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A METHOD FOR STAR IMAGE SEPARATION FROM BACKGROUND IN AUTOMATIC COORDINATE MEASUREMENTS WITH A MICRODENSITOMETER

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A method for star image separation from a background is suggested. The applicability of the method is illustrated on example of automatic coordinate measurements of double star images on astroplates with the help of the "Fantazia-2" Automatic Measuring Complex (AMC), which is based on a high-precision microdensitometer. It is shown that the method provides the minimum dispersion of distances between the components of a binary if the center of mass statistics of a discretized image is used for measurements. Preliminary results of analysis of the center of mass statistics in automatic coordinate measurements with the AMC are presented. It is shown that the center of mass cloud not be considered as an accurate characteristic of stellar image position. Strong correlation between images of the components of the double star ADS 3353 (angular distance $r = 4$ arcsec, magnitudes of components 6.5 and 6.5) is discovered, so that, in image processing of a double star with $r = 4$ arcsec, the components of binary should not be processed separately. In that case experiments to check up if there are any reasons to revise previously estimated so-called isoplanatic square size are of interest. Some properties of the ORWO astroplates NP-27, WP-1, WO-3 are discussed. This research program is coordinated with the Pulkovo program of photographic positional observations of double stars with the 26-inch Pulkovo Refractor.

KEY WORDS Automatic measurements, microdensitometer, photographic astrometry

INTRODUCTION

Investigations in photographic astrometry have shown that a qualitative visual analysis of an image cannot provide objective measurements of the object parameters. Increasing of the objective nature of image analysis is promoted by the quantitative methods. Image description in quantitative terms allows to apply strict mathematical methods. To promote quantitative methods in image processing, automatic systems were designed. Depending on the complexity level, these systems are capable

of not only presenting measuring functions but also of carrying out image analysis. One of implementations of automatical systems for image processing is the analysis of microscopic images with the size of their typical substructures comparable to the wavelength of the visible light. According to the definition of a microscopic image given above, photos made on a high-resolution plate belong to the same class of objects. In 1986, an Automatic Measuring Complex (AMC) "Fantazia-2" was installed at Pulkovo Observatory, which was designed on the basis of the AMC "Zenit-2" in the Novosibirsk Institute for Automatics and Electronics (Bury *et al.*, 1974; Kiritchuk *et al.*, 1984). The fundamental principle of the AMC activities is the adequate conversion of input image represented by continuous two-dimensional field into a digital code by space sampling and assignment to the every element of discrete field of the number defined by its optical density. Technical characteristics of the AMC provide space sampling with accuracy in one dimension up to 1 micron along with 156 levels of optical density discretization. High technical properties of the AMC, which were devised according to the Pulkovo scientist's demands (Kisselev and Zatziorsky, 1974; Kisselev *et al.*, 1981), provide adequate transformation of the information registered on the plate for future analysis. However, as shown by Makarov (1987), digital image processing in astronomy has its own character, which follows from the availability of a *priori* information and also from the existence of unremovable random noise (for instance, atmospheric turbulence). Therefore in digital image processing in astronomy it is necessary to develop algorithms according to the limitations mentioned above. This paper is aimed at the development of one of such algorithms.

FORMULATION OFF THE PROBLEM

The first in order and one of the most important steps in image processing in astronomy is the separation of an image from background. In accordance with their local problems, many researchers suggested various methods for image separation (see, i.g., Kosykh and Pusovskyh, 1982; Gonzalez and Wintz, 1986; Frieden, 1983; Nikolsky and Saddykov, 1982). Modern methods for image separation use the estimate of noise defined by a threshold. In many cases (Frieden, 1983) the choice of the threshold depends entirely on the experience of the researcher and, therefore, is subjective. Let us consider, for instance, one example taken from the excellent textbook written by Gonzalez and Wintz (1986). To get rid of the noise, the following procedure is suggested: the input discrete image $I(l, k)$ should be transformed as follows:

$$g(l, k) = |I(l, k) - I(L + 1, k)| + |I(l, k) - I(l, k + 1)|,$$

$$g_1(l, k) = \begin{cases} g(l, k), & \text{if } g(l, k) \geq T, \\ 0, & \text{if } g(l, k) < T, \end{cases}$$

$$l = 1, \dots, N - 1; \quad k = 1, \dots, M - 1,$$

where $g(l, k)$ is an intermediate product, $g_1(l, k)$ is the new image and T is the threshold level, which should be predefined by the user. There also exist methods based on the estimate of the local maxima of the transmission histogram. In this country, a method of averaging and then direct subtraction is of common use even in accurate CCD and photographic observations.

We see at least three reasons making the image separation so important. First, the common method to determine image coordinates on the plate (Kisselev *et al.*, 1984) is based on the center of mass (CM) definition, and the CM position depends the threshold. Secondly, the image, which has been formed in the presence of random atmospherical noise, includes some information about the noise at its border (Krassilnikov, 1986), and, therefore, the optimum choice of the threshold could improve our knowledge about the specific realization of the stochastic process which describes the model of image formation. Thirdly, image separation and, therefore, noise estimate are essential at next stages of image processing, for instance, in image restoration. Finally, our problem can be formulated as follows: search for the optimum threshold in optical density between a star image and noise.

THE ALGORITHM

The problem of the choice of the threshold between an object and noise is considered. According to Fu *et al.*, (1989), one could consider the histogram of intensity distribution, which consists of a sum of probability density values. Considering two types of intensity, we have for the approximating function

$$p(d) = S_1 \cdot p_1(d) + S_2 \cdot p_2(d), \quad (1)$$

where intensity d is a stochastic variable, $p_1(d)$ and $p_2(d)$ are the probability density functions. S_1 and S_2 are a *priori* probabilities. In our case a *priori* probabilities involve two types of intensity in our image: those of the object and of the background. In the decision theory (Tu and Gonzalez, 1978) it is stated that the mean error of identification of the pixel as a background or an object's one is minimized according to the following rule. For a pixel with the optical density d_0 , we compare $S_1 \cdot p_1(d_0)$ and $S_2 \cdot p_2(d_0)$. If $S_1 \cdot p_1(d_0)$ is greater than $S_2 \cdot p_2(d_0)$, we conclude that this pixel belongs to the image, and conversely. The threshold is determined by the equation:

$$S_1 \cdot p_1(T) = S_2 \cdot p_2(T), \quad (2)$$

where T is the threshold value. If n types of intensity appear on the plate, equation (2) can be generalized in the following manner:

$$S_k \cdot p_k(T_{kj}) = S_j \cdot p_j(T_{kj}), \quad k, j = 1, \dots, n, k \neq j. \quad (3)$$

Equation (3) allows to separate any two types of intensity on the plate if the relative squares S_k and S_j and the probability density functions are known.

In order to test this method, a number of experiments were performed with the photoplates ORWO WP-1, NP-27, WO-3, and WO-1 on the 26-inch Refractor ($d = 65$ cm, $f = 10,41$ m) at the Main Astronomical Observatory, Pulkovo.

Table 1.

| <i>Astroplate type</i> | <i>Average fog optical density, D_f</i> | <i>Dispersion, σ_f^2</i> |
|------------------------|--|--|
| ORWO NP-27 | 43.181 | 0.009 |
| ORWO WO-3 | 26.461 | 0.004 |
| ORWO WP-1 | 41.966 | 0.003 |

MEASUREMENTS, EXPERIMENTS AND CALCULATIONS

Preliminary analysis showed that the astroplates ORWO WO-1, WO-3 and NP-27 used in observations all have large values of optical density in the free-of-image areas. So we were forced to consider the plate fog, which achieves sometimes 45 units of optical density (UODs). We recall that the AMC discretizes optical density within 0-256 UOD range and, therefore, in accordance with the equation, representing relationship between the optical density D and the coefficient of transmission τ : $-\ln \tau = D/256$, one could see that 45 UODs mean that even an unexposed plate may hold 16 percents of falling light. Fog is non-zero optical density in unexposed areas, formed by chemicals and/or scattered emission (daylight, X-rays, cosmic rays, etc.). Studying of fog has its own value in connection with some other problems of image processing in astronomical photography. Some properties of astroplate fogs were studied by Breido (1964). We suppose the existence of three intensity levels on the plate while shooting a star: fog, background and star itself. Our first experiment was to determine fog of ORWO plates. One astroplate of each type was exhibited and than measurements of optical density in several arbitrary areas on the plate with typical resolution 4 microns in each dimension were performed. The results obtained are presented in Table 1.

For background and image measurements, astroplates with the double star ADS 3353 were used.

On each plate we measured with the help of the AMC optical densities of background (three arbitrary areas on each plate) and images with resolution of 4 microns in one dimension. Results of background measurements are presented in Table 2. A typical double star image is presented in Figure 1.

Table 2.

| <i>Astroplate type</i> | <i>Average background optical density, D_b</i> | <i>Dispersion, σ_b^2</i> | <i>Total exposure time, minutes</i> |
|------------------------|---|--|-------------------------------------|
| ORWO WO1/1 | 48.336 | 0.024 | 1.5 |
| ORWO WO-1/2 | 42.723 | 0.022 | 1.0 |
| ORWO WO-3 | 32.668 | 0.006 | 6.0 |

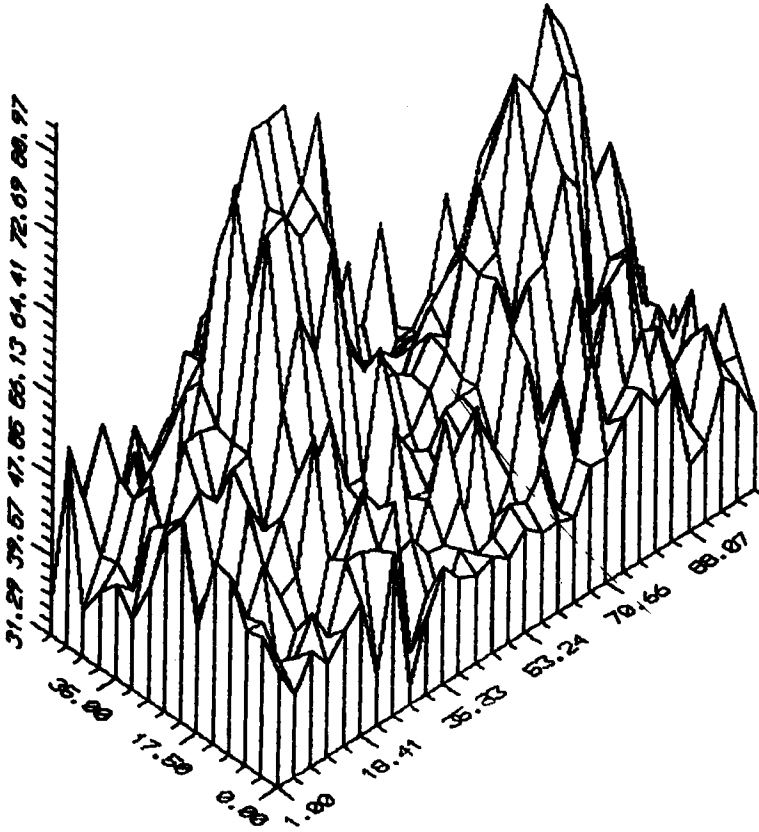


Figure 1 Image of the binary ADS 3353.

It should be pointed out that a sum of fog and background is shown in Table 2 as they are measured in practice. It is well known that fog and background are additive if measured in UODs. Correlations among fog, background and image is a question of special consideration and will be discussed elsewhere. To illustrate the efficiency of the separation method described above let us extract from background an image of the binary ADS 3353. According to Gasper and Palma (1980), the probability density function for the background can be expressed as a Gaussian. To find the threshold between the star and the background, let us assume that the stellar image also has, to a first order of approximation, a Gaussian probability density function. We denote the first and the second components of the binary by subscript O_1 and O_2 . Then equation (3) reduces to

$$\begin{aligned}
 & [P_{O_i}/(\sigma_{O_i} \cdot (2\pi)^{1/2})] \cdot \exp[-(T_{bO_i} - D_{O_i})^2/(2 \cdot \sigma_{O_i}^2)] \\
 & = [P_b/(\sigma_b \cdot (2\pi)^{1/2})] \cdot \exp[-(T_{bO_i} - D_b)^2/(2 \cdot \sigma_b^2)], \quad (4)
 \end{aligned}$$

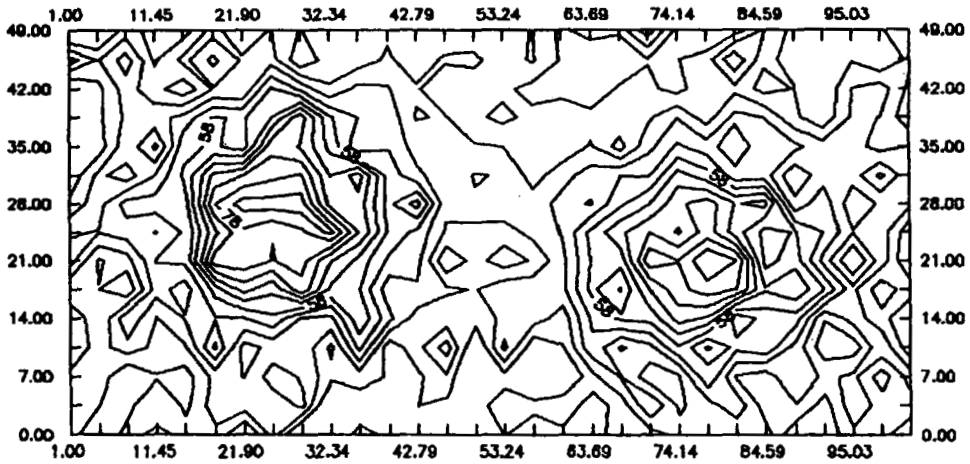


Figure 2 Image of the binary ADS 3353; XY plane; Threshold 43 UODs.

$$i = 1, 2,$$

where the subscript b corresponds to background, T_{bO_i} means the threshold between the background and the image O_i , P_{O_i} and P_b are the relative areas of image and background, normalized by the total scan area. Solving (4) for T_{bO_i} , we have

$$T_{bO_i} = \{2S \pm [4S^2 - (A_{O_i} - A_b) \cdot Q]^{1/2}\} / (A_{O_i} - A_b), \quad (5)$$

where

$$\begin{aligned} S &= A_{O_i} D_{O_i} + A_b D_b, \\ Q &= A_{O_i} D_{O_i}^2 + A_b D_b^2, \\ A_{O_i} &= -\ln \sigma_{O_i} / \sigma_{O_i}^2, \\ A_b &= -\ln \sigma_b / \sigma_b^2. \end{aligned}$$

The choice of the sign in the numerator of (5) should be made according to the following conditional inequalities:

$$T_{bO_i} > 0, \quad (6a)$$

$$D_f \leq T_{bO_i} \leq D_{O_i}, \quad (6b)$$

Because of indefiniteness of the threshold for stellar images it seems that direct calculation of the mean values and dispersions could be performed with difficulty. Let D_{O_i} and $\sigma_{O_i}^2$, $i = 1, 2$ to be unknown. Solving the maximum likelihood equation for the normal distribution with unknown average and dispersion (Koroluk *et al.*, 1985), we obtain:

$$D_{O_i} = [1/(n \cdot m)] \cdot \sum_{k=1}^n \sum_{l=1}^m D_{O_i}^{kl},$$

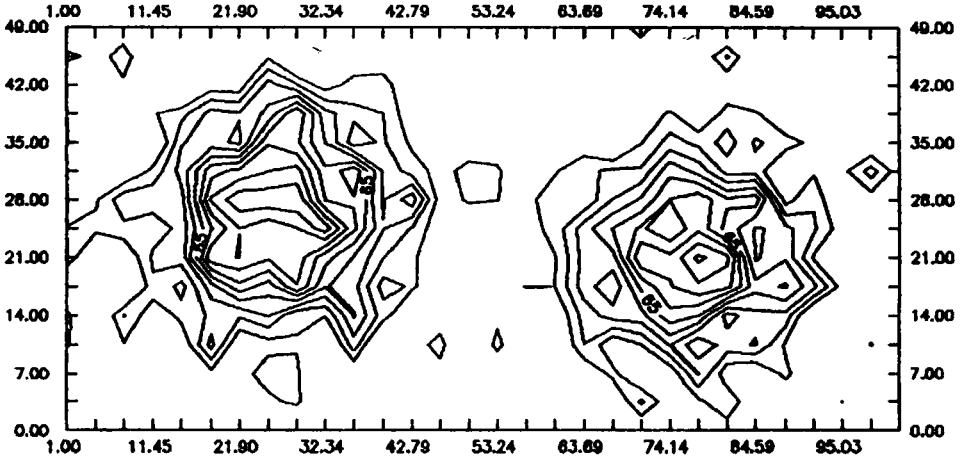


Figure 3 Image of the binary ADS 3353; XY plane; Threshold 49 UODs.

$$\sigma_{O_i}^2 = [1/(n-1)] \cdot [1/(m-1)] \cdot \sum_{k=1}^n \sum_{l=1}^m (D_{O_i}^{kl} - D_{O_i})^2,$$

where m and n are the mean numbers of pixels in the X and Y directions of the AMC scan; $D_{O_i}^{kl}$ is the optical density of the pixel having coordinates k and l which belongs to the i 'th image. In our practice, we determined preliminary estimates for m and n by the application of one well-known methods (for instance, we found the average value of optical density between the image and background) and then we evaluated scan parameters by visualization of the preliminary separation. Although we see some weakness here, practice showed that dependence of the threshold on the average and dispersion (i.e., on and) is very weak, so that variation in three UODs causes variation in the fourth digit after the decimal point in the threshold. And we should note that we are going to determine the threshold in terms of integers. For the ADS 3353 double star image derived from the ORWO WO-1/2 plate (see Table 2), averages and dispersions were as follows:

$$\begin{aligned} D_{O_1} &= 55.076, & \sigma_{O_1}^2 &= 0.122, \\ D_{O_2} &= 58.462, & \sigma_{O_2}^2 &= 0.115. \end{aligned}$$

Substituting averages and dispersions from the above and Table 2 into (5) and using (6), we have:

$$\begin{aligned} T_{b_{O_1}} &= 48.2487 = 48 \text{ UODs}, \\ T_{b_{O_2}} &= 48.2486 = 48 \text{ UODs}. \end{aligned}$$

Subtracting all the pixels with optical density less or equal than $T_{b_{O_i}}$, $i = 1, 2$ we separate the image from the background. Figure 2 represents the result of the subtraction of the average background value in accordance with one of well-known

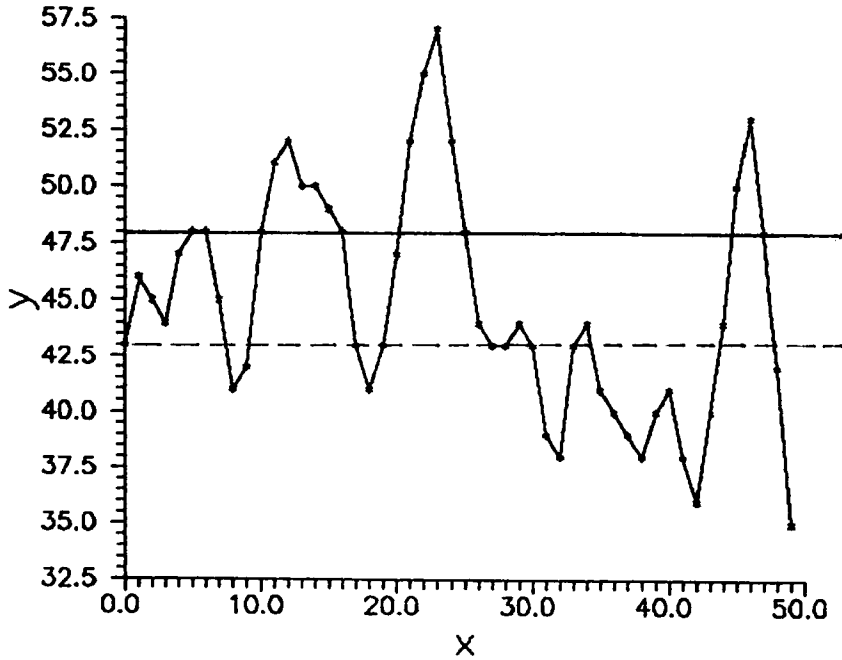


Figure 4a Crosssection of the binary ADS 3353 (4 microns from scan border).

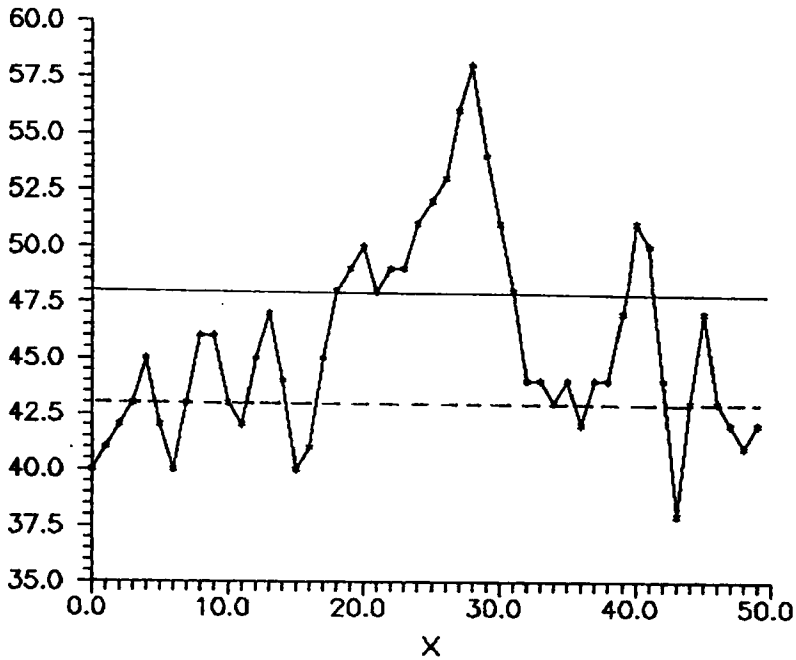


Figure 4b Crosssection of the binary ADS 3353 (40 microns from scan border).

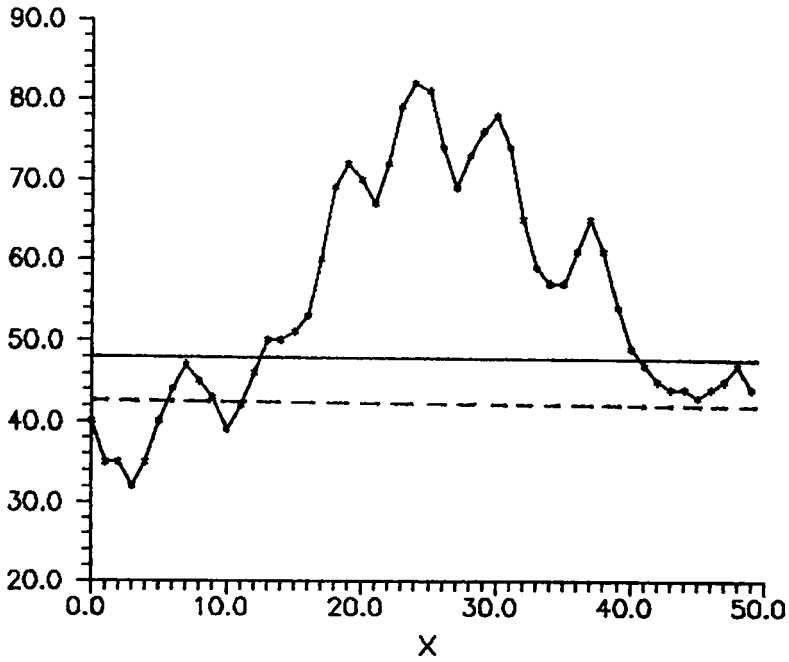


Figure 4c Crosssection of the binary ADS 3353 (76 microns from scan border).

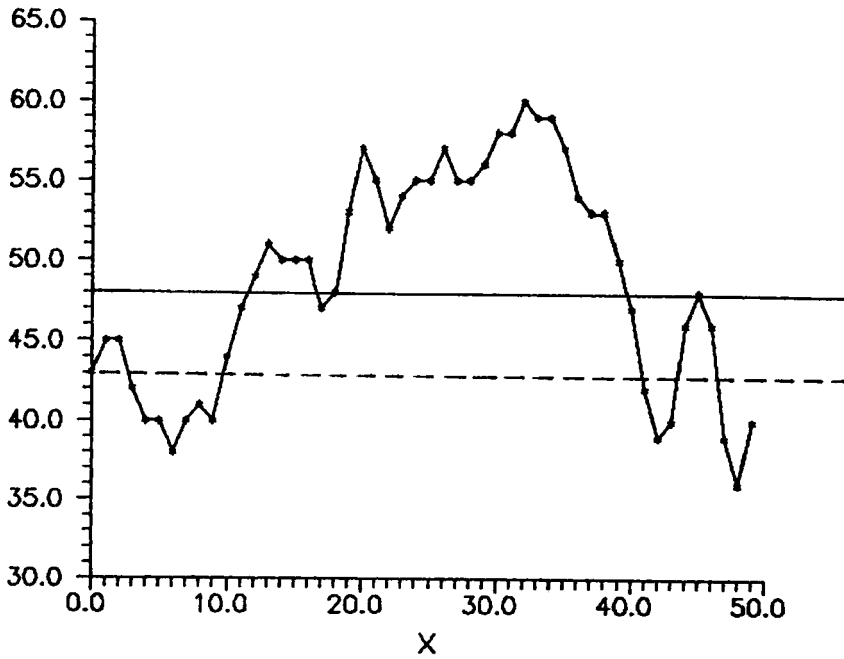


Figure 4d Crosssection of the binary ADS 3353 (120 microns from scan border).

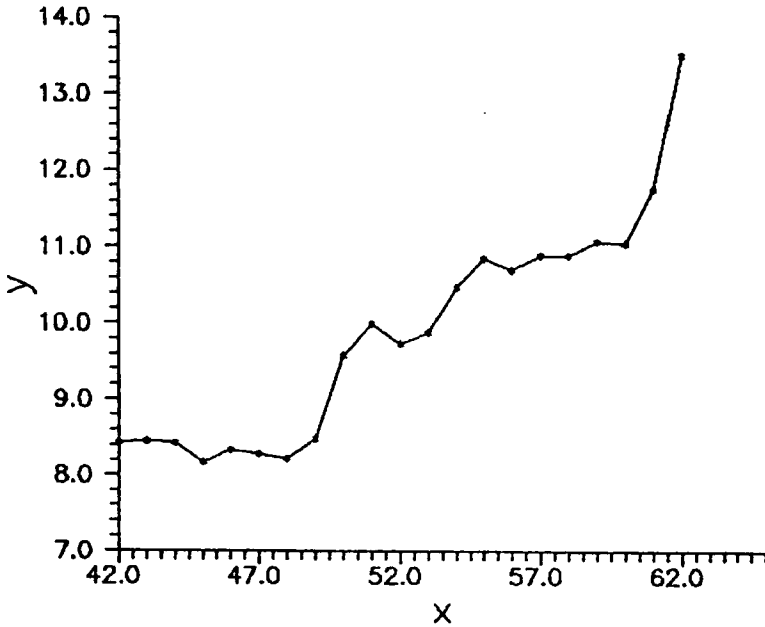


Figure 5 Dispersion of distances between components in the binary ADS 3353 as a function of the threshold.

methods; the minimum equidensity contour corresponds to 43 UODs. Projection of the image on the plate plane is shown in Figure 3, all the values with optical density less or equal 48 UODs are removed in same image. In Figures 4a–d, several side views of the image are shown. On the X -axis, scan coordinates are shown and these should be multiplied by four to express them in microns. The Y -axis is labelled in UODs; dotted line shows the threshold values of 43 UODs; solid line corresponds to the threshold value of 48 UODs.

DISCUSSION

It is interesting to assess the quality of the method suggested as applied to the problems of photographic astrometry we deal with in Pulkovo, i.e. accurate measurements of double star positions, and, consequently, angular distances between components in a close binary in order to determine trigonometric parallaxes and orbits with the AMP method (Kisselev, 1986). At the 26-inch refractor in Pulkovo, we obtain approximately 10 binary images on each plate. Calculations show that there is nearly no difference among thresholds for all the 10 star images of each component on the same plate because of typical equality of their time exposure, and, therefore, approximately equal mean values of density and dispersion. Therefore, considering the plate and not taking into account possible correlations among

fog, background and image we could assume that the threshold defined above could be extended over the plate for all star images with the same mean values of density. We took four binary images of the binary ADS 3353 on the plate ORWO WO-1 and calculated according to the well-known procedure the mean value of distance between the components and its dispersion as a function of the threshold. Although it cannot be considered as an accurate criterium, the minimum of the dispersion of stellar positions should be considered as a preliminary estimate of the threshold quality. Results are shown in Figure 5, where the X -axis is the threshold value and the Y -axis corresponds to dispersion of distances (in microns). One could see that the threshold defined as above provides minimum value of dispersion in angular distance between components. An inverse problem, that is determination of threshold using the minimum dispersion criterium seems to be a valuable method for the separation, but this approach requires an *a priori* knowledge of parameters of the atmospheric stochastic process and some assumption about the image model.

THE CENTER OF MASS POSITION AND THE THRESHOLD

Let us consider one interesting and important application of the threshold defined above. We are going to investigate how does the position of the center of mass of a blurred image depend on the chosen threshold. This problem is interesting to solve because of a wide range of users who apply just for simplicity the center of mass statistics in automatic coordinate measurements both in microdensitometer and CCD fields. Well-known formulae for the mass center position are

$$\begin{aligned} X_0 &= \frac{\sum_i \sum_j x_i D(i, j)}{\sum_i \sum_j D(i, j)}, \\ Y_0 &= \frac{\sum_i \sum_j y_i D(i, j)}{\sum_i \sum_j D(i, j)}, \end{aligned} \quad (7)$$

where x_i and y_i are coordinates and $D(i, j)$ is the optical density. It is easy to determine by direct calculations the position of the center of mass for each component in the binary as a function of the threshold. In Figures 6 and 7, one can see an example of the typical trajectory of the center of mass in relative scan coordinates for the first and second components of the binary ADS 3353, respectively; the number of points on the grid and their connections correspond to the number of sequential thresholds (the starting value of the threshold is 41 UODs, the last one is 61). Each figure, taken alone, shows that for accurate positional observations and, then, measurements the center of mass statistics provides an unsatisfactory estimate of star position even when considering only the problem of the threshold definition; the maximum angular displacement, as one can see from Figures 6 and 7, is equal to 0.3 arcsec if different levels of the threshold are compared. In a parallel way with automatic coordinate measurements, it is possible to consider another one kind of relationships between the center of mass and the threshold. To clarify the physical nature of this question, we have to make some suggestions about the image

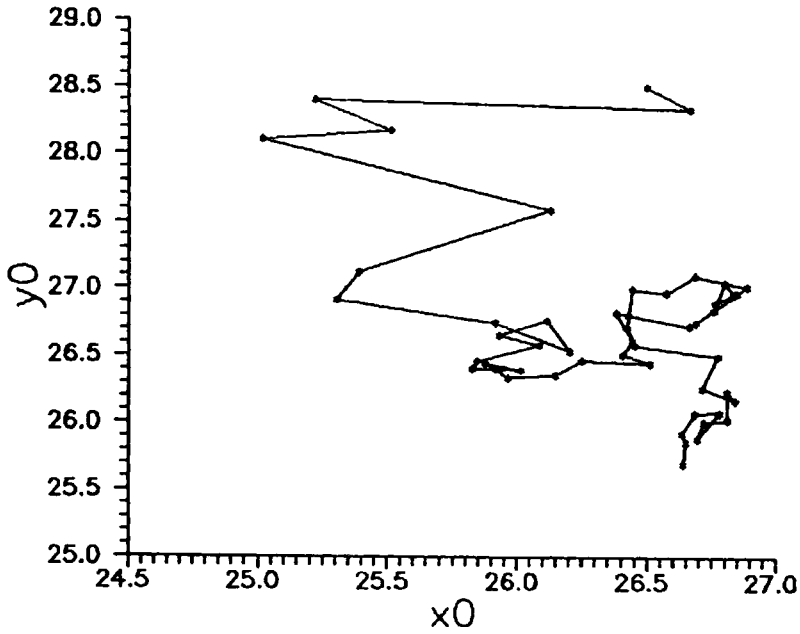


Figure 6 Centre of mass position as a function of the threshold; image of the binary ADS 3353; component A.

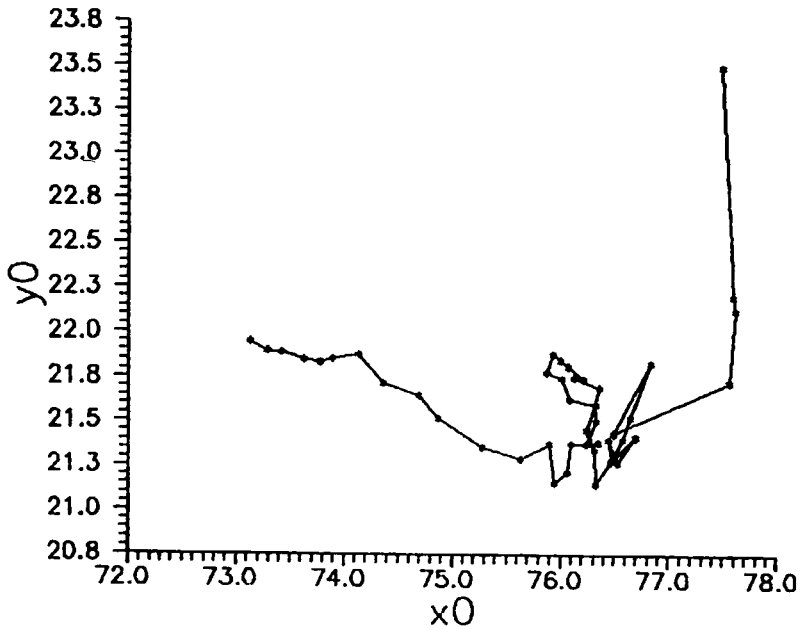


Figure 7 Centre of mass position as a function of the threshold; image of the binary ADS 3353; component B.

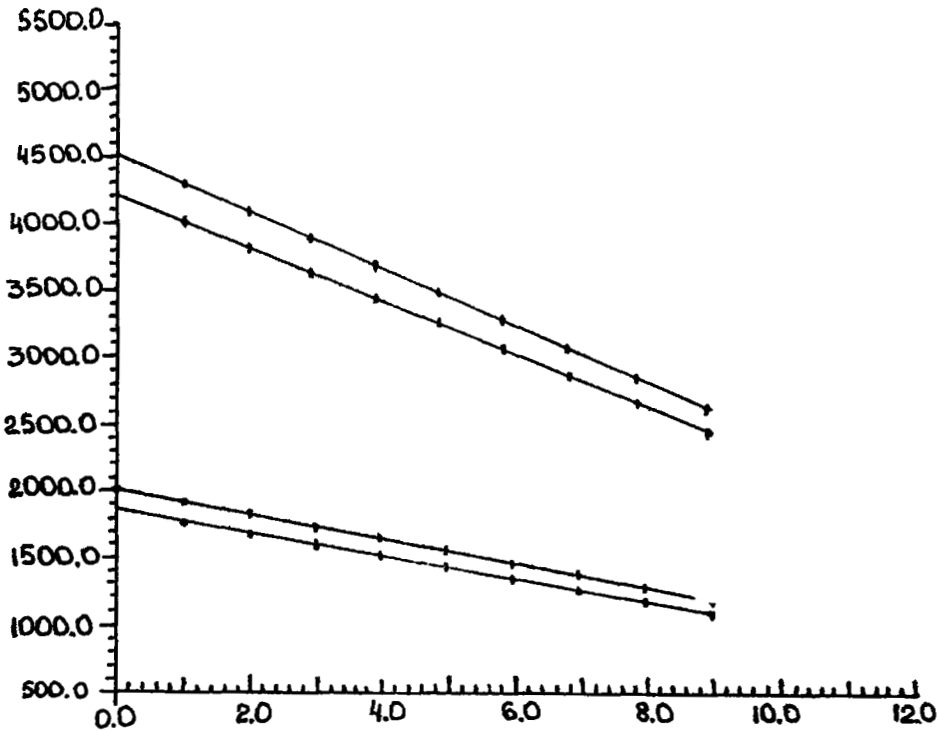


Figure 8 Correlation function between X and Y coordinates of centre of mass: binary ADS 3353, upper and lower pictures – 5 and 3.5 second exposures respectively.

formation process. This process is very complicated and could be hardly described mathematically. But here we consider just a qualitative picture of this phenomenon. We could assume that the image formation occurs layer by layer. This suggestion is consistent with ideas expressed by Kriss (1980). It follows naturally that the CM position of a star image as a function of a chosen threshold level is also some kind of a time function. Therefore, if studying the CM position as a function of the threshold, we study at the same time the process of star image formation. However, if the time dependence in the star image formation process is not evident and at least is non-linear, for stars with equal magnitudes it is naturally to assume that their formation processes are identical in time expansion. This process is stochastic by its nature, so it is possible to raise up the question of existence of correlation between two images of components in a binary, or in our terminology, two realizations of the stochastic process of the image formation. This problem is very close to that of the determination of the so-called isoplanatic square size. In terms of binaries, the isoplanatic square size could be defined as a size which is minimum for appearance of noticeable correlation between processes of formation of images of components. Koltchinsky (1988) estimated this size to be 1–2 arc seconds. We calculated correlation function between x_i and y_i , $i = 1, 2$ coordinates of centers of

mass of components for two images of the binary ADS 3353 (angular distance between components $r = 4$ arcsec). Results are shown in Figure 8. Our first intention was to revise estimates of Koltchinsky, but we are also looking ahead to check up our results other binaries with different angular distances. Possible target tracing error of our refractor can be excluded because of the relatively short exposure-time of star images, 3.5 second per image.

CONCLUSION

(1) A method for stellar image separation from noise is suggested. The applicability of the method is shown on example of automatic coordinate measurements of double stars: the method described in this paper provides minimum dispersion in distance between the components of a binary. In the nearest future it seems to be interesting to work out criteria for the evaluation and comparison of various methods for the threshold definition in astronomical coordinate measurements.

(2) Preliminary results of analysis of the center of mass statistics in automatic coordinate measurements with microdensitometer are presented. It is shown that the center of mass cannot be considered as an accurate characteristic of the star image position.

(3) Strong correlation between images of the components in the double star ADS 3353 (angular separation $r = 4$ arcsec) is discovered, so that, in the image processing, components of the double star with $r = 4$ arcsec should not be processed separately. Investigation of binaries with other angular distances for the purpose to check up if there are any reasons to revise the isoplanetic square size is in progress.

(4) Fog on the plates ORWO WP-1, WO-3, NP-27 can be explained by the fact that these plates are now far from their expiry date. For the Kodak plates, as shown by Walker (1971), it was possible to neglect the fog because of its small value. Nevertheless, it should be recommended to estimate fog for each type of plates for the purpose to determine possible fog trend, especially in the long-term observations on the same type of material. Experiments with fog are of interest in connection with investigation of possible correlations among fog, background and image. When getting 10 images onto the plate one by one with a stable exposure of X minutes, the last image is fixed on a plate with a $9 \times X$ min exposed background. If the plate is high-sensitive, this may cause unreasonable increasing of the mean value of the optical density of the image. In that case "smoothing of exposures" must be applied. Thorough experiments are in progress.

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