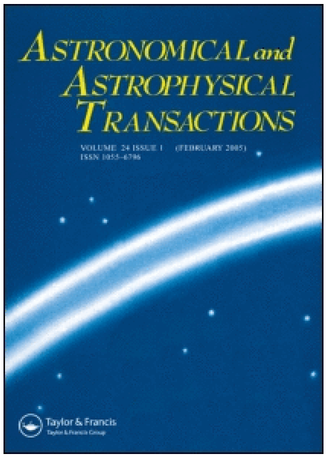

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DETERMINATION OF 'DIFFUSION COEFFICIENTS' AND STELLAR WIND VELOCITIES FOR X-RAYS BINARIES

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The distribution of neutron stars (NS's) is determined by stationary solution of the Fokker–Planck equation. In this work, using the observed period changes for four systems: Vela X-1, GX 301-2, Her X-1, and Cen X-3, we determined D, the 'diffusion coefficient' parameter, from the Fokker–Planck equation. Using strong dependence of D on the velocity, we determined the stellar wind velocity for Vela X-1 and GX 301-2, systems accreting from a stellar wind. For different assumptions for a turbulent velocity we obtained $V = (660 - 1440)$ km s^{-1}. It is in good agreement with the stellar wind velocity determined by other methods.

We also determined the specific characteristic time scale for the 'diffusion processes' in X-ray pulsars. It is of the order of 200 sec for wind-fed pulsars and 1000–10000 sec for the disk accreting systems.

KEY WORDS Accretion: neutron stars–stars: stellar wind–stars

1 INTRODUCTION

The most precisely determined characteristic for accreting neutron stars (NS's) is their period. Thus using observations of the period we can determine different properties of the observed object.

Period changes show fluctuations. These fluctuations were discussed in de Kool & Anzer (1993). The authors determined noise level for these systems and characteristic time scales. Using these results we can estimate diffusion coefficients in the Fokker–Planck equation (see below) and the stellar wind velocities (only for the wind-accreting systems).

During accretion the angular momentum of plasma is transferred to the NS. But the process of the momentum transfer is not stationary. The transferred angular momentum fluctuates and therefore the period changes of the NS will also show fluctuations.

Processes with fluctuations are well known (see for example Haken, 1978). Some applications of stochastic processes in astrophysics, especially in accreting systems,

Table 1.

	p, sec	p_{orb}, sec	L, erg/sec	$\mu/10^{30}$ Gs cm^3	$t_{su\ observ}$, yrs	$t_{su\ min}$, yrs
Vela X-1	283	7.7×10^5	1.5×10^{36}	3	3000	3000
GX 301-2	696	3.6×10^6	10^{37}	120	> 100	100
Her X-1	1.24	1.5×10^5	10^{37}	0.6	3×10^5	8000
Cen X-3	4.84	1.8×10^5	5×10^{37}	5.7	3400	600

were discussed in Lipunov (1987), Hoshino & Takeshima (1993) and Lipunov (1992). In Hoshino & Takeshima (1993) the authors, using simple models of MHD turbulence, try to explain aperiodic changes in X-ray luminosity of X-ray pulsars. Luminosity fluctuations are explained as the result of density fluctuations due to turbulence in the plasma flow. The authors used a 2D model for accretion disk and 3d model for the wind accreting systems. Detailed exploration of this question is very difficult in both theoretical and observational ways (there is no good theory of MHD turbulence, and the resolution of modern equipment of satellites is not high enough for power spectra of X-ray pulsars; see Hoshina & Takeshima, 1993). But detailed exploration of the density fluctuations will help to understand period fluctuations. It will be very interesting to compare X-ray luminosity fluctuations of the period of the NS.

There are different methods of describing these processes. In this work we use differential equations for the distribution function. For the frequency changes we can write the Langeven equation, which describes the process with fluctuations:

$$\frac{d\omega}{dt} = F(\omega) + \Phi(t). \tag{1}$$

Here, $F(\omega)$ is the constant angular momentum. For $F(\omega)$ in the most general form we can write (Lipunov, 1982):

$$F(\omega) \cdot I = \begin{cases} \dot{M}\eta_k \Omega R_G^2 - k_t \frac{\mu^2}{R_c^3}, & \textit{wind accretion} \\ \dot{M}\sqrt{GMR_d} - k_t \frac{\mu^2}{R_c^3}, & \textit{accretion disk,} \end{cases}$$

where R_d is the disk radius, k_t and η_k are dimensionless parameters ($k_t \approx 1, \eta_k \approx 1$), I is the moment of inertia of the NS, M is the mass of the NS, Ω is the orbital frequency of the system, R_G is the radius of gravitational capture ($R_G = \frac{2GM}{v^2+v_{orb}^2}$, we include here the orbital velocity), and R_c is the corotational radius, $R_c = \left(\frac{GM}{\omega^2}\right)^{1/3}$.

We assume that the 'force' is conservative and in this case we can write $F(\omega)$ in the form: $F(\omega) = -\nabla_\omega V$, where V is a scalar potential. Φ is a fluctuating moment; i.e. $\langle\Phi(t)\rangle = 0$ (Lipunov, 1987).

The distribution of frequency, ω, is described by the distribution function $f(\omega)$. This function satisfies the Fokker–Planck equation (Haken, 1978):

$$\frac{df}{dt} = \frac{dfF(\omega)}{d\omega} + D\frac{d^2f}{d\omega^2}, \tag{2}$$

Table 2.

	A	L_{max}
Vela X-1	–9.1	$10^{36.8}$
GX 301-2	–8.5	10^{37}

where D is the 'diffusion coefficient', which is determined by the correlation of the stochastic force Φ:

$$\langle\Phi(t)\Phi(t')\rangle = 2D\delta(t-t') \tag{3}$$

The stationary solution of Eq. (2) is the following:

$$f(\omega) = N\exp(-V(\omega)/D) \tag{4}$$

where N is determined from the normalization condition:

$$\int_{-\infty}^{\infty} f(\omega)\,d\omega = 1 \tag{5}$$

Using expression for $V(\omega)$ from Lipunov (1992), we can write D in the form:

$$D = \frac{k_t\mu^2}{3GMI}\cdot\frac{\omega^3}{\gamma} \tag{6}$$

where k_t is a constant parameter ($k_t \approx 1$), I is the moment of inertia of the NS and γ is evaluated as $\gamma \approx \frac{t_{su}}{\Delta t}$, here t_{su} is time of spin-up and Δt is the characteristic time for period changes (see for details Lipunov, 1987 or Lipunov, 1992).

Using Eq. (6), in Section 2.1 we shall determine the value of the 'diffusion coefficient', D. With these D, in section 2.2 we shall make the estimates of the stellar wind velocities for Vela X-1 and GX 301-2.

2 RESULTS

2.1 Determination of the 'Diffusion Coefficient'

At first we shall estimate D using Eq. (6). In this equation all variables, except Δt, are known (in principle). Their values taken from Lipunov (1992) are shown in Table 1.

Nagase (1992) gives results of GINGA observations of the cyclotron lines in X-rays pulsars. For Vela X-1 and Her X-1 values of magnetic field, B, coincide with the values μ that we use when radii of NS's are 10 km and 6 km correspondingly. The value for B obtained from observations of Vela X-1, $B = 2.3 \times 10^{12}$ Gs, coincides quite well with assumption that there is no stable accretion disk in this pulsar.

Table 3.

	Δt, *sec*	γ	D *sec*$^{-3}$	v_{sw}, *km/s*, ($v_t = 0.1a_s$)	v_{sw}, *km/s*, ($v_t = a_s$)	t_{char}, *sec*
Vela X-1	1.5×10^4	6.3×10^6	8.7×10^{-24}	848	1442	200
GX 301-2	1.1×10^3	2.9×10^6	2×10^{-21}	656	1120	245
Her X-1	4×10^3	6.0×10^7	4×10^{-19}	-	-	960
Cen X-3	2×10^4	9.5×10^5	4.1×10^{-17}	-	-	8500

Characteristic time Δt for the wind-accreting systems can be determined from the equation:

$$\Delta t \approx 1.7 \times 10^4 \alpha^{-2} 10^{2(A+8.5)} L_{37}^{-\frac{12}{7}} \mu_{30}^{-\frac{4}{7}} \text{ sec}, \tag{7}$$

where α is the fraction of the specific angular momentum of the Kepler orbit at the magnitospheric radius, A is the noise level (see Table 2) (de Kool & Anzer, 1993).

In Eq. (6) we must use minimum values for t_{su}. These times were calculated using equations for pure spin-up from Lipunov (1992), $t_{su\ min}$ are shown in Table 1.

We took $k_t = 1/3, M = 1.5M_\odot, i = 10^{45}$ g cm^2. From Eq. (7) we can get Δt for Vela X-1 and GX 301-2. For Her X-1 and Cen X-3, Δt is determined from the graph in de Kool & Anzer (1993) (see Table 3). So we can write the equation for D in the form:

$$D = 5.55 \times 10^{-19} \mu_{30}^2 \omega^3 \gamma_6^{-1} I_{45}^{-1} \left(\frac{M}{1.5M_\odot}\right)^{-1} \text{ s}^{-3}. \tag{8}$$

Values of D for four systems are presented in Table 3.

From the theory of diffusion we can write:

$$D = \omega_{\text{char}} \dot{\omega}, \tag{9}$$

where ω_{char} is the characteristic length in the frequency space.

For characteristic time in this space we can write:

$$t_{\text{char}} = \omega_{\text{char}} / \dot{\omega} = \frac{Dp^4}{4\pi^2 \dot{p}^2}. \tag{10}$$

We can give a physical interpretation for t_{char} for wind-fed pulsars as a characteristic time of the momentum transfer:

$$\frac{R_G}{v_{\text{sw}}} = 400 \left(\frac{M}{1.5M_\odot}\right) \left(\frac{v_{\text{sw}}}{10^8\,\text{cm/s}}\right)^{-3} \text{ sec}. \tag{11}$$

For $v_{\text{sw}} = 1178$ km/s we obtain $\frac{R_G}{v_{sw}} = 245$ sec and for $v_{\text{sw}} = 1281$ km/s we obtain $\frac{R_G}{v_{sw}} = 200$ sec. These velocities are close to v_{sw} obtained using estimates of the 'diffusion coefficient'.

These characteristic time scales are also given in Table 3.

2.2 Determination of the Stellar wind Velocity

The fluctuating moment Φ (see Eq. (1)) can be estimated as:

$$\left(\frac{\dot{M} v_t R_t}{I}\right).$$

So for D we can write an equation which differs from Eq. (6) (Lipunov and Popov, 1995):

$$D = \frac{1}{2}\left(\frac{\dot{M} v_t R_t}{I}\right)^2 \frac{R_G}{v_{\mathrm{sw}}}, \tag{12}$$

where v_t is the turbulent velocity, R_t is the characteristic scale of the turbulence. This scale is of thé order of the gravitational capture radius, R_G:

$$R_t \approx R_G = \frac{2GM}{v_{\mathrm{sw}}^2} \approx 4 \times 10^1 0 v_8^{-2}\left(\frac{M}{1.5 M_\odot}\right) \text{ cm}, \tag{13}$$

where $v_8 = \frac{v}{10^8 \text{ cm/s}}$.

The turbulent velocity is less or equal to the sound speed, a_s (in opposite case a great bulk of energy will dissipate in the form of shock waves), that's why we can write:

$$v_t = \eta \cdot a_s, \quad \eta \leq 1 \tag{14}$$

where $a_s = ((5RT)/(3\mu))^{1/2} = 1.18 \times 10^6 T_4^{1/2} \mu^{-1/2}$ cm/s and $T_4 = T/(10\,000\,K)$.

From Eq. (12) we can get:

$$D = 4.38 \times 10^{-23} \dot{M}_{16}^2 \eta^2 T_4 \mu^{-1} v_8^{-7} I_{45}^{-2}\left(\frac{M}{1.5 M_\odot}\right) \text{ s}^{-3}, \tag{15}$$

here $\dot{M} = \frac{\dot{M}}{10^{16} \text{ g/s}}$, $I_{45} = \frac{I}{10^{45} \text{ gcm}^2}$.

As we see there is a strong dependence of D on v. So we can evaluate v (in this case it is the stellar wind velocity, v_{sw}):

$$\begin{aligned} v_{\mathrm{sw}} &= 1700 \cdot D_{-24}^{-1/7} \dot{M}_{16}^{2/7} \eta^{2/7} T_4^{1/7} \mu^{-1/7} \\ &\times I_{45}^{-2/7}\left(\frac{M}{1.5 M_\odot}\right)^{2/7} \text{ km/s}. \end{aligned} \tag{16}$$

Values of v_{sw} are presented in Table 3.

CONCLUSIONS

The obtained values of v_{sw} coincide well with the characteristic value of this quantity for supergiants of early spectral types: $v_{sw} \approx (600\text{–}3000)$ km/s (de Jager, 1980). We also use the well known equation for the terminal velocity of a stellar wind which is well confirmed by observations:

$$v_{\infty} \approx 3 \cdot v_{\rm esc}, \tag{17}$$

where $v_{\rm esc} = \left(\frac{2GM}{R_*^2}\right)^{1/2}$ is the escaping velocity on the surface of the star, $r = R_*$. We use this dependence and equation (de Jager, 1980):

$$v(r) = V_{\infty}\left(1 - \frac{R_*}{r}\right)^{\alpha} \tag{18}$$

where $\alpha \approx (0.35\text{–}0.5)$.

For the optical component of GX 301-2 we have: $M = 35\,M_{\odot}, R = 43\,R_{\odot}$, and $e = 0.47$ (Watson *et al.*, 1982). We took $r = r_{\rm min} \approx 2R_*$, because in our calculations we used maximum luminosity, i.e. luminosity in periastror. For these values we have: $v_{\rm esc} \approx 560$ km/s and for v_{∞} we have $v_{\infty} = 1680$ km/s (for ρLeo, B1 Iab, in de Jager (1980) we find $v_{sw} = 1580$ km/s).

From Eq. (18) we get: $v_{sw}(r = 2R_*, \alpha = 0.5) \approx 1180$ km/s. It coincides well with our maximum value $\approx$1150 km/s (see Table 3). For Vela X-1 there exist estimates (Haberl, 1991): $v_{\infty} = 1700$ km/s, $r = 1.7R_*$ and $\alpha = 0.35$. We get: $v_{sw} \approx 1250$ km/s. It also coincides with our estimates of the maximum stellar wind velocity: $\approx$1450 km/s. With different assumptions for the turbulent velocity (for example, $v_t = \frac{1}{3}a_s$) we can get excellent coincidence of our results with the stellar wind velocities estimated above using other techniques.

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References

Haberl, F. (1991) *A&A*, 252, 272.
Haken, G. (1978) *Sinergetics*, Springer–Verlag.
Hoshino, M. and Takeshima, T. (1993) *Ap. J.*, 411, L79.
de Jager, C. (1980) *The Brightest Stars*, D. Reidel Publishing Company, Dordrecht, Holland.
de Kool, M. and Anzer, U. (1993) *MNRAS*, 262, 726.
Lipunov, V. M. (1982) *Ap&SS*, 82, 343.
Lipunov, V. M. (1987) *Ap&SS*, 132, 1.
Lipunov, V. M. (1992) *Astrophysics of the Neutron Stars*, Springer-Verlag.
Lipunov, V. M. and Popov, S. B. (1995) (in preparation).
Nagase, F. (1992) In *Ginga Memorial Symposium (ISAS Symposium on Astrophysics 1992)*, p. 1.
Watson, M. G., Warwick, R. S. and Corbet, R. H. D. (1982) *MNRAS*, 199, 915.