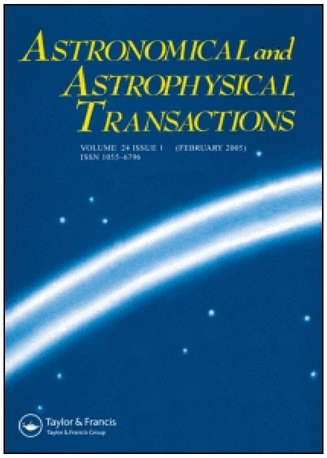

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Cosmological tests for determining real spatial and time dimensionalities of our Universe

A. D. Popova [a]; G. N. Kulik [a]
[a] Sternberg Astronomical Institute, Moscow

Online Publication Date: 01 June 1996
To cite this Article: Popova, A. D. and Kulik, G. N. (1996) 'Cosmological tests for determining real spatial and time dimensionalities of our Universe', Astronomical & Astrophysical Transactions, 10:2, 113 - 119
To link to this article: DOI: 10.1080/10556799608203016
URL: http://dx.doi.org/10.1080/10556799608203016

Astronomical and Astrophysical Transactions, 1996,
Vol. 10, pp. 113-119
Reprints available directly from the publisher
Photocopying permitted by license only

COSMOLOGICAL TESTS FOR DETERMINING REAL SPATIAL AND TIME DIMENSIONALITIES OF OUR UNIVERSE

A. D. POPOVA and G. N. KULIK

Sternberg Astronomical Institute, Moscow

(Received October 31, 1994)

In the framework of the homogeneous and isotropic cosmological model with arbitrary (noninteger) space and time dimensionalities, we derive the three classical cosmological tests: visual bolometric magnitude, angular distance and the number of sources versus redshift. We also obtain the deceleration parameter and the age and modern radius of the Universe.

There are promising tendencies in modern physics stemming from theories of fractals and theories of dynamical chaos to describe physical reality as structural and chaotic. For us, they suggest an idea to transfer features of such a description to space and time themselves. We are accustomed to consider our space and time as continuous, however this can turn out to be incorrect. The continuality of our space cannot be confirmed by a finite number of physical experiments (Harmuth, 1989); but a noncontinuous space can have another (arbitrary) dimensionality. Thus, the main concepts which could be revised are those of the dimensionalities of space and time. We also agree with an opinion that the dimensionalities of space and time might be noninteger and vary on various scales.

Recently, we argued for that the spatial dimensionality decreases form 3 on laboratory scales up to 2 on cosmological scales. This could help to avoid discrepancy between the availability of luminous matter and its dynamics and that between the relatively short age of the Universe in the standard FRW cosmology and the age of globular stellar clusters and galaxies. Now we make an attempt to include into consideration noninteger dimensionalities of both space and time taking as a basic a FRW-type model and to infer simple observable consequences.

Thus, in our constructions, we think that the cosmological dimensionalities m and n of time and space differ from the laboratory values $m = 1$ and $n = 3$. Unfortunately, we have no formalism to take into account a possible smooth change of m and n when passing to cosmological scales and have to consider m and n constant. We introduced (Popova, 1994) a relative radius R_0 between any two bodies where the space dimensionality for them changes by leap from 3 to n. We note that our constructions do not require an analogous characteristic quantity for time. As it has been explained, we are mostly interested in values $2 < n \leq 3$ (the case $n = 2$ is in some sense distinct), and it is enough to impose $m > 0$.

As for observational predictions, we consider the three classical cosmological tests: visual bolometric magnitude, angular distance and the number of sources versus redshift and also obtain expressions for the age and modern radius of the Universe and the deceleration parameter in some approximations. Certainly, this consideration does not give a simple way for determining real m and n from observations because there are too many parameters to determine from available tests, however we emphasize that we propose another conception for the interpretation of suitable observed dependencies.

Consider the metric interval which is a topological product of a homogeneous and isotropic n-dimensional "spatial" space and a flat m-dimensional "temporal" space:

$$ds^2 = c^2\left(dt_1^2 + dt_2^2 + \ldots + dt_m^2\right) - a^2(t)\, dl^2_{(n,k)},$$

where $dl^2_{(n,k)} = dr^2 + \tilde{\phi}^2(r)\, d\sigma^2_{(n)}$ is the metric interval of the n-dimensional space with constant Gauss curvature (whose sign is $k = -1, 0, +1$) with $\tilde{\phi}(r) = \sin r$ for $k = +1$, $\tilde{\phi}(r) = r$ for $k = 0$ and $\tilde{\phi}(r) = \sinh r$ for $k = -1$, $d\sigma_{(n)}$ is the element of angular separation. The scale factor a depends only on the radial time coordinate

$$t = (t_1^2 + t_2^2 + \ldots t_m^2)^{1/2}.$$

To accomplish our model, we assume that the Universe is filled with hydrodynamic matter, however, in order to avoid further restrictions than necessary, we should introduce a new parameter p' (Popova and Kulik, 1994) which plays the role of pressure (perhaps, tension) in the temporal space. We denote $A, B = 1, \ldots, m$ and $i, k = 1, \ldots, n$, and then we can write nonvanishing components of our stress-energy tensor

$$T_{AB} = (\epsilon + p')u_A u_B - p' g_{AB}, \quad T_{ik} = (\epsilon + p)u_i u_k - p g_{ik}.$$

Here ϵ, p and p' depend only on t. The quantities $u_A = t_A/t$ represent the time components of an $(m+n)$-velocity vector field (with the unit norm) comoving with matter, so that $u_i = 0$.

After that we can derive the set of the Einstein-like equations (Popova and Kulik, 1994):

$$n\left[\frac{m-1}{t}\frac{\dot{a}}{a} + \frac{n-1}{2}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right)\right] = \varkappa_{(m,n)}\epsilon, \tag{1}$$

$$n\left[\frac{\ddot{a}}{a} + \frac{m-2}{t}\frac{\dot{a}}{a} + \frac{n-1}{2}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right)\right] = -\varkappa_{(m,n)}p', \tag{2}$$

$$(n-1)\left[\frac{\ddot{a}}{a} + \frac{m-1}{t}\frac{\dot{a}}{a} + \frac{n-2}{2}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right)\right] = -\varkappa_{(m,n)}p, \tag{3}$$

where $\varkappa_{(m,n)}$ is an Einstein-like constant, and dot denotes differentiation with respect to t. As usual, Eqs. (1)–(3) are not independent due to the contracted Bianchi identities (and a conservation law) and require an additional condition in the form of "the equation of state": $\Phi(\epsilon, p', p) = 0$ with Φ a function. For the purpose of deriving cosmological tests, we confine ourselves to a dust-filled Universe [$p = 0$, however p' remains a free parameter determined by (2)]. After that Eq. (3) serves as the equation for finding the scale factor:

$$\frac{\ddot{a}}{a} + \frac{m-1}{t}\frac{\dot{a}}{a} + \frac{n-2}{2}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right) = 0. \tag{4}$$

Starting with integer m and n, we further make the following crucial step: after deriving Eqs. (1)–(4), we here consider m and n as real (noninteger) parameters, and the only restrictions $m > 0$ and $n > 2$ are as yet sufficient.

Generally, solutions to Eq. (4) cannot be expressed via elementary functions. Its partial solutions (Popova and Kulik, 1994) have diverse properties. For example, in the case $k = 0$ with $m \neq 2$ we have

$$a(t) \propto t^{2(2-m)/n}. \tag{5}$$

It is easily seen from (5) that the value $m = 2$ plays the role of some critical time dimensionality which separates expanding ($m < 2$) and contracting ($m > 2$) solutions. For $m = 2$, there is an expanding logarithmic solution. From the substitution of the solution (5) into the set (1)–(3), surprisingly follows "the temporal equation of state": $p' = \epsilon$.

As for the propagation of light, we assume that light propagates along the time direction u_A. (In this time "lives" the dust fluid.) On the $(m+n)$ isotropic and homogeneous background, independently of m and n, the eikonal equation in the high-frequency approximation has a solution $\nu(t)a(t) = \text{Const}$ with ν the frequency of light. Whence, in recalling the definition of the redshift $z = (\nu - \nu_0)/\nu_0$ where ν is frequency at the moment of emission, and ν_0 is frequency at the moment of observation, we can write as usual

$$a = a_0(1+z)^{-1}. \tag{6}$$

Here and below the index "0" denotes quantities which correspond to the modern epoch of the Universe.

The Hubble parameter is defined as usual, $H = \dot{a}/a$. The critical energy density ϵ_{cr} can be determined from (1) when imposing $k = 0$, this leads to the standard definition of the density parameter $\Omega = \epsilon/\epsilon_{cr}$. Then, with the aid of (1), we obtain an expression for the modern (physically) dimensionless radius, DR, of the Universe

$$(a_0 H_0 c^{-1})^2 = \left[|\Omega_0 - 1|\left(2\frac{m-1}{n-1}\frac{1}{t_0 H_0} + 1\right)\right]^{-1}, \tag{7}$$

which is useful by itself and is necessary for deriving the cosmological tests. In our model with $m \neq 1$, Eq. (7) involves the quantity $t_0 H_0$ which is a dimensionless age, DA, of the Universe. This is not however the case for $m = 1$. The same is true for the deceleration parameter (see below).

Now we come to the derivation of the tests. To do this, we must have the dependence of the coordinate radius r on z (see Zel'dovich and Novikov, 1975). For a light ray coming from a light source to us, the following differential relation holds:

$$dr = -\frac{cu_A\, dt^A}{a(t)} = -\frac{c\, dt}{a(t)} = -(1+z)\frac{c}{a_0}\left(\frac{dt}{dz}\right) dz. \tag{8}$$

Thus, the next step is to find the dependence of dt/dz on z. Equation for $t(z)$ follows from (4) with the use of (6) and (7), if one comes to the inverse function $t(a)$ instead of $a(t)$:

$$\frac{d^2t}{dz^2} + \frac{n+2}{2}\frac{1}{1+z}\frac{dt}{dz} - \frac{m-1}{t}\left(\frac{dt}{dz}\right)^2 + \frac{n-2}{2}$$
$$\times(\Omega_0 - 1)H_0^2\left(2\frac{m-1}{n-1}\frac{1}{t_0H_0} + 1\right)(1+z)^3\left(\frac{dt}{dz}\right)^3 = 0. \tag{9}$$

We see that the age, i.e. $t_0 = t(0)$, also enters Eq. (9), meaning that Eq. (9) is in some sense an intergro-differential one. However, in approximations which are considered below, this fact does not give additional difficulties for solving Eq. (9).

Now, let us denote $(d^2t/dz^2)_0 = H_0^{-1}\Psi$ and make sure that $(dt/dz)_0 = -H_0^{-1}$ for $z = 0$. Then, without any approximations, we have from (10):

$$\Psi = \frac{n+2}{2} + \frac{m-1}{t_0H_0} + \frac{n-2}{2}(\Omega_0 - 1)\left(2\frac{m-1}{n-1}\frac{1}{t_0H_0} + 1\right). \tag{10}$$

Note that the modern value of the deceleration parameter, which is defined in a standard way as $q = -\ddot{a}a/\dot{a}^2$, can be also expressed via Ψ:

$$q_0 = H_0\left(\frac{d^2t}{dz^2}\right)_0 - 2 = \Psi - 2. \tag{11}$$

Now we present expressions for the cosmological tests in the first two non-vanishing orders in z.

(i) *Visual magnitude vs. redshift.* The distance modulus $\bar{m} - M$ is expressed as

$$\begin{aligned}\bar{m} - M \;&=\; 5\lg\frac{cz}{H_0} + \frac{5}{4}\lg e\cdot[n+3-(n-1)\Psi]z - \frac{5}{2}\lg\left(\frac{4\Gamma(n/2)}{\sqrt{n}\Gamma((n+1)/2)}\right) - 5\\ &+\; \frac{5}{2}(3-n)\lg\left(\frac{R_0H_0}{cz}\right) - \frac{5}{16}\frac{(n-1)(3-n)}{n+1}\lg e\cdot\left(\frac{R_0H_0}{cz}\right)^2,\end{aligned} \tag{12}$$

where $\bar{m}$ is a visual magnitude, M is an absolute visual magnitude, Γ denotes the gamma function, and R_0 is a distance where the space dimensionality changes by leap. We have supposed in (12) that the quantity $R_0/r \approx R_0H_0/cz$ is sufficiently small.

(ii) *Angular distance vs. redshift.* The second test is usually expressed as the distance along the curved n-dimensional sphere which can be determined from the linear (l) and angular (θ) sizes of an extended source:

$$\tilde{R}(z) = \frac{l}{\theta} = \frac{cz}{H_0}\left[1 - \frac{z}{2}(1+\Psi)\right]. \tag{13}$$

(iii) *The number of sources vs. redshift.* The differential dN of the sources of a required sort within the redshift interval dz is

$$dN = \frac{2\pi^{n/2}}{\Gamma(n/2)}\bar{n}_0\left(\frac{c}{H_0}\right)^n z^{n-1}\left[1 - \frac{n+1}{2}(\Psi - 1)\right] dz \tag{14}$$

where $\bar{n}_0$ is the density of these sources at $z = 0$.

One can check that for $m = 1$ and $n = 3$, the quantities (11)–(14) acquire familiar forms coinciding with those in Zel'dovich and Novikov (1975). For $m = 1$ and arbitrary n, they coincide with those obtained by Popova (1994a).

We see that knowledge of (10) is a key for calculating the deceleration parameter (11) and the tests (12)–(14) in required approximations. As mentioned above, Eq. (10) is exact in the sense that it is obtained without any approximations. However, to obtain it finally, we must know the value t_0 that is solve (9) for $t(z)$ and take $t_0 \equiv t(0)$. We have a success in doing this in the two different approximations when (1) $(\Omega_0 - 1)$ is small and (2) $(m-1)$ is small.

(1) $(\Omega_0 - 1) \equiv \omega \ll 1$. This approximation can be obtained only for expanding solutions with $\dot{a} = \infty$ at $t = 0$: This means that $(2 - n/2) < m < 2$. In this case, DA is approximately

$$t_0 H_0 = \frac{2(2-m)}{n}\left[1 - \omega\frac{n-2}{n-1}\,\frac{n-(2-m)}{2n-(2-m)(4-n)}\right]. \tag{15}$$

Evidently, for $\Omega_0 = 1$, DA effectively increases if n tends to 2 and m is smaller than unity. As for the ω-correction in (15), it is negative at least for $2 < n \leq 3$: The Universe is older when $\omega < 0[k=-1]$ and younger when $\omega > 0$ $[k = +1]$.

Here, DR is given by

$$(a_0 H_0 c^{-1})^2 = \frac{1}{|\omega|}\,\frac{(n-1)(2-m)}{n-(2-m)}\left(1 - \omega\frac{(m-1)n(n-2)}{(n-1)[2n-(2-m)(4-n)]}\right). \tag{16}$$

Certainly, $a_0 H_0 \to \infty$ when $\omega \to 0$ (flat n-dimensional space). The ω-correction enters (16) with the factor $(m-1)$, the latter can have both negative and positive signs. Therefore, the sign of a correction in (16) is determined by the sign of $-\omega(m-1)$.

Expression for Ψ acquires the form

$$\Psi = 1 + \frac{n}{2(2-m)} + \omega\frac{(n-2)[n-(2-m)][8-4m-3n]}{2(n-1)(2-m)[(2-m)(4-n)-2n]}. \tag{17}$$

(2) $(m-1) \equiv \mu \ll 1$. This approximation has a feature that the quantity $m-1$ enters Eqs. (7) and (10) only in combination with the DA $(t_0 H_0)$, that is why for the latter we can take its value for $m = 1$:

$$(t_0 H_0)\big|_{m=1} = 2(n\sqrt{\Omega_0})^{-1} F_0(n, \Omega_0) \tag{18}$$

where $F_0(n, \Omega_0) \equiv F(n, \Omega_0; 0)$,

$$F(n, \Omega_0; z) \equiv {}_2F_1\left(\frac{n}{2(n-2)}, \frac{1}{2}; \frac{3n-4}{2(n-2)}; \frac{\Omega_0 - 1}{\Omega_0} \frac{1}{(1+z)^{n-2}}\right).$$

and ${}_2F_1$ is the hypergeometric function. Some numerical values of (18) were presented by Popova (1994a).

Thus, using (18), expressions for Ψ and $a_0 H_0 c^{-1}$ can be easily obtained from (7) and (10):

$$\Psi = 2 + \frac{n-2}{2}\Omega_0 + \mu \frac{n}{2(n-1)} \frac{\sqrt{\Omega_0}}{F_0(n, \Omega_0)}[(n-2)\Omega_0 + 1], \tag{19}$$

$$(a_0 H_0 c^{-1})^2 = \frac{1}{|\Omega_0 - 1|}\left(1 - \mu \frac{n}{n-1} \frac{\sqrt{\Omega_0}}{F_0(n, \Omega_0)}\right). \tag{20}$$

Expression for the dimensionless age is more cumbersome,

$$\begin{aligned} t_0 H_0 \quad = \quad & \frac{2}{n} \frac{1}{\sqrt{\Omega_0}} F_0(n, \Omega_0) - \mu \frac{n}{2}\left(\sqrt{\Omega_0} \int_0^\infty dz\,(1+z)^{n-4} \xi^{-3/2}(z)\right. \\ \times & \int_0^z dz'\, \xi^{1/2}(z')(1+z)^{-n/2} F^{-1}(n, \Omega_0; z') + (n-1)^{-1}(\Omega_0 - 1) \\ \times & \left.\int_0^\infty dz\, \xi^{-3/2}(z)(1+z)^{-2}[(1+z)^{n-2} - 1]\right). \end{aligned} \tag{21}$$

where

$$\xi(z) = \Omega_0(1+z)^{n-2} - \Omega_0 + 1.$$

We should add that Eqs. (15), (16) and (17) are in agreement with Eqs. (21), (20) and (19), respectively, if we make the approximation in μ in the former ones and that in ω in the latter ones.

References

Harmuth, H. F. (1989) *Information Theory Applied to Space-Time Physics*, Moscow, Nauka, Russian translation.

Popova, A. D. (1994) *Astron. and Astroph. Trans.* 5, 31.

Popova, A. D. (1994a) *Friedmann Cosmology in Alternative Spatial Dimensions. Solutions and Tests.* To appear in *Astron. and Astroph. Trans.*

Popova, A. D. and Kulik, G. N. (1994) *On Homogeneous and Isotropic Cosmological Models with Arbitrary Numbers of Time and Space Dimensionalities.* To appear in *Astron. and Astroph. Trans.*

Zel'dovich, Ya. B. and Novikov, I. D. (1975) *The structure and Evolution of the Universe*, Moscow, Nauka (in Russian).