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#### A criterion for identification of OB associations and star complexes

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# A CRITERION FOR IDENTIFICATION OF OB ASSOCIATIONS AND STAR COMPLEXES

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A new objective automated method for identification of OB associations and star complexes is proposed. It is suitable for a reliable comparison of the stellar groups in nearby galaxies. The method was applied for the galaxy M33. The criterion identifies three hierarchic groups in M33: OB association, aggregates, and star complexes. The select stellar groups show strong correlations with the HII regions in M33. The average size of the stellar associations, aggregates, and star complexes are 80 pc, 190 and 600 pc, respectively.

**KEY WORDS** Galaxies: individual, M33 – open clusters and associations, general – HII regions – methods, data analysis

## 1 INTRODUCTION

M33 is a nearby galaxy of Sc type with a suitable inclination between the galactic disk and the plane of the sky ( $57^\circ$ ). The photographic catalogue of Ivanov, Freedman & Madore (1993) (IFM) is suitable for identification of stellar groups in M33. The computer version of the catalogue contains 5095 blue stars. M33 is an excellent candidate for a study of star complexes and OB associations. Efremov *et al.* (1987) (EIN) and Ivanov (1987, 1991) (hereafter Papers I and II) found four categories of star formation by eye estimation: OB associations with a scale length (SL) 80 pc, aggregates (SL $\approx$ 250 pc), and star complexes (SL $\approx$ 600 pc). They correspond to the real scale of star formation by cascade fragmentation of the giant gas clouds (Elmegreen, 1991).

Many clustering methods have been proposed over the last two decades by Murtagh & Heck (1987), Battinelli (1991) (B91), and Wilson (1991) (W91). Ivanov (1995; hereafter Paper III) proposed a new criterion and identified the stellar associations in eight galaxies with CCD photometry. However, the criteria of B91, W91, and Paper III are not suitable for identification of star complexes.

A new automated criterion for the identification of hierarchic stellar structures is proposed in this study. The method is described in the Section 2. It is similar to

those of B91 and W91 and of Paper III. The preference of the present method is the objective determination of the scale distance parameter  $d_S$  from the observational data. The properties of the star complexes and OB associations are presented in Section 3. A short discussion is given in Section 4.

## 2 IDENTIFICATION CRITERION

The new criterion suggests that blue stars will be assigned to one and the same group if they show a statistically significant peak of density above the mean level of the surrounding blue stars. The surface density of the blue stars is the main property that can isolate stellar groups from the foreground of blue stars.

### 2.1 The Search Neighbour Distance $d_S$

Suppose a random Poisson distribution of stars in a galaxy on the sky. The mean value of the distance to the nearest neighbour  $d$  is (see the Appendix in Paper III):

$$\langle d \rangle \approx 3/[2(\delta)^{1/2}] = 3(\pi)^{1/2}D/[4(n)^{1/2}], \quad (1)$$

where  $D$  is the diameter and  $n$  is the number of stars in a stellar group and  $\delta$  is stellar surface density.

In the case uniform density

$$\langle d \rangle \approx 2/(\pi\delta)^{1/2} = D/(n)^{1/2}. \quad (2)$$

Eq. (1) shows that the mean angular distance  $\langle d \rangle$  to the nearest neighbours is a measure for the surface density in a stellar group. The search distance  $d_S$  can be defined as a distance scale which can isolate groups of a hierarchical level. The present criterion assumes that the blue stars belong to a group of some hierarchy if they have similar neighbour distances  $d_i \leq d_S$ . The first group includes the blue stars which are close enough. Then new groups are formed from stars that are remote from the existing groups but close to one another. Such approach was used by B91 and W91. The effectiveness of the present identification method depends on the suitable choice of the search distance  $d_S$ .

### 2.2 The Least Number of Stars in a Group

Stellar groups of different hierarchic level have widely different stellar densities and numbers of blue stars. We suppose that OB associations are the smallest stellar groups with the highest stellar density. A group is candidate for an OB association if it consists of three or more stars,  $N_{\min} \geq 3$ . The groups belonging to a higher hierarchic level (for instance, star complexes) contain two or more OB associations, i.e. more than six blue stars, for a given average distance  $\langle d \rangle$ . The OB associations can be eliminated in the process of selecting star complexes using a

large number (for instance  $N_{\min} > 6$ ). Let us accept the least number of stars  $N_{\min}$  per stellar group of a given hierarchic level. Small groups with numbers of blue stars  $n < N_{\min}$  are eliminated. The stellar groups of a given rank hierarchy can be selected using suitable parameters  $d_S$  and  $N_{\min}$ . The choice of the two parameters is very important for the present study. The parameters  $d_S$  and  $N_{\min}$  must be specified from the observational data.

### 2.3 The Choice of the Parameters $d_S$ and $N_{\min}$

The search distance  $d_S$  was defined as a limiting distance scale typical for a given hierarchic level. The present criterion assumes that the blue stars belong to the same kind of stellar groups if they have a significant maximum of stellar density of blue stars above the mean level of the surrounding background. The best choice of the parameters  $d_S$  and  $N_{\min}$  is supposed if stellar groups have a maximum of stellar density above the blue background.

The fluctuation of stellar density of groups was defined as:

$$f(\delta_i) = (\delta_i - \delta_B) / ((\delta) - \delta_B), \quad (3)$$

where

$$\delta_i = n / \Sigma_g \quad (4)$$

is the individual surface density of a stellar group,  $n$  is the number of stars, and  $\Sigma_g$  is the surface of the stellar group in  $\text{arcsec}^2$ ,  $(\delta)$  is the mean density of blue stars around the stellar group, i.e. blue background density in the vicinity of the group, and  $\delta_B = N / \Sigma_{\text{galaxy}}$  is the mean density of all blue stars in the galaxy. The function  $f(\delta_i)$  gives individual fluctuations of stellar density of the selected groups above the surrounding blue background.

A reliable function for statistical evaluation of hierarchy of stellar groups is the Fourier transform of the stellar density fluctuations  $f(\delta_i)$ :

$$F(\nu_k) = \sum_{j=1}^{N_g} f(\delta_j) \exp(i2\pi s_j \nu_k), \quad (5)$$

where  $s_j = (j-1)(sz_j - szm)$  is the interval of discretion of sizes of selected stellar groups,  $\nu_k = (k-1)/(szx - szm)$ ,  $szx$  and  $szm$  are the largest and smallest size, respectively, of the selected groups,  $j, k = 1, 2, 3, \dots, N_g$ , and  $N_g$  is the number of the groups. The amplitude of the Fourier transform is:

$$FF(\nu_k) = \left\{ \left[ \sum_j f(\delta_j) \cos(2\pi s_j \nu_k) \right]^2 + \left[ \sum_j f(\delta_j) \sin(2\pi s_j \nu_k) \right]^2 \right\}^{1/2}. \quad (6)$$

Let the maximum of this function  $FF_m = \max\{FF(\nu_k)\}$  correspond to the sample of groups for which the amplitude of the stellar density is maximal. The functions  $FF_m$  can be obtained for different search distances  $d_S$ . The highest maximum of the function  $FF(\nu_k)$  was defined as  $FF_m$  which indicates that the scale distance  $d_S$

separates the maximum number of stellar groups with the largest sum of amplitudes of densities above the local blue background by Eq. (6).

The present criterion assumes that the blue stars form groups of the same kind of stellar hierarchy if the function  $FF_m$  has the highest maximum for the given search distance  $d_S$ . If the groups with small numbers of blue stars belong to another hierarchic class, they can be eliminated using a large number  $N_{\min} > 3$ . If such groups are rejected using larger  $N_{\min}$ , the function  $FF_m$  shows another maximum for another  $d_S$ . For instance, a stellar association can contain at least three blue stars. However a star complex contains several stellar associations, at least two associations, i.e.  $N_{\min} > 6$  blue stars per complex. A larger distance scale  $d_S$  and  $N_{\min} > 6$  eliminate many groups with lower hierarchic rank. However, the number  $N_{\min}$  depends on the completeness of photometry, the distance to the galaxy, and the rate of star formation. It can be derived from the observational data at the maximum of the function  $FF_m$  for different  $N_{\min}$  numbers. The statistics in this Section is a suitable basis which gives an average distance  $d_S$  and the most probable  $N_{\min}$  characteristic for each hierarchic class. The choice of this parameter for some hierarchic rank of stellar groups is based upon the observational data.

#### 2.4 Elimination of Random Clumps

In order to exclude stars randomly associated in groups, the numbers of stars in each group were compared with the expected number of such events derived from the Poisson distribution. If a group with  $n$  stars covers an area  $s$  in arcsec<sup>2</sup>, then the probability to find at least  $n$  stars from a random distribution on the same area  $s$  is (see the Appendix in the Paper III):

$$P_n(R_n < r) = 1 - (P_0 + P_1 + P_2 + \dots + P_{n-1}) = 1 - \sum_{k=0}^{n-1} \frac{u^k}{k!} \exp(-u), \quad (7)$$

where  $R_n$  is the distance to the  $n$ -th star from an arbitrary point inside the association,  $P_0, P_1, P_2, \dots, P_n$  are the probabilities to have 0, 1, 2,  $\dots, n$  stars, respectively, within a radius  $r$ ,  $u = \pi r^2 \delta_B$ , and  $\delta_B$  is the average surface stellar density of the blue background. The number of stars in associations has to be significantly greater than the expected background number of stars. The area  $s$  of a stellar group with  $n$  stars has to be significantly smaller than the area occupied by a random clump with  $n$  stars. This means that the probability  $P_n(R_n < r)$  has a very small value for a real stellar group. In the present paper, a group would be considered as a real one if  $P_n(R_n < r) \leq 0.01$ . This value is quite small. It guarantees that random clumps are eliminated at the level  $\approx 5\sigma$ .

#### 2.5 The Highest Individual Fluctuation of Stellar Groups

The criterion described in Section 2.3 is a suitable hierarchic agglomerative algorithm. The least number of stars  $N_{\min}$  for a given hierarchical level can be obtained

from the observational data by searching the highest maximum of  $FF_m$  for different  $N_{\min}$ . However, Eq. (1) shows that the mean angular distance ( $d$ ) to the nearest neighbours is a measure of the surface density in a stellar group. The stellar density depends substantially on the rate of star formation (the number of OB stars per unit area). The stellar groups in the central region of M33 are richer in OB stars than in the outer regions of the galaxy. For this reason, the stellar groups of the same hierarchical level have widely different stellar densities. It is not possible, with a fixed  $d_S$ , to select the total sample of groups for a given hierarchic level. For instance, in the crowded central region of M33, stellar associations (groups with the highest fluctuations of stellar density) have a smaller  $d_S$  than those in the outer region of the galaxy. It is better to select the groups with a fixed  $N_{\min}$  and different  $d_S$ . Let a stellar group be selected with some  $N_{\min}$  and  $d_S$ . We want to know if a neighbour blue star is a member of the group. Is it possible for this star to be assigned or rejected as a group member? We can look for the blue stars closest to the group. The function  $f(\delta)$  of the group can increase or decrease when a neighbour is added. The neighbour stars are added to the group if the function  $f(\delta)$  increases. We can verify neighbour stars for membership. On the other hand, we can improve the analysis by implementing a discrimination of member stars. A member star may be rejected if the function  $f(\delta)$  increases when eliminating this star.

The procedure which selects the group members is quite simple in practice. The highest individual fluctuation for each group was searched. The function  $f(\delta)$  given by Eq. (3) was obtained for different  $d_S$  and a fixed  $N_{\min}$ , consequently for different samples of blue stars. The maximum of the function is

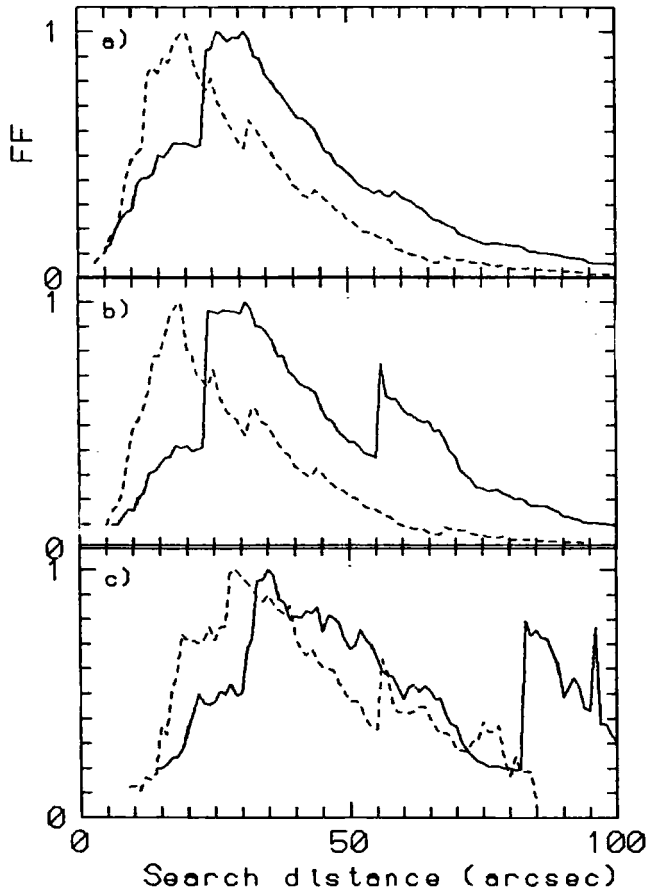
$$f_i^{\max}(\delta) = \max\{f_i(\delta)\} \quad (8)$$

for different  $d_{S,i}$   $\{i = 1, 2, \dots, N_g\}$  where  $N_g$  is the number of selected groups.  $f_i^{\max}(\delta)$  is a general characteristic of different groups from a given hierarchic rank. The fixed  $N_{\min}$  and an individual  $d_{S,i}$  determine the highest fluctuation of stellar density  $f_i^{\max}(\delta)$  in some region of the galaxy. The value of  $d_{S,i}$  was accepted as the most probable individual characteristic for a stellar group from given hierarchic rank. For each group, an individual threshold value  $d_{S,i}$  was estimated. This algorithm in constructing the nearest neighbour approach and a threshold  $d_{S,i}$  for each group guarantees that we can select the most compact group.

The highest fluctuation of stellar density  $f_i^{\max}(\delta)$  in some region of the galaxy is the best criterion for partition of a group from their neighbours. However, a threshold value  $f^{\lim}$  of the function  $f_i^{\max}(\delta)$  must be estimated for each hierarchic level so that

$$f_i^{\max}(\delta) \geq f^{\lim}. \quad (9)$$

This relation guarantees that we select the most compact and the best separated groups of a given hierarchic rank.



**Figure 1** Distributions of the  $FF$  function versus the distance parameter  $d_S$  for 1880 OB stars in M33 (continuous line) and for total sample of 5095 blue stars (dashed line). Panel "a" corresponds to the minimum number of stars per group  $N_{\min} = 3$ ; panel "b", to  $N_{\min} = 4$ ; and panel "c", to  $N_{\min} = 7$ .

### 3 STELLAR GROUPS IN M33

The computer version of the IFM catalogue contains 5095 blue stars with  $(U - B) < 0.0$ . The UBV data are more selective than  $B - V$  ones. Following Massey *et al.* (1989), FitzGerald (1970) and Flower (1977) obtained:

$$E_{B-V} = B - V - 0.33Q + 0.17, \quad E_{U-B}/E_{B-V} = 0.72, \quad (10a)$$

$$\log T_{\text{eff}} = 3.994 - 0.267Q + 0.367Q, \quad (10b)$$

$$BC = 23.493 - 5.926 \log T_{\text{eff}}. \quad (10c)$$

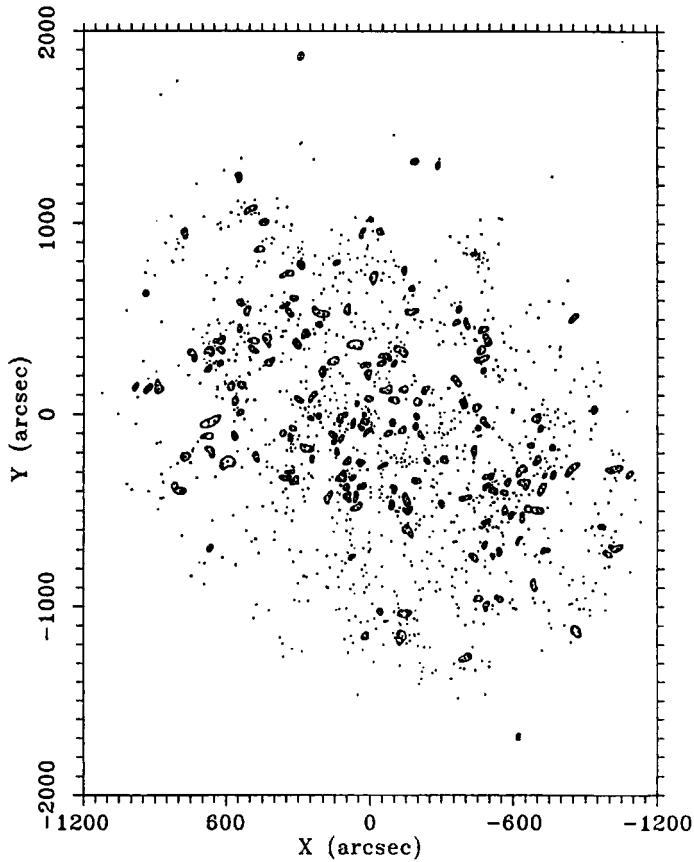


Figure 2 OB associations in M33 selected with the parameters:  $N_{\min} = 3$ , and the limiting fluctuation  $f^{\text{lim}} = 14$ . This means that the 179 selected groups have at least three OB stars and stellar density 14 times higher than the surrounding blue background.

The stars with  $(B - V)_0 \leq -0.2$ ,  $(U - B)_0 \leq -1.0$ , and  $-10 < M_{\text{bol}} < -6$  are typical for massive stars between  $15\text{--}60M_{\odot}$ . Following this criterion, 1880 stars in M33 were selected. Hereafter they are called OB stars.

### 3.1 OB Associations and Star Complexes in M33

Figure 1 shows the distributions of the normalized FF function given by Eq. (6) versus the search distance  $d_S$ . The maximum of the distribution for  $N_{\min} = 3$  is at the search distance  $d_S \approx 25$  arcsec (Figure 1a). Consequently this value is the probable search distance which separates the smallest stellar groups with the highest stellar density. The search distance  $d_S \approx 25$  arcsec corresponds to the smallest hierarchy of stellar groups in M33.



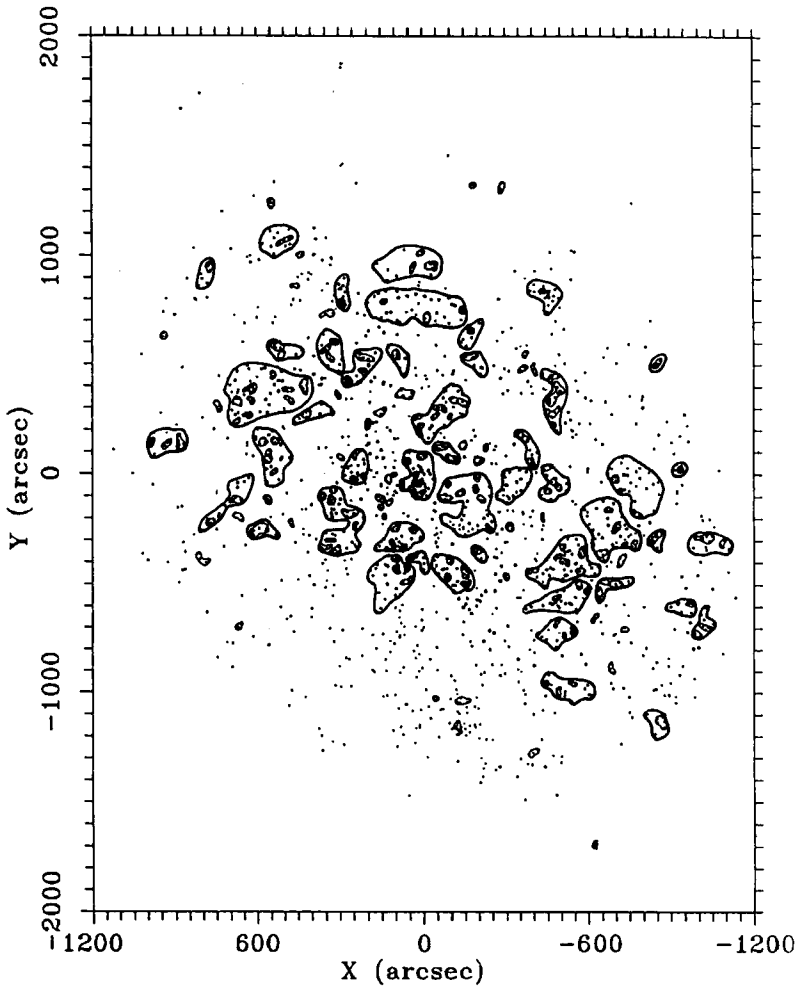


Figure 3 Star complexes in M33 (thick line) selected with the parameters:  $N_{\min} = 7$ ,  $f^{\text{lim}} = 3$ . OB associations in Figure 2 are outlined with a thin line. They seem like cores within the star complexes.

The highest individual fluctuation  $f_i^{\text{max}}(\delta)$  above the blue background was obtained for  $M_{\min} = 3$ . Using the individual parameters  $d_{S,i}$  for selected groups and  $N_{\min} = 3$ , the quantity  $f^{\text{lim}} = 14$  was obtained. 179 stellar groups with the average size 80 pc were identified. Their contours are shown in Figure 2. The sizes of selected OB associations were measured along the major axis of M33. This is the only dimension which is not affected by the projection effect due to the inclination between the galactic disk and the plane of the sky. The groups in Figure 2 correspond to the OB associations selected in the previous papers (Papers I, II, and III).

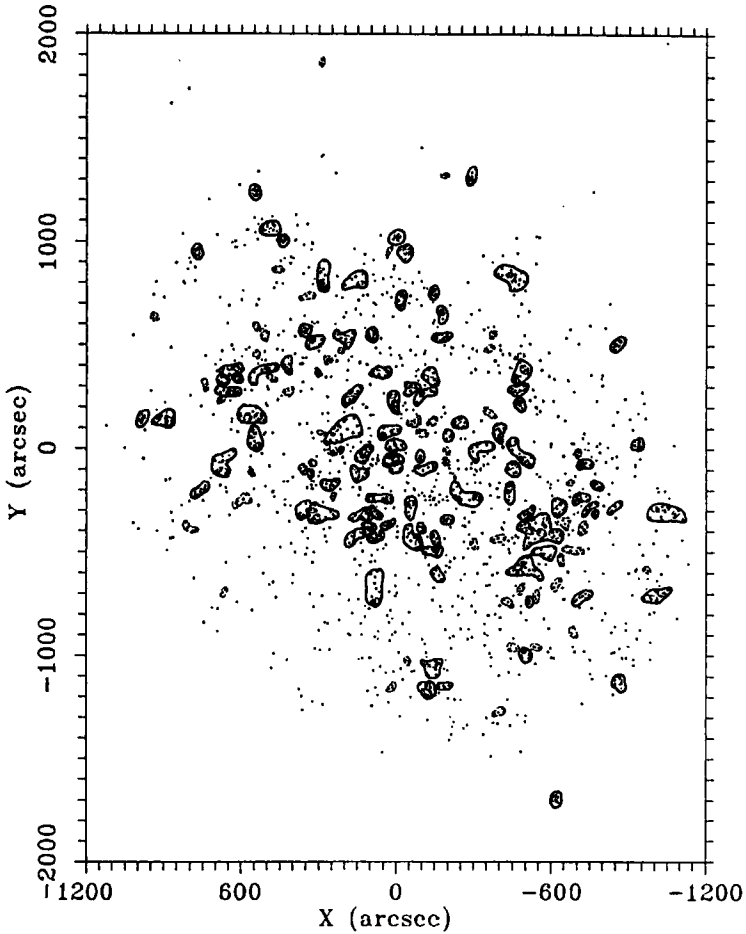


Figure 4 Stellar aggregates in M33 selected with the parameters:  $N_{\min} = 4$  and  $f^{\text{lim}} = 3$ . OB associations are outlined with a dashed line. Thick dots are members of stellar associations.

Varying the parameters  $d_S$  and  $N_{\min}$ , a new maximum in the distributions of the  $FF$  function was found in Figure 1c at  $d_S = 83$  arcsec and for  $N_{\min} = 7$  (panel “c”). About 50 stellar groups with an average size  $\approx 600$  pc were identified. The selected groups with these parameters are a real homogeneous sample with significant density above the mean background density level. Their contours are shown in Figure 3. The selected groups correspond to the star complexes (see Efremov, 1988).

Our analysis defines an intermediate class of objects at  $d_S = 56$  arcsec and for  $N_{\min} = 4$  (Figure 1; panel “b”). Using  $N_{\min} = 4$ ,  $f^{\text{lim}} = 3$ , about 100 stellar groups with an average size  $\approx 200$  pc were identified. The selected groups with these parameters correspond to stellar aggregates (Efremov, 1988). The contours of these groups are shown in Figure 4.

### 3.2 Comparison with HII Regions in M33

A half of OB associations (56% of their total number) have an intersection area with one or more HII regions. On the other hand, a small number of HII regions of Courtes *et al.* (1987) have a common area with OB associations (corresponding to 23% of the total number). The percentage of HII regions outside OB associations is high. This is why most HII regions are excited by a single OB star or a couple of them.

A simple way to evaluate the correlation between the OB associations and HII regions is to compare the percentage of the associated objects with that expected from the random distribution. Let  $N_1$  extended objects of one population have a characteristic radius  $r_1$  and a surface density  $\delta_1$ . Another population of extended objects  $N_2$  has a mean radius  $r_2$  and a density  $\delta_2$ . The two populations occupy the same area. The probability for the centre of one object to fall within radius  $r$  (in the case of Poisson distribution of the objects) is given by Eq. (7) for  $n = 1$ . The probability to find at least one object of the population "1" and at least one object of the population "2" together within the radius  $r = r_1 + r_2$  (i.e., for the two objects to have a common area) can be written as:

$$P(d < r) = P_1(r_1 < r)P_2(r_2 < r) \quad (10)$$

where  $d = (r_1 + r_2)/2$ .

The expected number of coincidences between OB associations and HII regions from the random distribution is  $N_E = N_1 N_2 P(d < r)$ . The correlation between OB associations and HII regions can be evaluated by the ratio:

$$R_N = N_{\text{ass}}/N_E. \quad (11)$$

From the observational data,  $R_N \approx 52$  was found. The number of OB associations connected with the HII regions,  $N_{\text{ass}}$ , is about fifty times larger than that when the two populations were assumed to be randomly distributed.

Another way to evaluate the correlation between OB associations and HII regions follows the overlap correlation technique of Deul & Hartog (1990). The expected overlap area from random distribution is:  $E_{\text{ov}} = \Sigma_{\text{OB}}\Sigma_{\text{HII}}/\Sigma_{\text{tot}}$ , where  $\Sigma_{\text{OB}}$  and  $\Sigma_{\text{HII}}$  are the total surface area of OB associations and HII regions respectively, and  $\Sigma_{\text{tot}}$  is the surface area covered by the two populations. The total overlap area between HII regions and OB associations is  $\Sigma_{\text{ov}}$ . The ratio of observed area is:

$$R_s = \Sigma_{\text{ov}}/E_{\text{ov}}. \quad (12)$$

The data give  $R_s \approx 5$ . This result can be interpreted as tight correlation between OB associations and HII regions.

A measure of coincidence between two populations can be evaluated as:

$$R_c = \Sigma_{\text{ov}}/\Sigma_{\text{max}}, \quad (13)$$

where  $\Sigma_{\text{max}}$  is the expected overlap area if the centers of the two populations fully coincided.  $R_c = 0.33$  was found. Probably a fraction of association members are

Table 1. Properties of the stellar groups in M33

<i>Hierarchic class</i> (1)	$d_S$ arcsec (2)	$N_{min}$ (3)	$f^{lim}$ (4)	$N_g$ (5)	<i>Size</i> arcsec (6)	$N_{HII}$ % (7)	$R_N$ (8)	$R_s$ (9)	$R_c$ (10)
OB assoc.	5-25	3	14	179	20.7	0.56	52	5	0.33
Aggregates	26-56	4	3	101	49.4	0.73	10	3	0.16
Complexes	59-83	7	3	47	155.1	0.84	3	2	0.10

exiting stars embedded in the HII regions. Table 1 lists the properties of stellar groups in M33; the contents of the table are as follows:

Column (2) – The range of individual threshold values  $d_{S,i}$  for the groups.

Column (3) – The minimum number of stars per group ( $N_{min}$ ).

Column (4) – The limited surface density  $f^{lim}$  of groups given by Eq. 12.

Column (5) – Number of groups ( $N_g$ ).

Column (6) – The size of associations along the major axis of the galaxy in arcsec.

Column (7) – The number of coinciding with HII regions ( $N_{HII}$ ).

Column (8) – The correlation between OB associations and HII regions given by Eq. (12) ( $R_N$ ).

Column (9) – The correlation between OB associations and HII regions given by Eq. (13) ( $R_s$ ).

Column (10) – The measure of coincidence between OB associations and HII regions given by Eq. (14) ( $R_c$ ).

#### 4 DISCUSSION

The hierarchical structures, from the star complexes to the associations, are a phenomenon in spiral galaxies (Efremov, 1988). It seems that the shock wave is the main triggering mechanism for formation of giant low-density clouds with a size of 1 kpc and masses of the order of  $10^7 M_\odot$ . Elmegreen (1981) considered Parker–Jeans instability as a possible mechanism for forming large dense molecular clouds with a mass of  $\approx 10^6 M_\odot$  and a scale length of 300 pc. Probably the magnetic fields connect the clouds in extensive regions of 0.5–1 kpc (Elmegreen, 1982). They are progenitors of the star complexes with a mean size of 600 pc and a mass of  $10^5 M_\odot$ . Cascade fragmentation in the giant clouds forms smaller and denser subclouds, and then the gravitational instability inside them produces denser cores with smaller sizes. The star formation process in the dense core of a molecular cloud produces an OB association.

The combination of the shock wave, Parker–Jeans instability, and propagating star formation can explain the origin of the star complexes and OB associations. Elmegreen (1991) considered the combined effect including magnetic, thermal, and

gravitational instabilities which can lead to star formation in galaxies. The existence of the same physical processes in the gas layers of galaxies can lead to the similar size distributions for the OB associations with the characteristic dimension  $\approx 100$  pc.

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