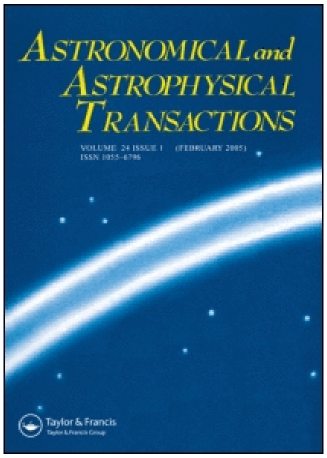

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The cosmic microwave background anisotropy and very large structures in the universe: The problem of agreement

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THE COSMIC MICROWAVE BACKGROUND ANISOTROPY AND VERY LARGE STRUCTURES IN THE UNIVERSE: THE PROBLEM OF AGREEMENT

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The general formula for correlation function of CMB temperature fluctuations, measured in the real experiments on angular scale $\gtrsim 1^\circ\!.5$, is obtained. Predictions by the CDM model with different slope of the primordial spectrum and of the hybrid HDM+CDM (HC) one for COBE, ULISSE and SP93 experiments are calculated. It is shown that the SP93 upper limit on $\frac{\Delta T}{T}$ does not agree with large scale peculiar velocities of galaxies and with existence of very scale structures even in preferable models of the flat University without reionization. A reconstructed spectrum explaining the COBE and ULISSE results, large scale peculiar velocities, two point correlation functions, concentrations of bright galaxies and rich clusters are proposed.

KEY WORDS Anisotropy of the CMB radiation, large scale structure of the Universe, correlation functions

Data of observational cosmology concerning the large scale structure of the Universe which has been obtained during the recent years give a chance for progress in solution of the reverse problem of cosmology to reconstruct the initial power spectrum of the density perturbations from combined observable characteristics of the large scale structure of the Universe. The goal of this paper is to reconstruct the primordial density perturbation spectrum from the following observable characteristics of the large scale structure of the Universe:

- Anisotropy of the relic background radiation (from COBE, ULISSE, and South Pole experiments),
- Correlation functions of galaxies and clusters (Davis & Peebles, 1983; Guzzo *et al.*, 1991; Jing & Valdarnini, 1992; Loveday *et al.*, 1992; Bahcall & Soneira, 1983; Einasto *et al.*, 1992,
- Large-scale peculiar velocities of galaxies (Bertchinger *et al.* 1990; Courteau *et al.*, 1993),

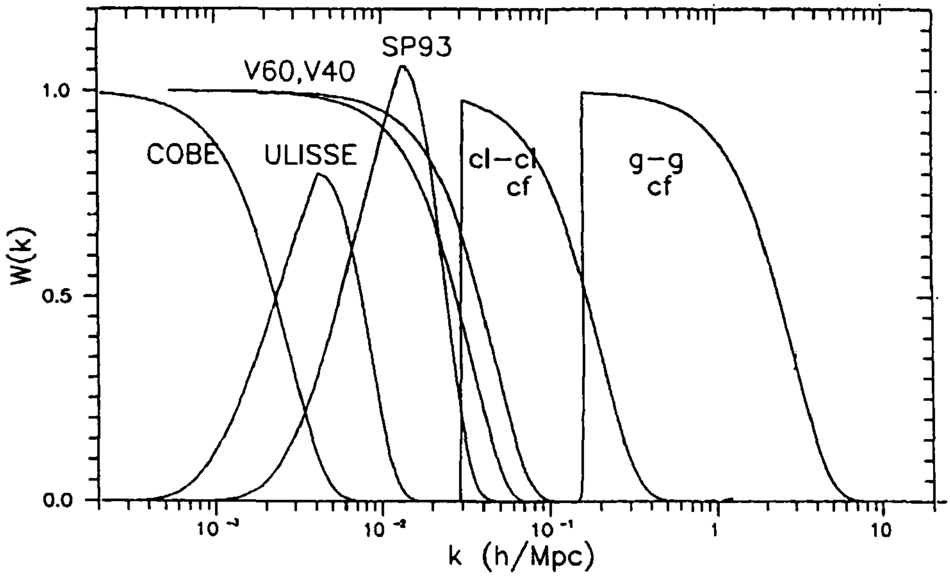

Figure 1 Spectral windows for the CMB anisotropy experiments (COBE, ULISSE, and SP93), for the bulk motion (V40, V60), cluster–cluster (cl–cl) and galaxy (g–g) correlation function measurements.

- Mean distances between bright galaxies and clusters or their number densities (CfA, IRAS, APM, Abell, and ACO catalogues).

The basic assumption about Gaussian statistic allows us to calculate, for a given spectrum, all these characteristics of the large scale structure of the Universe and to confront them with observations. Each of them is same integral of the spectrum and is defined by a certain part of it. Their regions of sensibility, as window functions, are shown in Figure 1.

To study the requirements to the initial power spectrum of density perturbations, seeking conformity between theoretical and observed large-scale features of the Friedman flat Universe, we analyzed the primordial power spectra of density perturbations $P(k) = Ak^n$ with $n = 1$, 0.8 and 0.7 and transfer function $T(k)$ for cold dark matter (CDM) dominated model ($\Omega_{\rm CDM} = 0.9$, $\Omega_b = 0.1$), hybrid hot+cold dark matter ($\Omega_{\rm CDM} = 0.6$, $\Omega_{\rm HDM} = 0.3$, $\Omega_b = 0.1$) (Holtzman, 1989), and phenomenological spectrum CDM+Z by Hnatyk *et al.* (1991, 1993). All spectra were normalizied using standard normalization – to r.m.s. mass fluctuations in top-hat sphere of radius $8h^{-1}$ Mpc: $\frac{\Delta M}{M}$ (8 h^{-1} Mpc)$= 1/b_g$, where b_g is the galaxy biasing parameter. They are shown in Figure 2, their observed parameters are presented in the Table 1. The biasing phenomenological parameter depends on the shape of perturbation spectrum and on the threshold ν_t (bright galaxies are formed in the peaks with $\nu \equiv \frac{\delta}{\sigma_0} \geq \nu_t$), which is specified by the observable concentration of the objects. Theoretical and calculational aspects of the biasing can be found

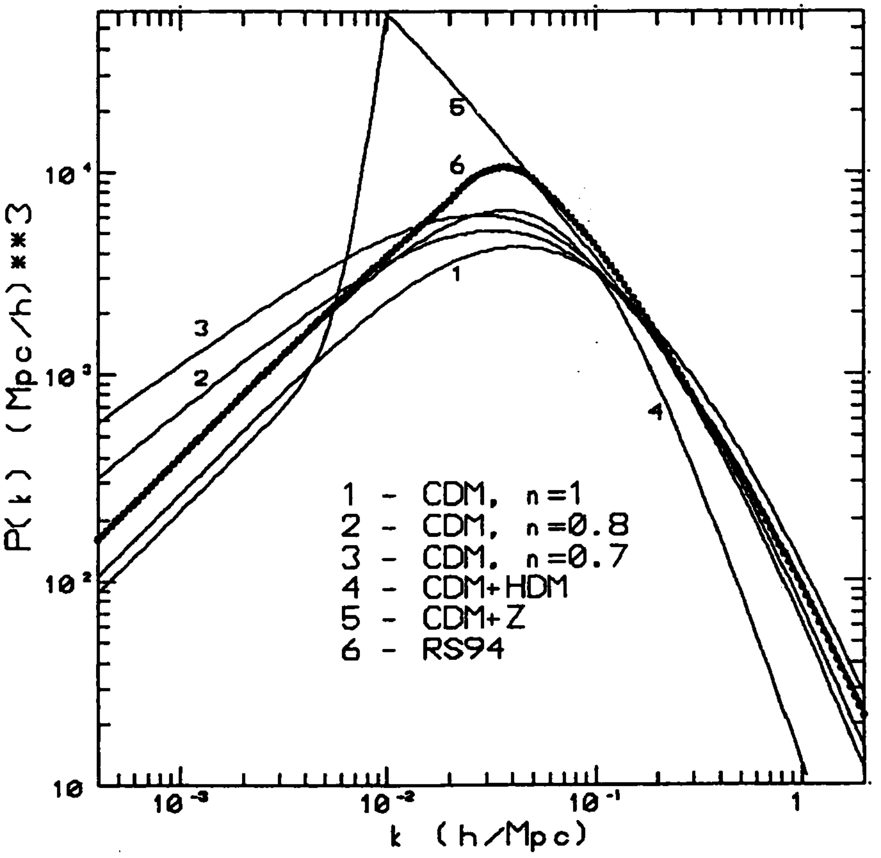

Figure 2 Power spectra of density fluctuations for CDM models with $\Omega_b = 0.1$ and $n = 1, 0.8, 0.7$, for the hybrid model (HC) with $\Omega_{CDM} = 0.6$, $\Omega_b = 0.1$, and for spectra CDM+Z and RS94 proposed for explanation of the data on the very large scale structures.

in Bardeen *et al.* (1986) and Hnatyk *et al.* (1991), respectively. It ranges within $b \in 1.4$–1.8 for different models (see Table 1). Mean height of peaks $\langle\nu\rangle$ from which bright galaxies and rich clusters are formed in any models of the Universe as well as the moment of the first counterflow appearance in dark matter and generation of shock wave in baryon component ($z_g = \langle\nu\rangle\sigma_0/1.69 - 1$) are presented, too.

All linear correlation functions can be easily derived if the power spectrum $P(k)$ is known:

$$\xi_{ab}(r) = \frac{b_a b_b}{2\pi^2} \int_0^\infty dk\, k^2\, P(k) W(kR_a) W(kR_b) \frac{\sin(kr)}{kr},$$

where both indexes, a and b, may denote galaxies and clusters. The filtering function $W(kR)$ isolates the objects of scale R in the spectrum $P(k)$; b_a, b_b are the parameters of biasing for these objects. Here we take $R_g = 0.35\ \mathrm{h}^{-1}$ Mpc and $R_{\mathrm{cl}} = 5\ \mathrm{h}^{-1}$ Mpc, which correspond to the total masses $M_g \simeq 2 \times 10^{11} h^{-1} M_\odot$ and $M_{\mathrm{cl}} \simeq 5 \times 10^{14} h^{-1} M_\odot$.

The observed and theoretical values of the cl–cl correlation function at 20 and 50 h^{-1} Mpc for different spectra can be found in the Table 1.

For the mean square galactic velocity in a sphere of radius R, we have:

$$V_{\mathrm{rms}}^2(R) = \frac{H^2}{2\pi^2} \int_0^\infty P(k) W^2(kR)\, dk.$$

The Table 1 displays the results of these calculations for radius 5 (Gaussian filtering) and top-hat spheres with radii 40 and 60 h^{-1} Mpc with preliminary Gaussian smoothing for the scale $R_f = 12\ \mathrm{h}^{-1}$ Mpc R_5, R_{40}, and R_{60}, respectively.

The temperature fluctuations of the cosmic microwave background were calculated as $\langle (\frac{\Delta T}{T})^2 \rangle^{\frac{1}{2}} = (C(0;\sigma))^{\frac{1}{2}}$, where $C(\alpha,\sigma) \equiv \langle \frac{\Delta T}{T}(\vec{n}_1) \frac{\Delta T}{T}(\vec{n}_2) \rangle$ is the correlation function for such fluctuations, $\vec{n}_1 \vec{n}_2 = \cos\alpha$ (for general expressions (A4–A5) see Appendix), σ is the smoothing scale (by the antenna or/and method of data processing). The calculations were carried out for Sachs–Wolfe, Doppler, and Silk effects and excluding monopole and dipole components according to Eq. (A4). For comparison with ULISSE and SP93 data, we have filtered the low spatial frequency in the density perturbation power spectra according to their observational strategy (Gorsky, 1993) as it is shown in Figure 1. Calculating the quadrupole $Q_2 = (\frac{5}{4\pi} a_2^2)^{\frac{1}{2}} f_2(\sigma)$, we take into account only the Sachs–Wolfe effect, and the harmonic coefficient a_2 is calculated using formulae after Peebles (1982). The factor f_2 – the decomposing coefficient of the smoothing angular function with respect to Legendre polinomeans – in the case of the COBE experiment is approximately equal to $f_2(\sigma) \simeq \exp(-\frac{25}{8}\sigma) \simeq 0.909$. The results of calculations of $\frac{\Delta T}{T}$ for angular scales (FWHM) of 1°.5, 5°.2, 10° and the quadrupole component Q_2 as well as SP93, ULISSE, and COBE experimental data (for comparison) are presented in the last four rows of the Table 1.

As we can see, theoretical predictions of $\frac{\Delta T}{T}$ for FWHM = 1°.5 and the SP-experiment spectral function strongly disagree with SP93 data for all spectra. Such contradiction may be eliminated for some spectra if we suppose that density and velocity fluctuations in the baryon component at last scattering surface are essentially reduced and $\frac{\Delta T}{T}$ are caused by fluctuations of the gravitational potential only. However, if we account for the acoustic mode (Naselsky & Novikov, 1993), which may exist on this scale, then the problem obviosly remains. It is possible that erasing effects take place lower z and this explain the low upper limit on $\frac{\Delta T}{T}$ in the SP93 experiment.

The results presented in the Table 1 show that the CDM models with $n = 1$ and 0.8 predict too low large-scale bulk motion and cluster-cluster correlation, whereas the CDM with $n = 0.7$ do not conform with the COBE result or with number density of bright galaxies and the epoch of their formation if we increase the biasing parameter b_g to 2.4. The HC model predicts a low value of the cluster-cluster

Table 1.

	Observed	*CDM* $n=1$	*CDM* $n=0.8$	*CDM* $n=0.7$	*HC* $n=1$	*CDM+Z* $n=1$	*RS94* $n=1$
b_g		1.4	1.5	1.5	1.8	1.4	1.4
σ_g		3.6	2.8	2.6	1.3	3.3	3.2
$\langle\nu\rangle_g$		2.5	2.4	2.4	1.9	2.5	2.5
z_g		4.3	3.0	2.7	0.5	3.9	3.7
b_{cl}		3.9	4.0	4.1	3.9	3.9	3.7
σ_{cl}		0.55	0.52	0.53	0.46	0.57	0.58
$\langle\nu\rangle_{cl}$		2.8	2.8	2.8	2.6	2.8	2.7
z_{cl}		-0.1	-0.1	-0.1	-0.3	-0.06	-0.07
$V_5^{*)}$	$\simeq 580 \pm 60^{a)}$	500	520	550	540	830	640
V_{40}	$\simeq 335 \pm 38^{b)}$	250	280	310	305	620	360
V_{60}	$\simeq 360 \pm 40^{b)}$	200	230	260	250	555	290
$\xi_{cl\,cl}(20)$	$\simeq 1.5^{c)}, 0.9^{d)}, 1.3^{e)}$	0.5	0.6	0.7	0.8	1.1	1.0
$\xi_{cl\,cl}(50)$	$\simeq 0.3^{c)}, 0.1^{d)}, 0.1^{e)}$	-0.01	0.01	0.01	0.01	0.18	0.04
$\frac{\Delta T}{T}(1^\circ)$	< 1.6 (95%c.l.)$^{f)}$	1.83	1.92	2.04	2.01	3.17	2.40
$\frac{\Delta T}{T}(5^\circ)$	< 1.4 (95%c.l.)$^{g)}$	0.87	1.05	1.21	1.02	1.89	1.11
$\frac{\Delta T}{T}(10^\circ)$	$\simeq 1.1 \pm 0.2^{h)}$	0.92	1.38	1.77	1.11	1.24	1.11
Q_2	$\simeq 0.5 \pm 0.2^{h)}$	0.41	0.69	0.99	0.50	0.37	0.50

Note. $^{*)}$, velocities are presented in km/s, linear scales in h^{-1} Mpc, $\frac{\Delta T}{T}$ and Q_2 in units of 10^{-5}; $^{a)}$, Bertchinger *et al.*, (1990); $^{b)}$, Counteau *et al.*, (1993); $^{c)}$, Bahcall & Soneira (1983); $^{d)}$, Jing & Valdarnini (1993); $^{e)}$, Einasto *et al.* (1992); $^{f)}$, (SP93); $^{g)}$, ULISSE; $^{h)}$, (COBE).

correlation and too late epoch of galaxy formation. The phenomenological CDM+Z spectrum explains very well the existence of large-scale structures, bulk motions, correlations, the epoch of galaxy formation (Hnatyk *et al.*, 1994), but does not conform with ULISSE data and, obviously, with SP93. That is why we propose another reconstructed spectrum (RS94) explaining the COBE and ULISSE results, the large-scale peculiar velocity, two-point correlation functions, concentration of bright galaxies and rich clusters:

$$P(k) = A_0 k \left(1 - a_0 \left(\frac{k}{k_0}\right)\right)^{p_0}, \quad \text{if } k < k_0;$$

$$P(k) = P(k_0) + \frac{P(k_1) - P(k_0)}{(k_1/k_0)^{p_1} - 1}((k/k_0)^{p_1} - 1), \quad \text{if } k_0 \le k \le k_1;$$

$$P(k) = AkT^2(k)\left(1 + a_1\left(\left(\frac{k_2}{k}\right)^{p_2} - 1\right)\right), \quad \text{if } k_1 \le k \le k_2;$$

$$P(k) = AkT^2(k), \quad \text{if } k \ge k_2,$$

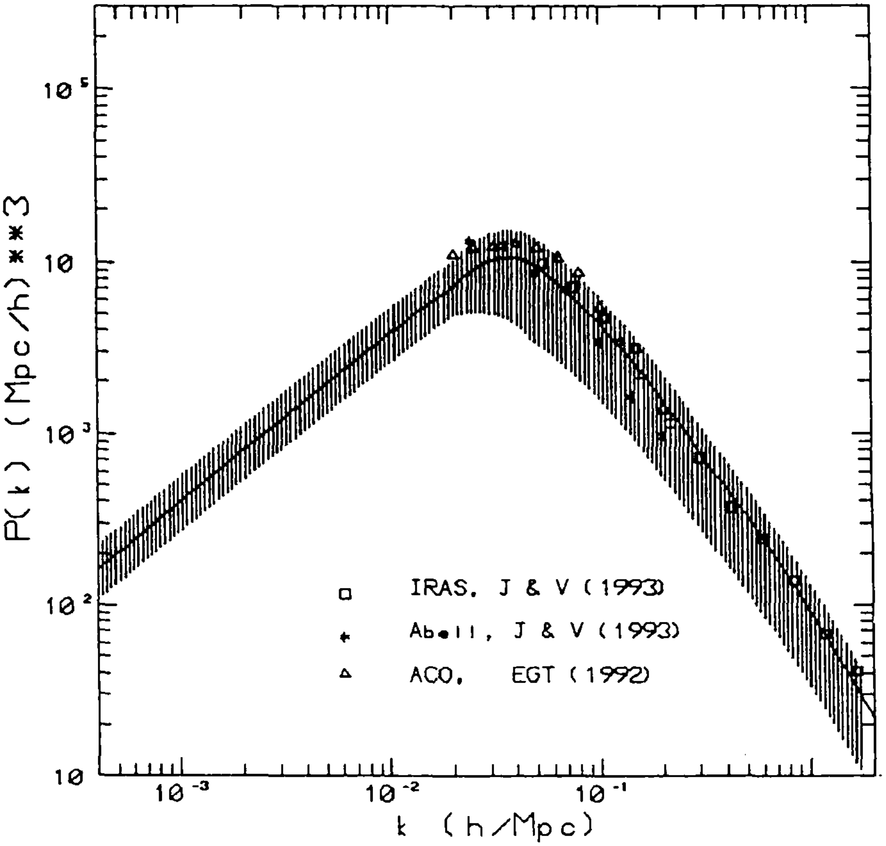

Figure 3 The power spectrum RS94 (solid line), proposed here to get agreement between the data on CMB anisotropy and on very large scale structures and motions, and reconstructed spectra from spatial distribution of rich clusters (triangles and asterisc) and of IRAS galaxies (squares) by different authors. The error corridor is determined by error bars of COBE data (at large scale and by uncertainties of normalization $\frac{\Delta M}{M}$ = (0.6 ~ 1.2)/b_g (at intermediate and small scale). The rich cluster power spectrum data by Jing & Valdarnini (1993) and Einasto *et al.* (1992) are devided by the cluster biasing parameter b_{cl} of RS94 model.

where $A_0 = 6.464 \times 10^6$, $k_0 = 0.02h/Mpc$, $a_0 = 0.1$, $p_0 = 1.0$, $k_1 = 0.04654h/Mpc$, $a_1 = 0.32$, $p_1 = 0.8$, $k_2 = 0.3333h/Mpc$, $p_2 = 1.0$, and $T(k)$ is the transfer function for the composite CDM+baryon ($\Omega_{CDM} = 0.9$, $\Omega_b = 0.1$) model with $h = 0.5$ (Holtzman, 1989). It is shown in Figure 2. The comparison of this spectrum with that reconstructed by other authors is shown in Figure 3. Error bars are determined by the error bars of the COBE result (for large scale) and uncertainties of normalization $\frac{\Delta M}{M}$ (8 h^{-1}Mpc)= $(0.6 - 1.2)/b_g$ for small scale. The correlation functions of galaxies for this spectrum and the observed ones, obtained by different authors, are given in Figure 4. Other calculated characteristics of the large scale structure of the Universe for this spectrum are presented in the Table 1.

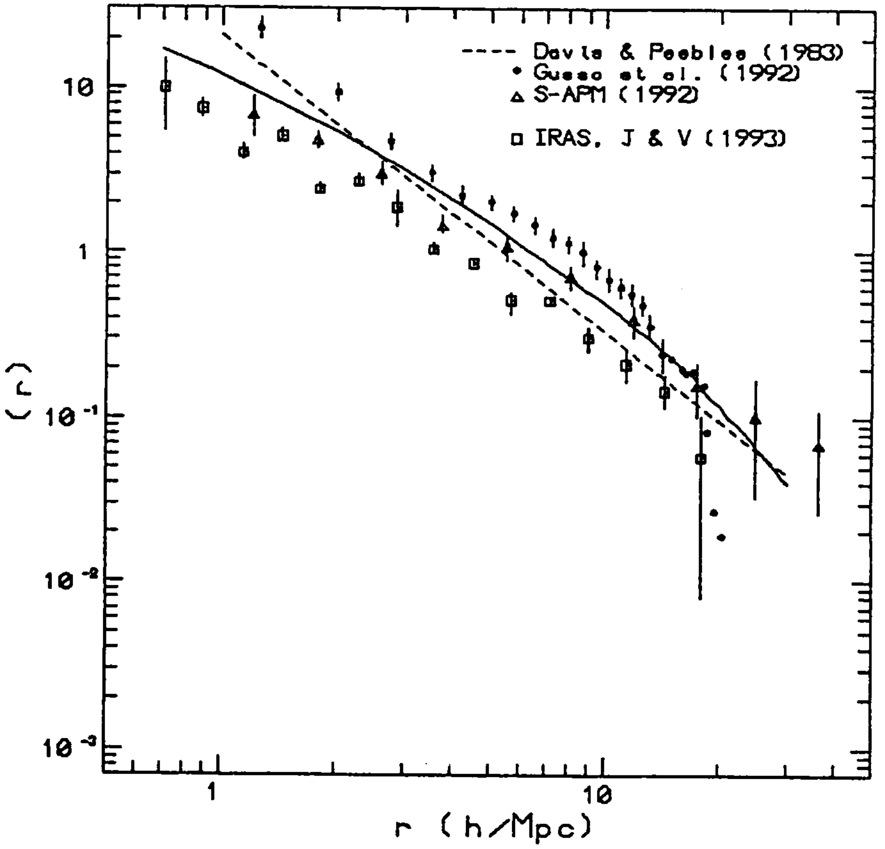

Figure 4 Galaxy-galaxy correlation functions for the RS94 spectrum and the observed one obtained by different authors.

Appendix. The Correlation Function of the Temperature CMB Fluctuations on the Large Angular Scales

The solution of the equations $\delta R_i^k - \delta_i^k R/2 = k\delta T_i^k$ for small scalar adiabatic fluctuations of density $\delta\rho/\rho_0$ and velocity δV^a of matter and curvature of space-time on the flat Fridmanian background in the quasi-newtonian gauge with metric $ds^2 = (1+2\phi)dt^2 - a^2(1-2\phi)\eta_{\alpha\beta}dx^\alpha dx^\beta$ are the following (for the growth mode only):

$$\phi = Y(x^\alpha),$$

$$\delta\rho/\rho_0 = \frac{2}{3H^2}\left(\frac{t}{t_0}\right)^{2/3}\Delta Y(x^\alpha) - 2Y(x^\alpha),\ \delta V^\alpha = -\frac{2}{3H}\left(\frac{t}{t_0}\right)^{1/3} Y_{,x^\alpha}, \qquad (A1)$$

where $a(t)$ is the scale factor of the Fridmanian background model, $Y(x^\alpha)$ is an arbitrary function of space coordinates, $t_0 = \frac{2}{3H}$ is the contemporary cosmological time, Δ is the Laplacian and $(,x^\alpha)$ denote the spatial derivative on x^α $(i, k, l \ldots =$

$0, 1, 2, 3,\ \alpha, \beta \ldots = 1, 2, 3)$. Here ϕ and $\delta\rho/\rho_0$ are respectively gauge-invariant values Φ_A and ϵ_g introduced by Bardeen (1980).

Let us use these solutions for analysis of angular variations in the microwave radiation. Integrating the geodesic equation, following Sachs & Wolfe (1967), and taking into consideration the adiabatic process (Silk, 1968; Peebles & Yu, 1970), we can connect the observed temperature fluctuations of CMB radiation with density, velocity, and metric perturbations at the last scattering surface:

$$\frac{\delta T}{T}(\mathbf{n}) = \phi(\mathbf{n}R_h) + n_\alpha \delta V^\alpha(\mathbf{n}R_h) + \frac{1}{3}\epsilon_g(\mathbf{n}R_h)$$
$$= \frac{1}{3}\phi(\mathbf{n}R_h) + n_\alpha \delta V^\alpha(\mathbf{n}R_h) + \frac{1}{3}\epsilon_m(\mathbf{n}R_h), \qquad (A2)$$

where $\mathbf{n}$ is unit vector; n_α, its space covariant components; R_h is the distance to the last-scattering surface; ϵ_m is gauge-invariant energy density perturbation relative to the spacelike hypersurface which represents everywhere the matter local rest frame (Bardeen, 1980). The formula (A2) is obtained also in the synchronous gauge, but in this case ϕ is a solution of the Poisson equation $\Delta\phi = \frac{3}{2}H^2\epsilon_m$. This means that $\frac{\delta T}{T}$ is a gauge-invariant value for the sum of three effects only, as it was noted earlier by Starobinsky (1988).

Let us find the correlation function $C(\alpha) = \langle \frac{\delta T}{T}(\mathbf{n}_1)\frac{\delta T}{T}(\mathbf{n}_2)\rangle$ on large angular scale first for an ideal antenna at first. Substitution of (A2) here gives

$$C(\alpha) = \frac{1}{9}\langle\phi(\mathbf{n}_1)\phi(\mathbf{n}_2)\rangle + \langle \mathbf{n}_1\mathbf{V}_1\mathbf{n}_2\mathbf{V}_2\rangle + \frac{1}{9}\langle\epsilon_m(\mathbf{n}_1)\epsilon_m(\mathbf{n}_2)\rangle + \frac{2}{9}\langle\epsilon_m(\mathbf{n}_1)\phi(\mathbf{n}_2)\rangle \quad (A3)$$

(the rest of the components will become zero after averaging because of their uncorrelated). Using the Fourier transform and solutions of (A1) for Eq. (A2) and (A3), we shall obtained:

$$C(x) = \frac{2}{\pi^2}\int_0^\infty dk \frac{P(k)}{R_h^4 k^2}\Big(W(R_f')j_0(2R_h kx) + \frac{R_h^2 k^2 W(R_f'')}{1+z_{\mathrm{rec}}}(xj_2(2R_h kx)$$
$$+(1-2x^2)j_1(2R_h kx))\Big) - \frac{2}{\pi^2}\int_0^\infty dk \frac{P(k)}{R_h^4 k^2}(j_0^2(R_h k) + 3(1-2x^2)j_1^2(R_h k)), \quad (A4)$$

where $x \equiv \sin(\alpha/2)$, z_{rec} is the redshift of the recombination epoch, the term $W(R_f) = \exp(-R_f^2 k^2)$ describes the damping due to the fuzziness of the last scattering surface with $W(R_f') = 4.4\ \mathrm{h}^{-1}$ Mpc, and $W(R_f'') = 5.9\ \mathrm{h}^{-1}$ Mpc (Bond, 1986), j_0, j_1, j_2 are spherical Bessell functions of 0, 1st and 2nd order, respectively. Here the second integral is monopole and dipole moments which are excluded from the general expression for $C(x)$.

Now let us find the correlation function of temperature CMB fluctuations measured with an antenna having a beam of certain width. Taking a Gaussian function for the receiver response, with dispersion $\sigma = 0.425\theta_{\mathrm{FWHN}} - W(\mathbf{n}, \mathbf{n}', \sigma) =$

$e^{-(\mathbf{n}-\mathbf{n}')^2/2\sigma^2}/2\pi\sigma^2(1-e^{-2\sigma^2})$ – so $\frac{\delta T}{T}(\mathbf{n}) = \int \frac{\delta T}{T}(\mathbf{n}')W(\mathbf{n},\mathbf{n}',\sigma)\,d\Omega$, the observed correlation function is then

$$C(\alpha,\sigma) = \int_0^1 \int_{-1}^1 xC(x)e^{y(1-2x^2\cos\alpha+\cos\alpha)/\sigma^2} \frac{I(x,y,\alpha)}{\sigma^4 \mathrm{sh}^2(\sigma^{-2})}\,dydx, \qquad (A5)$$

where

$$I(x,y,\alpha) = \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} e^{f_1(x,y,\gamma,\psi)\sin\alpha/\sigma^2 + f_2(x,y,\gamma)\cos\alpha/\sigma^2}\,d\psi d\gamma,$$

$$f_1(x,y,\gamma,\psi) = -2x\sqrt{1-x^2}(y\sin\gamma\cos\psi+\cos\gamma\sin\psi)+(1-2x^2)\sqrt{1-y^2}\cos\psi,$$

$$f_2(x,y,\gamma) = 2x\sqrt{1-x^2}\sqrt{1-y^2}\sin\gamma.$$

The general expression for $C(\alpha,\sigma)$ (A4–A5) should be applied to calculate the r.m.s. temperature fluctuations in a single beam experiment: $\langle(\frac{\Delta T}{T})^2\rangle^{1/2} = (C(0;\sigma))^{\frac{1}{2}}$ or in a beam switching experiment with beamthrow angle α and beam width σ: $\langle(\frac{\Delta T}{T})^2\rangle^{\frac{1}{2}} = (2(C(0,\sigma)-C(\alpha,\sigma)))^{\frac{1}{2}}$, and the coherence angle of these fluctuations $\theta_c = \sqrt{-C(0,\sigma)/C''(0,\sigma)}$, where ($''$) denotes the second derivative over α.

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