of view used for the interpretation of the extra diagonal components of 6D metric, so that they reproduce geometrically the properties of matter in the four-dimensional space-time. Thus, the general expressions for effective density of matter and pressure were found, and consequently their equations of state were obtained. Particularly, the states of dust and the radiation were shown.

For the case of flat models, the corresponding transition from the 6D cosmological models to the five-dimensional ones was studied. These 5D models are described analogously by the five-dimensional FRW metric. The effective density and pressure, which satisfy the equation of state of dust were obtained for these 5D models. Also complete correspondence of our results with those of Wesson, which started exclusively from 5D models [16, 18], was established.

Nevertheless, it should be noted that in early investigations made by Vladimirov [19, 20], it was shown that for the solutions of five-dimensional Einstein's equations, in the case of 5D homogeneous and isotropic cosmological models with constant positive and negative curvatures, the effective properties of matter do not correspond entirely to the well-known ones of 4D Friedmann models.

These peculiarities of non-flat multidimensional solutions make it necessary to consider the equation $R_{\mathrm{AB}} = 0$ more carefully, and hence this forces to extend the geometric significance of the properties of matter and consequently intensify the search for new multidimensional solutions.

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