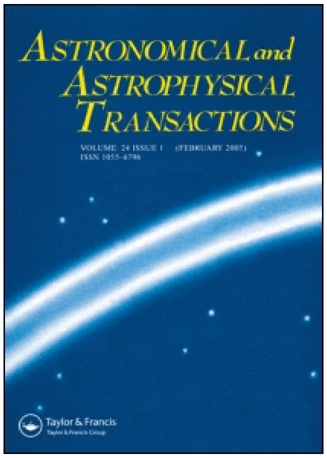

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

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Online Publication Date: 01 March 1996
To cite this Article: Fyodorov, M. V., Klimenko, V. V., Dovgalyuk, V. V. and Snytin, S. Yu. (1996) 'Sunspot cycle timing: A secular forecast', Astronomical & Astrophysical Transactions, 9:3, 225 - 232
To link to this article: DOI: 10.1080/10556799608208225
URL: http://dx.doi.org/10.1080/10556799608208225

Astronomical and Astrophysical Transactions, 1995,
Vol. 9, pp. 225–232
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SUNSPOT CYCLE TIMING: A SECULAR FORECAST

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(Received June 7, 1995)

The spectral analysis of sunspot minima dates time series demonstrates that a high degree of determinism is characteristic of these data. A simple regression model involving a linear trend (11.083 yrs/cycle) and 3 harmonic functions (with periods corresponding to 18.6, 8.8 and 7 Schwabe cycles) allows us to develop an extra-long sunspot cycle timing forecast.

KEY WORDS Sunspot, minima dates, forecast

1 INTRODUCTION

The quasi 11-year Schwabe cycle is the most prominent and persistent feature of the time series (TS) of Wolf numbers, which are widely used as an indicator of solar activity. Reliable prediction of this cycle characteristics could be of great importance for many fields of human activities, telecommunication, space flights, and climate forecasting being among them (e.g., Eddy, 1977; Klimenko *et al.*, 1994; Schove, 1983; Vampola, 1989).

We restrict our consideration to long-term forecasting, i. e. for periods longer than one Schwabe cycle (for prediction technique within a cycle see Withbroe, 1989). All previous attempts of such forecasting were unsuccessful, and we have to conclude that at present there are no reliable long-range methods of predicting solar activity (Withbroe, 1989; Meeus, 1991). One should not be surprised at this fact because the process under consideration is extremely complicated and involves a number of quasi-periodic components with periods ranging probably from 2 to 2300 years (Schove, 1983). The limited available accurate data on Wolf numbers do not make it possible to develop a reliable detailed forecast of this characteristic.

A more realistic task is forecasting only main sunspot cycle features, that is, $R_{\max}$, $R_{\min}$ (maximum and minimum smoothed Wolf numbers in the cycle), $t_{\max}$, and $t_{\min}$ (time of maximum and minimum). In this way the problem is divided into

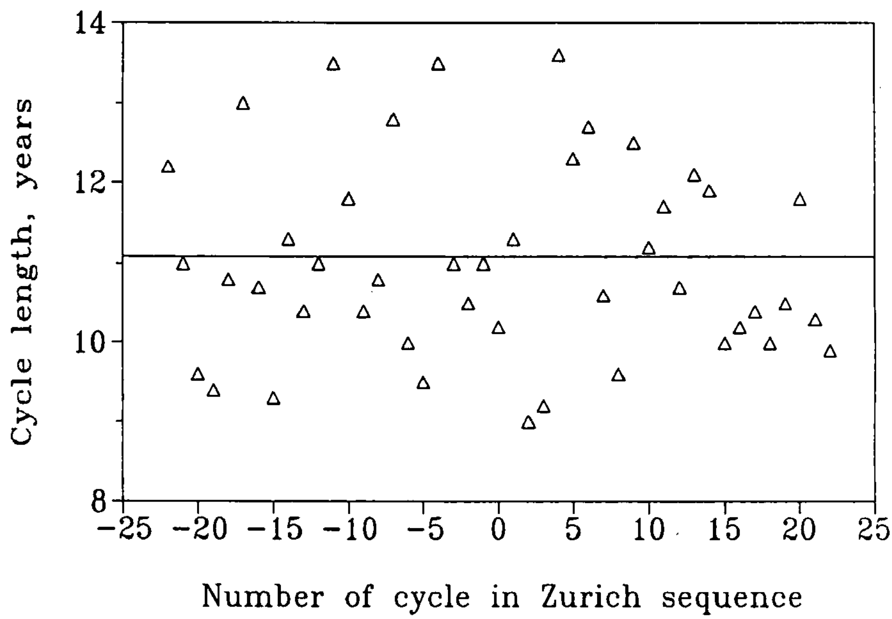

Figure 1 Sunspot cycle length since 1500 AD.

simpler parts, which can be solved by different methods, most suitable in each case. Our analysis of the behaviour of the above mentioned sunspot cycle characteristics convinced us that the level of determinism inherent to each of them is sufficient for forecasting on a quite long-term scale. Thus, cycle length or dates of sunspots minima, which are the subject of the present paper, show only slight fluctuations around the corresponding linear regression lines (Figure 1), and these fluctuations seem to contain stable periodic components (see discussion below).

For the period 500 BC–1980 AD we used data on $t_{\min}$ recommended by Schove (1955, 1983). For the period prior to 1500 AD, monthly $t_{\min}$ data are not available. In this case we supposed that the minimum occurred in the middle of the year. We filled six gaps in Schove minima dates prior to 300 BC assuming equal length for corresponding adjacent cycles. Data for 500 BC–1500 AD and for the Maunder minimum (1645–1700) are probably not very reliable, and for this reason they were used only for the evaluation of main periodicities in the TS. Dates of the initial minimum of the latest, 22nd cycle – 1986.8†, and of its maximum – 1989.6† were calculated by applying the Waldmeier (1961) smoothing procedure to the most recent monthly Wolf number data (N. S. Sidorenkov, personal communication). We also used an estimate for the minimum of the next, 23rd cycle – 1996.6, calculated according to the Schove rule based on the behaviour of sunspot cycles since 1610: "the minimum following a strong maximum often occurs about seven years afterwards" (Schove, 1983).

†Here as usual (e.g., Schove, 1983) 1986.8 means October 1986, 1989.6–August 1989 etc.

Table 1. Errors* in long-term predictions of sunspot minima dates

Authors/Cycle No.	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23***
Donjon (1920)	0.3	0.1	0.4	0.3	0.6	2.0	1.5	0
Schove (1955)					-1.8	−2.0	−2.7	−3.5
Bell, Wolbach (1965)					0.3	1.6	1.1	−0.3
Jose (1965)						−1.0	−3.7	−5.5
King–Hele (1966)						1.8	1.5	
Cohen & Lintz (1974)						−1.0	−1.2	

Note. *Error = actual time – predicted time, years.
**The last column was calculated using the 23th cycle minimum date estimate – 1996.6 (see text).

2 ACCURACY REQUIRED

Now let us decide what accuracy of the prediction is necessary. Since 1500 the average interval between a minimum date and the subsequent maximum date was 4.5 years. A forecast having an error comparable to this figure is obviously useless. Therefore we regard a prediction as useful if its error does not exceed a half of this interval – 2.3 years. Let us consider from this point of view some results of sunspot timing prediction (Table 1).

The predictions by Schove (1955) and Jose (1965) give too large errors – 2.7 and 3.7 years for cycle 22 and probably more than 3 and 5 years correspondigly for the next minimum date. We have also compared Schove's and Jose's predictions of $t_{\max}$ with actual data for the 22th cycle arriving to the errors of 5 and 6 years correspondigly.

A prediction by Cohen and Lintz (1974) is based on Fourier decomposition of the TS of Wolf numbers. It fails to predict $R_{\max}$ (300% error for the 21th cycle maximum), so their set of harmonics does not represent the real process beyond the base interval of 1793–1971 used by these authors.

Other correlations are based on the examination of the time series of cycle lengths or of $t_{\min}$ values and take into account main periodicities in these data. All of them seem to exhibit satisfactory results when compared with actual data. Therefore one can conclude that such methods give good ground for sunspot timing prediction. Nevertheless we can recommend none of these correlations for such extra-long forecast as a secular one.

3 PERIODIC COMPONENTS

To examine the periodic components of the TS of $t_{\min}$, we estimated their spectrum after removing the linear trend from the data (Figure 2). The spectrum estimates were obtained by the maximum entropy method (MEM) (e.g., Andersen, 1974) with

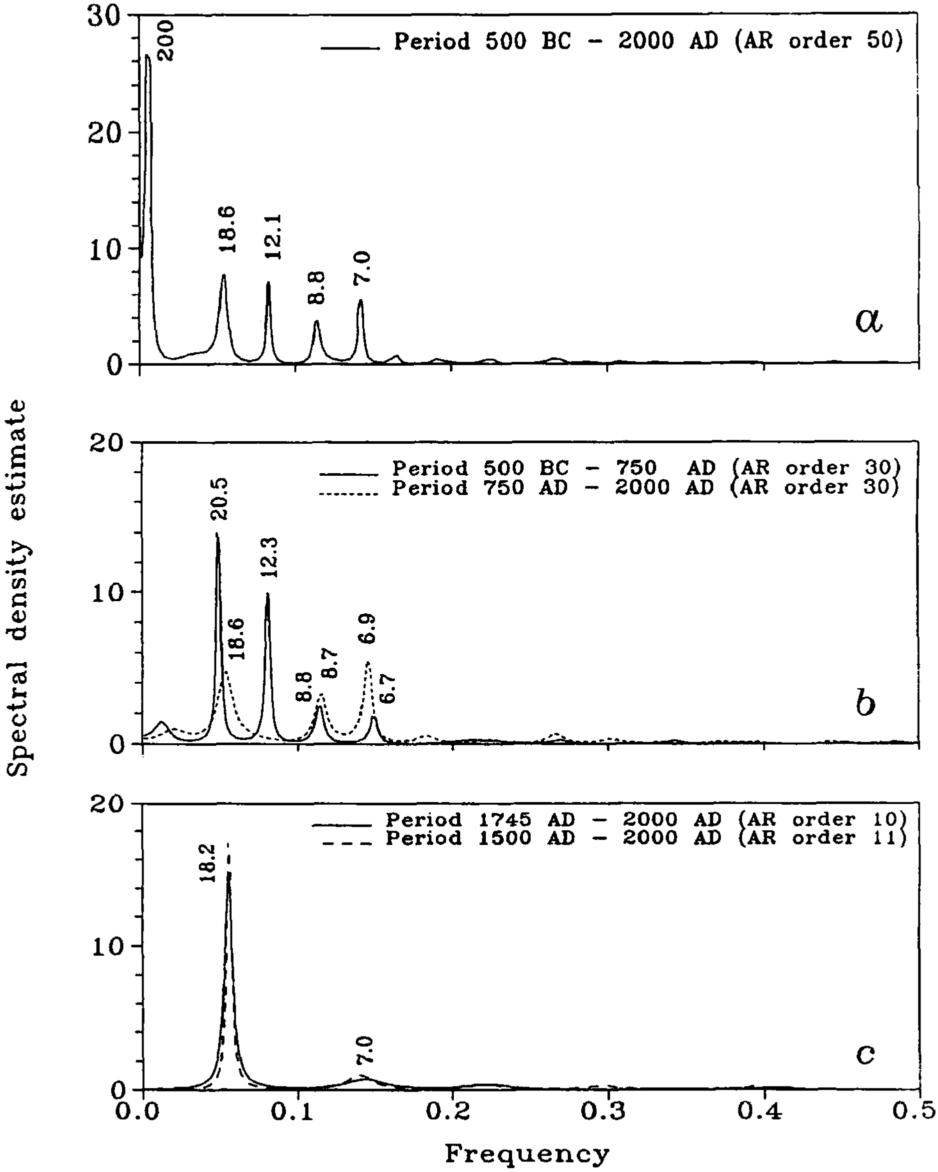

Figure 2 MEM power spectrum of the TS of sunspot minima dates after the linear trend extraction. Periods are indicated in numbers of Schwabe cycles, e.g. 7.0 corresponds to 77.6 years.

the autoregression (AR) order corresponding approximately to 1/3 of the TS length, as was recommended by Papitashvily and Rotanova (1979). As can be seen from Figure 2a, five periodic components were important in the past for the process under consideration. Their periods are 200, 18.6, 12.1, 8.8, 7.0 cycles. Are these oscillations accidental or do they reflect some real stable periodic processes in the Sun, thus opening a way for the development of a long-term forecast? Unfortunately the present level of solar physics is insufficient to answer this question. But the length of the TS allows us to check the stability of all periodic components except only the 200 cycle periodicity, by dividing the available data into two different parts and comparing spectrum estimates for each part. Such comparison for two equal portions of the whole 2500 year TS shows that at least 3 of the above-mentioned spectral peaks (with periods 18.6, 8.8, and 7 cycles) are most likely not accidental (Figure 2b). This conclusion is also consistent with the spectrum estimates for the most reliable data portions after 1500 and 1745 AD (Figure 2c). Let us remind that 1500 and 1745 are two turning points in the history of sunspot investigation, the former being the data from which accurate sunspot minima dates are known, and the latter, from which accurate monthly sunspot numbers are available.

The main peaks of these spectra are close to the 18.6 cycle (206 years) periodicity in the whole TS. The smaller peaks repeat exactly the near-7-cycle (77.6 years) periodicity which is observed in the whole TS. These results probably imply that early pre-instrumental data on sunspot minima are consistent with the more recent detailed observations starting from the mid-18th century and may be useful for determining long-term trends.

Let us turn to the problem of forecasting proper. First of all, we believe that the 200 cycle (that is, about 2200 years) periodicity has no significant influence on a secular time scale.

The most important for the secular forecast is the 18.6 cycle (about 206 years) periodicity, because oscillations with shorter periods are not very strong (see Figure 2). Surprisingly, none of the above-mentioned correlations takes into account this periodicity.

King–Hele (1966) used a seven-cycle (i.e. about 80 years) periodicity. These oscillations are important for the TS of cycle lengths, where the lowest frequencies are suppressed, since such data actually are a TS of $t_{\min}$ or $t_{\max}$ filtered with a first difference filter. Our aim is to give a reliable prediction for extrema dates, not for cycle length. In this case, the 200 year periodicity is more important.

Donjon (1920) and Bell and Wolbach (1965) used the following correlation:

$$t_{\min} = \text{const} + 10.87 \cdot n + 1.7 \cdot \sin[2 \cdot \pi/136 \cdot (t_{\min} - 1608)], \qquad (1)$$

where n is cycle number. The 136 year periodicity was deduced from the lunar eclipse data, which Donjon used because he had no confidence in sunspot data prior to 1823, and the TS of the rest of Wolf numbers was too short in 1920. Now we have a much longer reliable TS for $t_{\min}$ at our disposal, but their spectra are however free from any significant peaks near the $1/136\ \text{yr}^{-1}$ frequency. Moreover, the mean cycle length of 10.87 yrs seems to be too short on a long-term scale. For these reasons, Eq. (1) shows unsatisfactory results when comparing with data prior to the 19th century: the mean absolute deviations of the calculated dates from actual ones are 4.1, 5.8, and 6.9 years for the 18th, 17th, and 16th centuries, correspondingly.

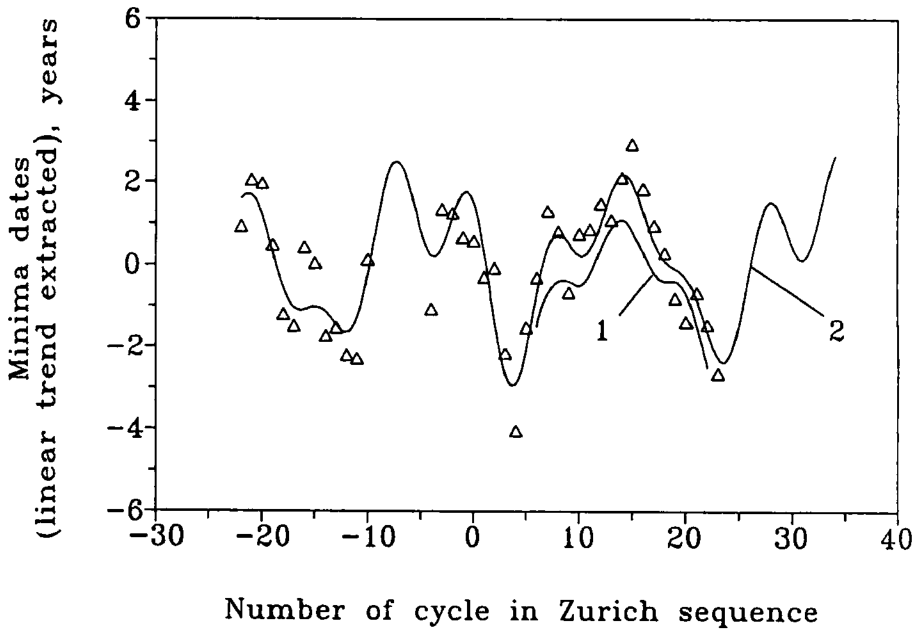

Figure 3 Comparison of the regression curve defined by Eq. (2) and the coefficients derived on the basis of different reference periods with the actual sunspot minima dates. Thin curves correspond to the forecast made on the restricted basis of 1500–1800 AD data (1) and on the basis of 1500–2000 AD data (2).

Therefore an improved correlation is necessary for long-term forecasting. To derive such a correlation, we used the following regression model:

$$t_{\min}(n) = \beta_1 + \beta_2 \cdot n + \sum_{i=1}^{3} \left[\beta_{2i+1} \cdot \sin\left(\frac{2\pi}{T_i} \cdot n\right) + \beta_{2i+2} \cdot \cos\left(\frac{2\pi}{T_i} \cdot n\right) \right] + z_n, \quad (2)$$

where β_i are regression coefficients, n is the cycle number in Zurich consequence, $T_1 = 18.6$, $T_2 = 8.8$, $T_3 = 7$; z_n are random errors with zero mathematical expectation. The coefficients of Eq. 2 were calculated on the basis of $t_{\min}$ data for 1500–2000 AD. We would like to emphasize that the shift of the initial point of the base $t_{\min}$ time series from 1500 to 1745 has virtually no influence on our main conclusions. Using the standard least-squares technique (e.g., Anderson, 1971), we arrived at the folowing set of the coefficients β_i:

$$1744.6,\ 11.083,\ -1.622,\ -0.237,\ -0.109,\ 0.663,\ -0.149,\ 0.913. \quad (3)$$

The corresponding forecast is presented in Figure 3 and Table 2.

It is difficult to draw a reliable estimate of the forecast variance as we do not know to what extent our model correspond to the reality. So we used the following simplified approach. First, we calculated the forecast variance estimation using the

Table 2. Forecast of sunspot minima dates

Cycle number	*Initial minimum date*	*Cycle number*	*Initial minimum date*
23	1997.0	28	2056.3
24	2008.2	29	2066.9
25	2020.1	30	2077.3
26	2032.5	31	2088.1
27	2044.8	32	2099.8

standard technique, assuming that the model is correct and z is a "white noise". The corresponding 95% confidence intervals are ± 2 years.

Second, we applied the proposed method of forecasting to the reduced TS of minima dates according to the following scheme:

Case No.	*Base interval, AD*	*Period of forecasting, AD*
1	1500–1800	1800–1900
2	1500–1900	1900–2000
3	1610–1900	1900–2000
4	1500–1800	1800–2000

Maximum errors for secular forecasts were 1.8 years for the 19th century (Case 1) and 1.5 years for the 20th century (Case 2). These results confirm the above-mentioned value of the confidence interval. The results of our forecast for the two centuries are shown in Figure 3. Maximum departure of a predicted date from the actual one in this case is higher – 2.1 years, but is still lower than the above mentioned limit of 2.3 years. This implies that our forecast never confuses maxima with minima. Therefore the regression equation (2) with the coefficients (3) may be useful even on the supersecular time scale.

Acknowledgements

The authors wish to thank the US Department of State (Grant No. 1753–300224) for providing financial support for this study. V. V. Klimenko would also like to acknowledge support from the Alexander von Humboldt Foundation (Germany).

Note added in proof. According to data for the second half of 1995, the average semiannual Wolf number is around 15. Since running semiannual Wolf number in the minimum following a strong maximum lies within 2–10, the next minimum may occur between June 1996 and February 1997. Therefore we were able to predict the beginning of solar cycle 23 (Table 2) within ± 0.5 year, this does not break the successful forecast sequence for the past two centuries.

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